“Staged” Strong Coupling Strategy for Fluid-Structure Interactions: Supersonic Flow Test Problem

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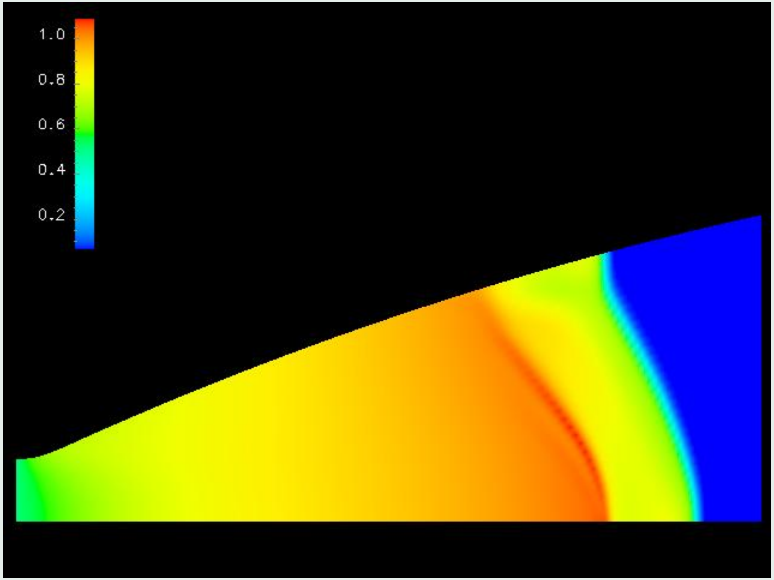
1. Introduction

- **One of the three tests proposed by "CIMEC CFD Team" to the E.S.A. (ESTEC)/OE to evaluate two CFD codes for its Strong Fluid-Structure Interaction project at supersonic/hypersonic regime:**

  - ▷ **FINE/HEXA-3D code by Numeca (C. Hirsch) and OOFELIE by O.E., Belgium.**
  - ▷ **PETSc-FEM code from CIMEC-INTEC, Argentina. (`http://www.cimec.org.ar/petscfem`).**

2. Supersonic gas exhaust in the nozzle of the Vulcain rocket engine

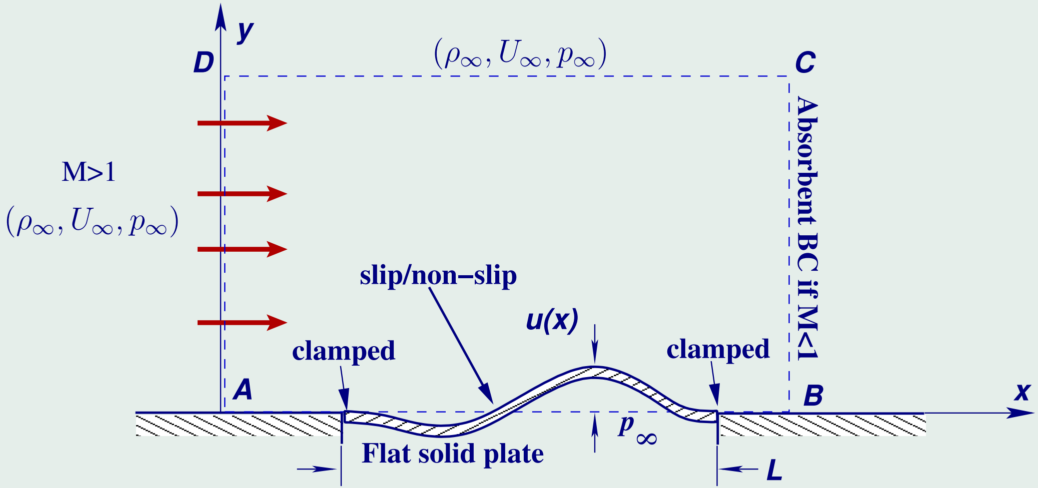


3. Introduction (contd...)

These tests involved different numerical techniques, namely:

- **Weak/Strong algorithm to couple Fluid and Structure (separate) codes (Presented in ENIEF 2006 by Mario Storti).**
- **Parallel code for 'Inviscid/Viscous Hypersonic Flows' in 'Beowulf' clusters of PC's.**
- **The Galerkin/SUPG formulation with added 'isotropic/anisotropic shock capturing' operators.**
- **ALE and 'Adaptive mesh refinement' techniques.**
- **The use of 'non-reflecting' boundary conditions on fictitious (and subsonic) walls.**
- **Finally, a 'Domain Decomposition' preconditioner for an efficient solution of the linear system.**

4. Test: Flutter of a flat solid plate aligned with the supersonic gas flow



- Viscous/inviscid supersonic flow (EULER/Compressible N-S eqs).
- Thin plate theory for structure: $m\ddot{u}(x,t) + D\frac{\partial^4 u(x,t)}{\partial x^4} = -(p - p_\infty) + f(x,t)$.
- Coupled through pressure and traction (viscous case only) on interface boundary.
- Undisturbed flow $(\rho, v, p)_\infty$ is a solution of the problem for zero initial condition (solid problem).

5. Theoretical Approximation: Houbolt's model [1958]

- **Fluid Problem**

$$p - p_\infty = C_x \frac{\partial u}{\partial x} + C_t \frac{\partial u}{\partial t},$$

$$C_x = \frac{\rho_\infty {U_\infty}^2}{\sqrt{{M_\infty}^2 - 1}}, \quad C_t = \frac{\rho_\infty U_\infty ({M_\infty}^2 - 2)}{({M_\infty}^2 - 1)^{3/2}}.$$

- Then, for the **Plate Problem** the deflection becomes

$$m\ddot{u} + D\frac{\partial^4 u}{\partial x^4} = -C_x \frac{\partial u}{\partial x} - C_t \frac{\partial u}{\partial t}.$$

- Using a global basis for displacements

$$u(x) = \sum_{k=1}^{N} a_k \psi_k(x),$$

$$\psi_k(x) = \frac{4\, x(L - x)}{L^2} \sin(k\pi x/L).$$

- **The basis functions satisfy the essential boundary conditions for plate equation $u = (\partial u/\partial x) = 0$ at $x = 0, L$.**

- **Replacing $u(x)$ in the Houbolt's approximation and using Galerkin method**

$$M\ddot{a} + Ka + H_x a + H_t \dot{a} = 0,$$

where

$$M_{jk} = \int_0^L m\, \psi_j(x)\, \psi_k(x)\, dx,$$

$$K_{jk} = \int_0^L D\, \psi_j''(x)\, \psi_k''(x)\, dx,$$

$$H_{x,jk} = \int_0^L C_x\, \psi_j(x)\, \psi_k'(x)\, dx,$$

$$H_{t,jk} = \int_0^L C_t\, \psi_j(x)\, \psi_k(x)\, dx.$$

- If the ansatz $\mathsf{a}(t) = \hat{a}e^{\lambda t}$ is proposed as a solution for the Plate Problem, the following eigenvalue problem is stated

$$\left(\lambda^2 M + \lambda H_t + K + H_x\right)\hat{a} = 0.$$

- Using time and mass non-dimensional parameters

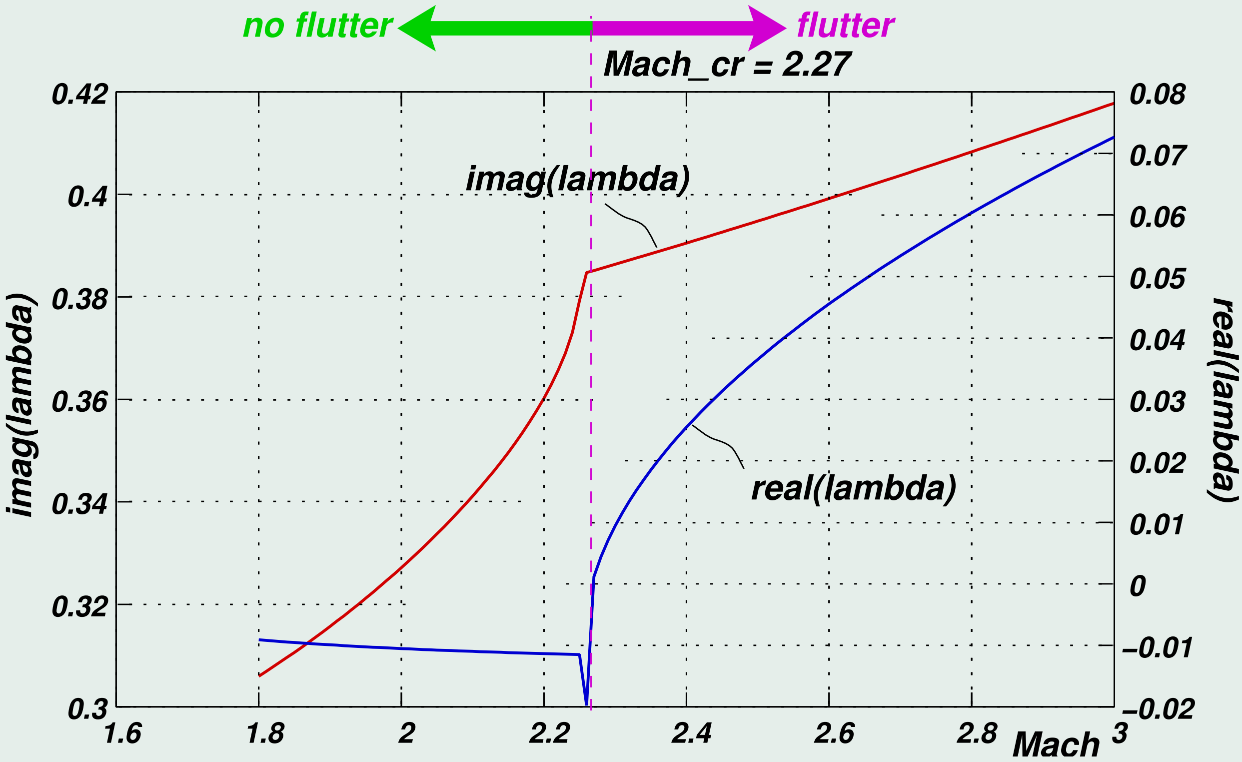
$$N_T = \left(\frac{T_{fl}}{T_{str}}\right)^2 = \frac{L/U_\infty}{\sqrt{mL^4/D}} = \frac{D}{mL^2 U_\infty{}^2} \text{ and}$$

$$N_M = \frac{\rho_\infty L^3}{mL^2} = \frac{\rho_\infty L}{m},$$

the space of parameters is full covered.

- Results for the Houbolt's model
  - ▷ $N = 20$, $N_x = 5000$, and a sweep in M_∞ while keeping constant ρ_∞, m, L and D,

    i.e., $N_M = \text{cte}$ and $N_T \propto M_\infty{}^{-2}$.



Flutter mode appears for $M_\infty \geq 2.27$
or $N_T = 4.3438 \times 10^{-5}$ and $N_M = 0.055$.

6. Staged FSI-FEM Results for flutter

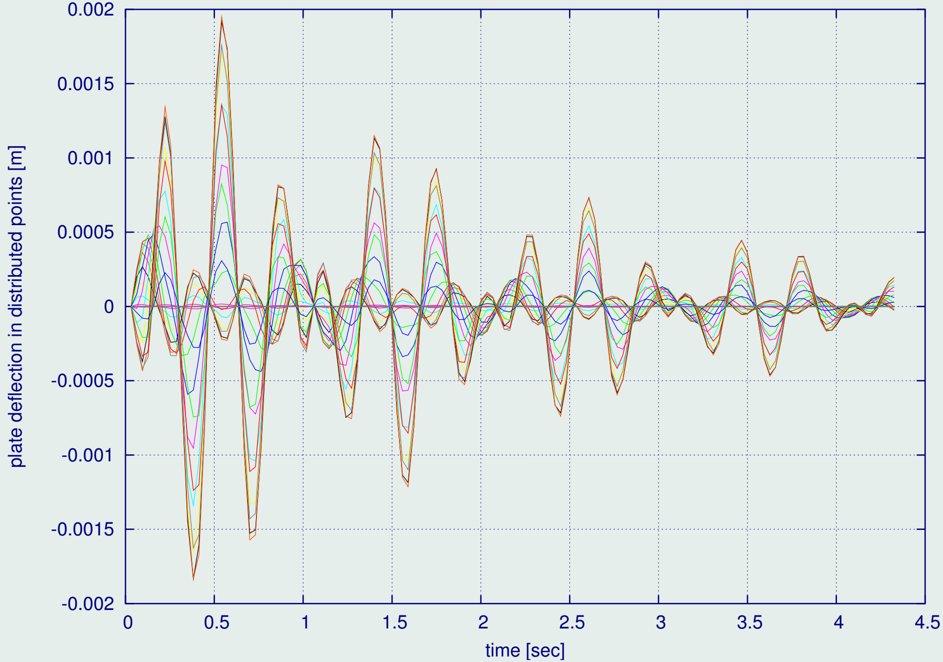


Figure 1: Structure response at M=1.8

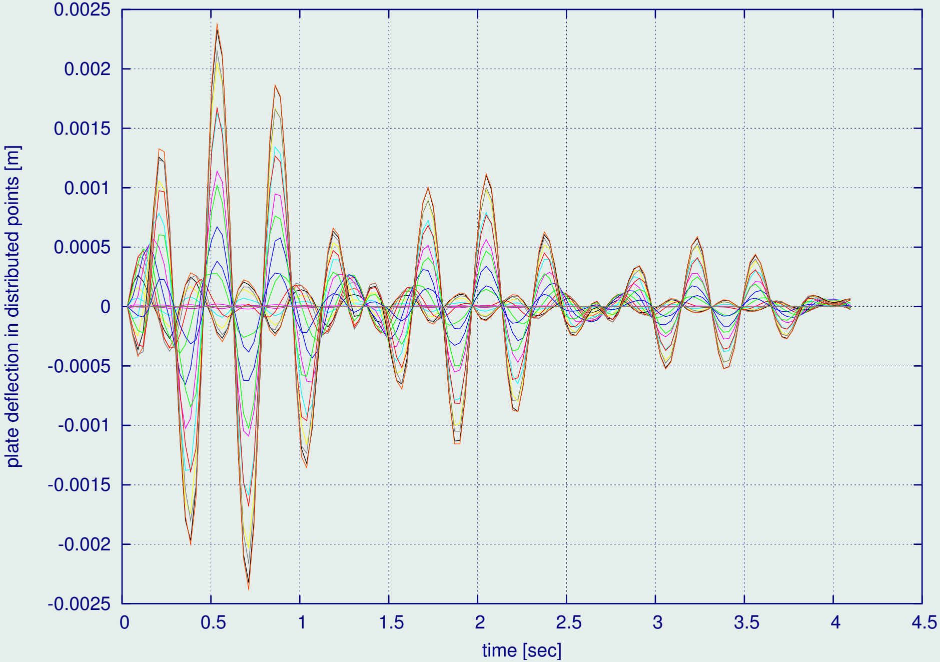


Figure 2: Structure response at M=2.0

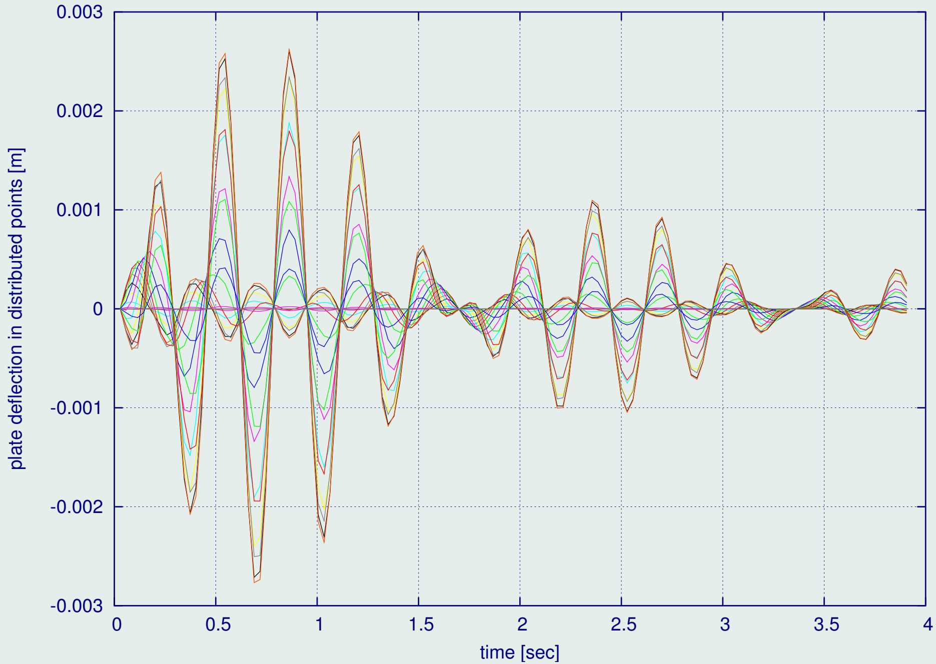


Figure 3: Structure response at M=2.1

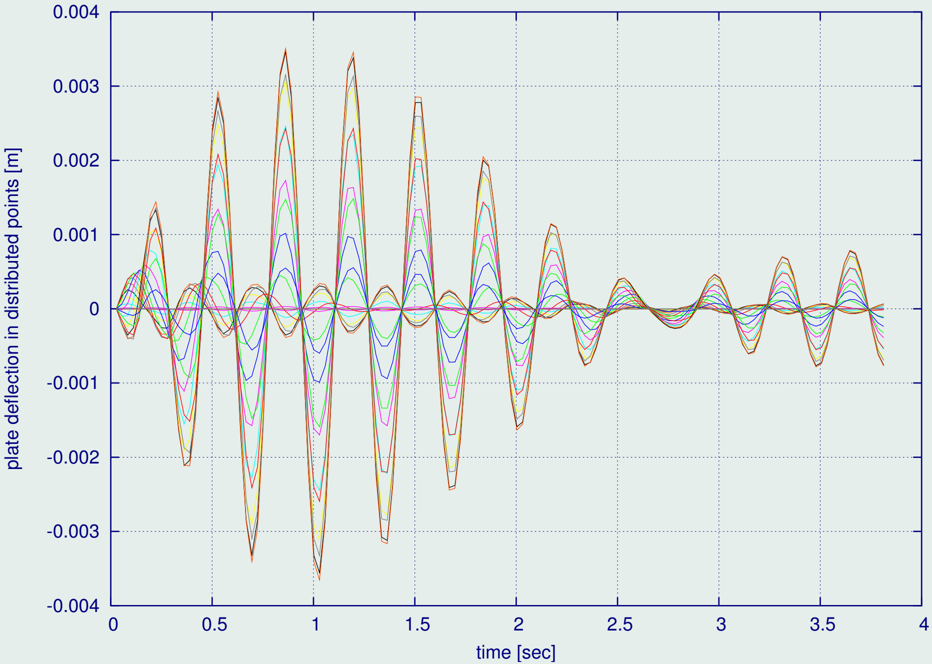


Figure 4: Structure response at M=2.2

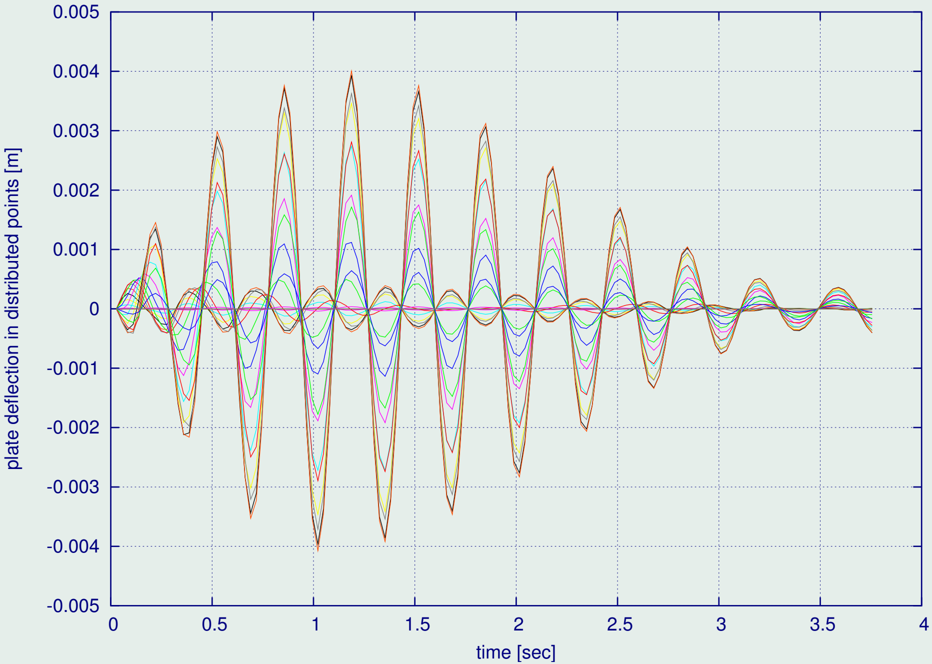


Figure 5: Structure response at M=2.225

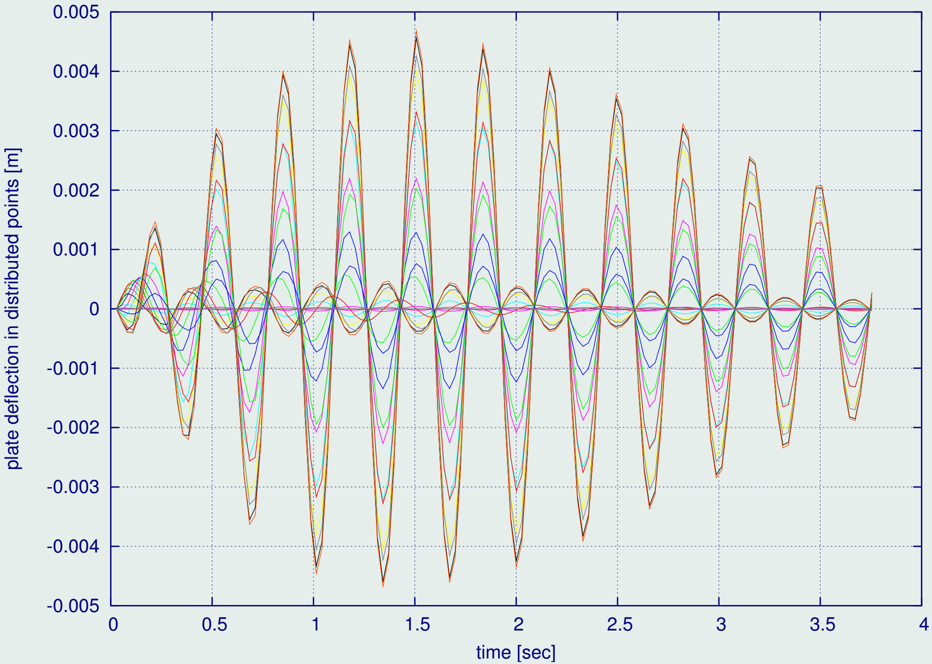


Figure 6: Structure response at M=2.25

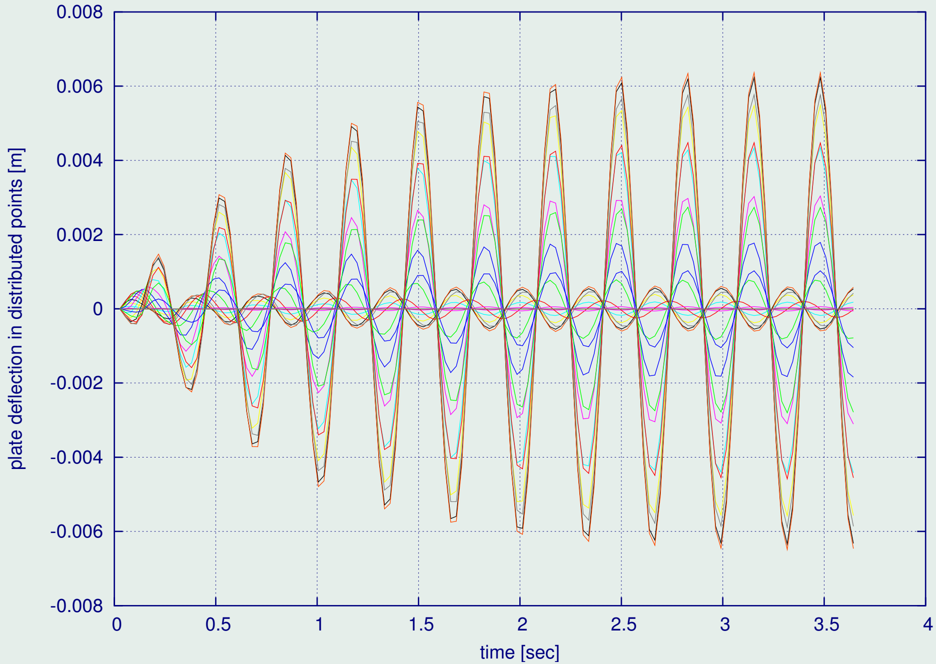


Figure 7: Structure response at M=2.275

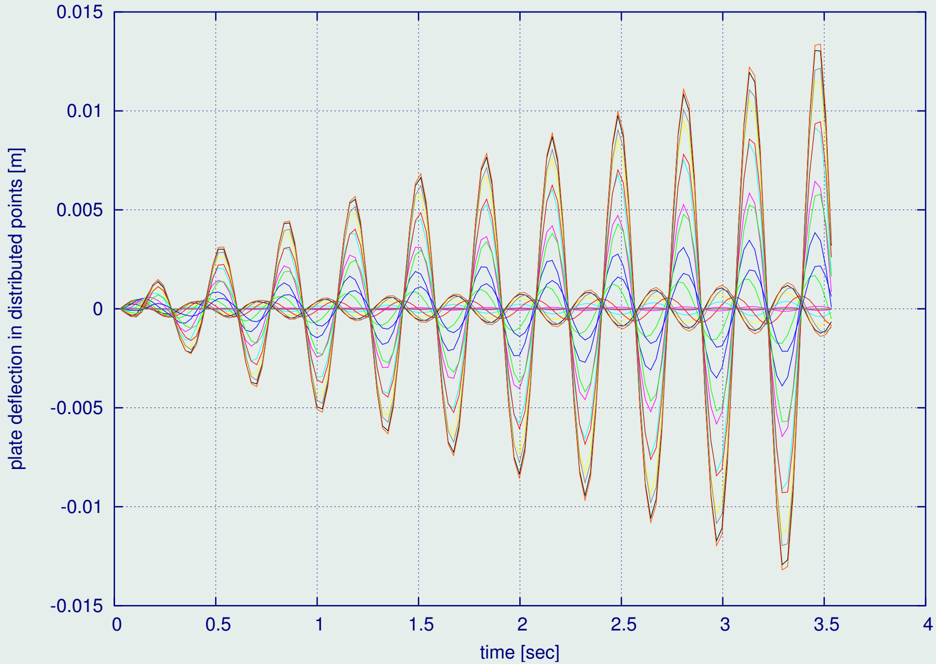


Figure 8: Structure response at M=2.3

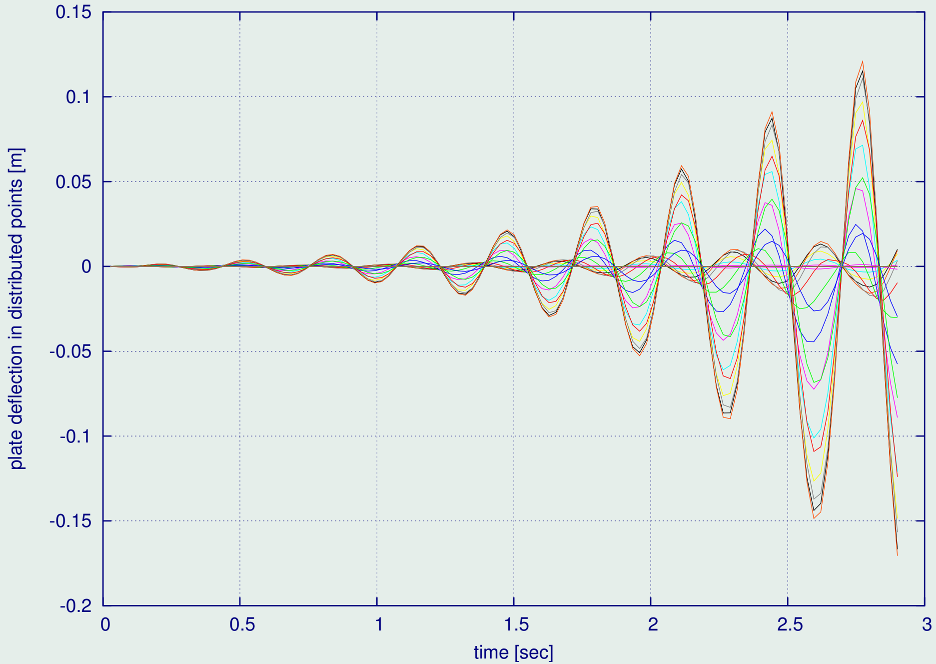


Figure 9: Structure response at M=2.5

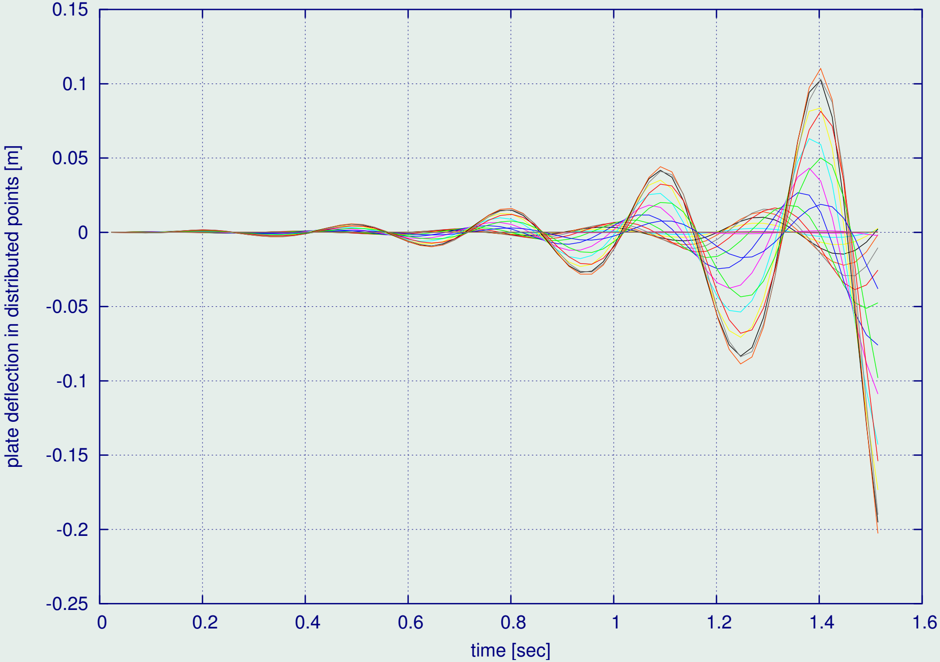


Figure 10: Structure response at M=3.0

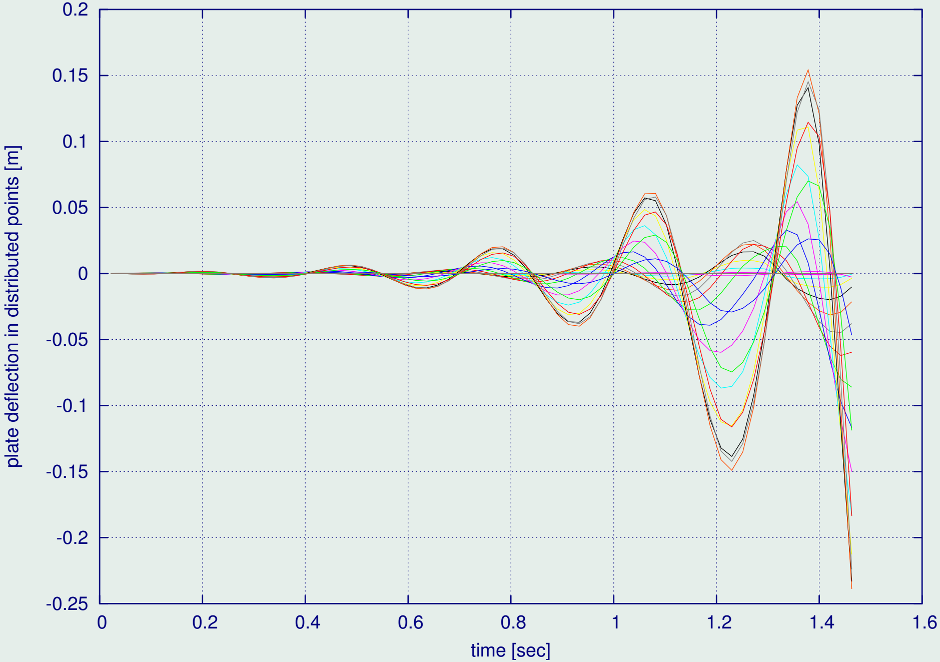


Figure 11: Structure response at M=3.2

7. Flat plate in Flutter

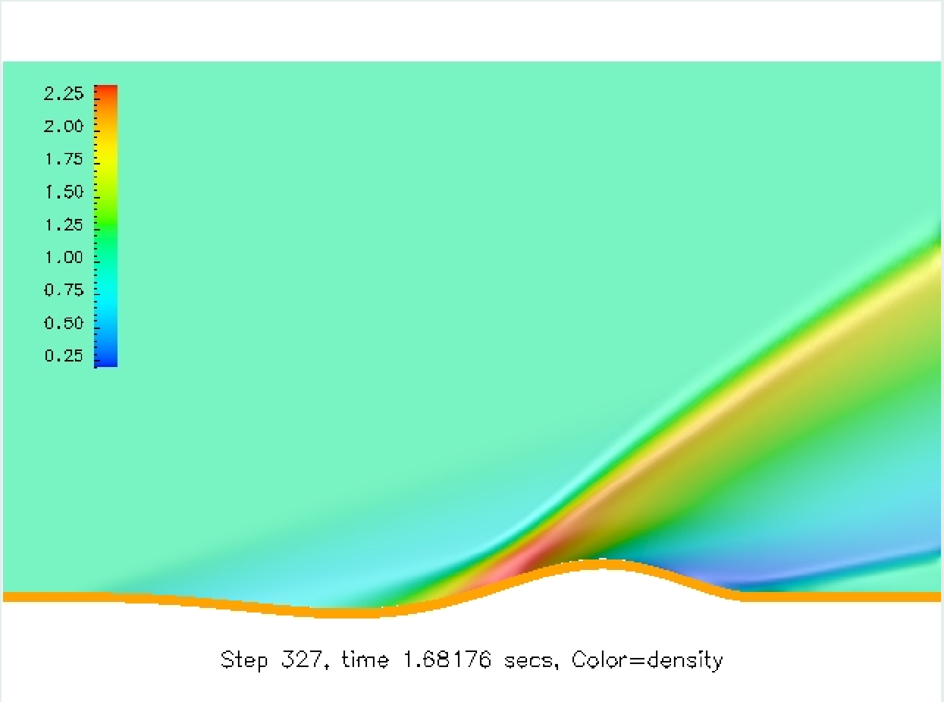


Figure 12: Fluid and Solid fields at $\mathbf{M_\infty = 3.2}$

- Being

$$\frac{N_M}{N_T M_\infty{}^2} = \frac{\rho_\infty L^3 c_\infty}{D},$$

sweeps in N_T, N_M and M_∞ estimated the flutter region as

$$\frac{N_M}{N_T M_\infty{}^2} < 200 \quad \text{no flutter for any Mach number,}$$

$$\frac{N_M}{N_T M_\infty{}^2} > 300 \quad \text{flutter for the lowest Mach considered } (M_\infty \geq 1.8).$$

- If $\left(\partial u/\partial t\right)$ is neglected (i.e., characteristic times of struct. are much lower than those of the fluid, $N_T \ll 1$),

$$\det(\bar{\lambda}^2 \bar{M} + K + H_x) = 0$$

$$\bar{\lambda} = \sqrt{m}\lambda,$$

$$\bar{M} = \frac{1}{\sqrt{m}} M,$$

the coefficients in $\bar{\mathbf{M}}$, $\mathbf{K}$, $\mathbf{H_x}$ do not depend on $\mathbf{m}$, neither do the eigenvalues of equation, then the λ eigenvalues are of the form

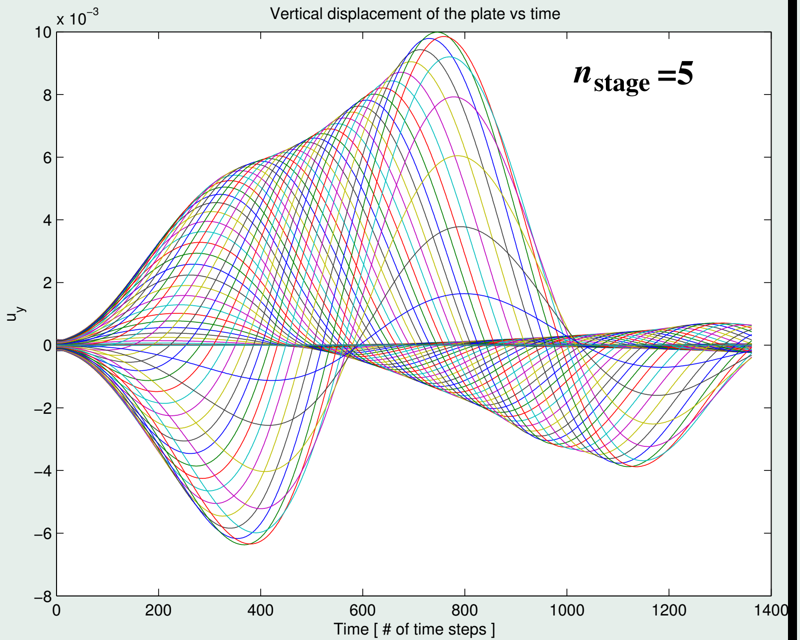
$$\lambda_j = \frac{\bar{\lambda}_j}{\sqrt{m}},$$

with $\bar{\lambda}$ not depending on $\mathbf{m}$. This means that the sign of the real part of the λ is independent of $\mathbf{m}$.

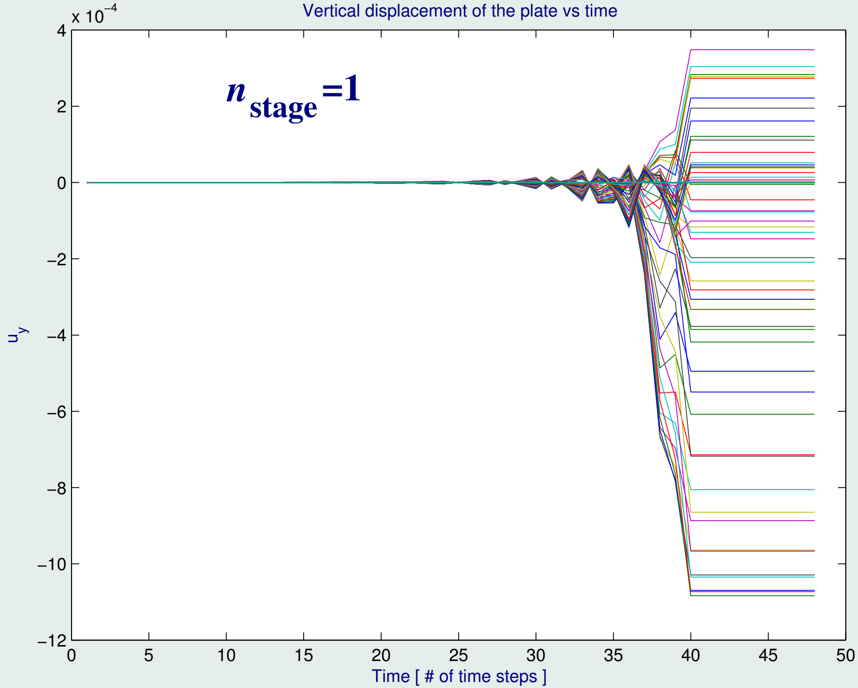
8. Stability of the staged algorithm (flutter region, contd...)

$$U_\infty = M_\infty = 2$$
$$t = 0.06$$
$$\nu = 0.33$$
$$m = 0.002$$
$$E = 39.6$$
$$D = 8.010^{-4}$$
$$N_T = \frac{D}{mL^2 {U_\infty}^2} = 0.025$$
$$N_M = \frac{\rho_\infty L}{m} = 1000.0$$
$$\frac{N_M}{N_T {M_\infty}^2} = 10000 > 300$$

(i.e., inside the **flutter region**)



9. Stability of the staged algorithm (flutter region, contd...)



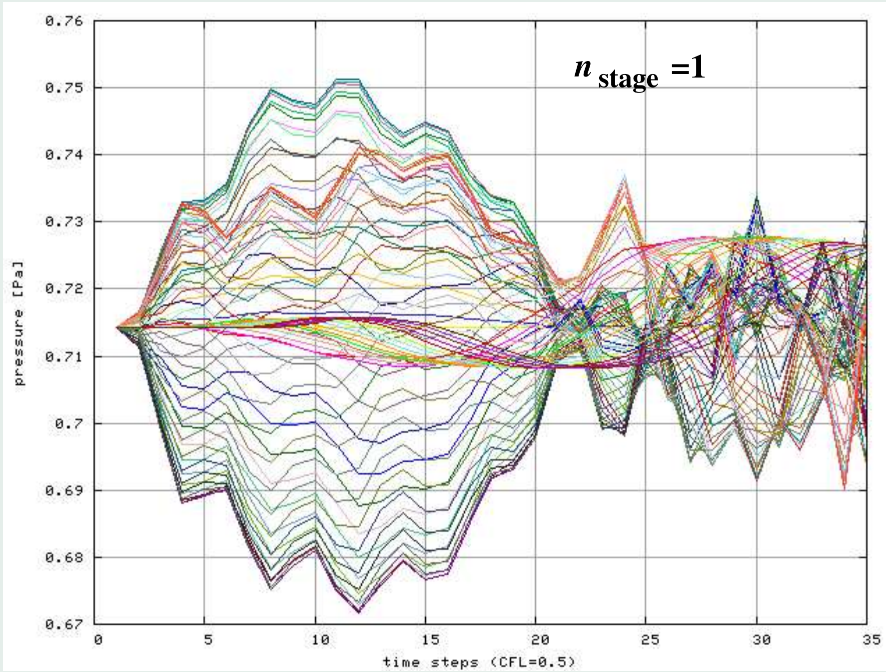
10. Stability of the staged algorithm outside the flutter region, i.e. $N_M/(N_T M_\infty^2) \ll 200$

- $N_M/(N_T M_\infty^2)$ do not depend on plate density m

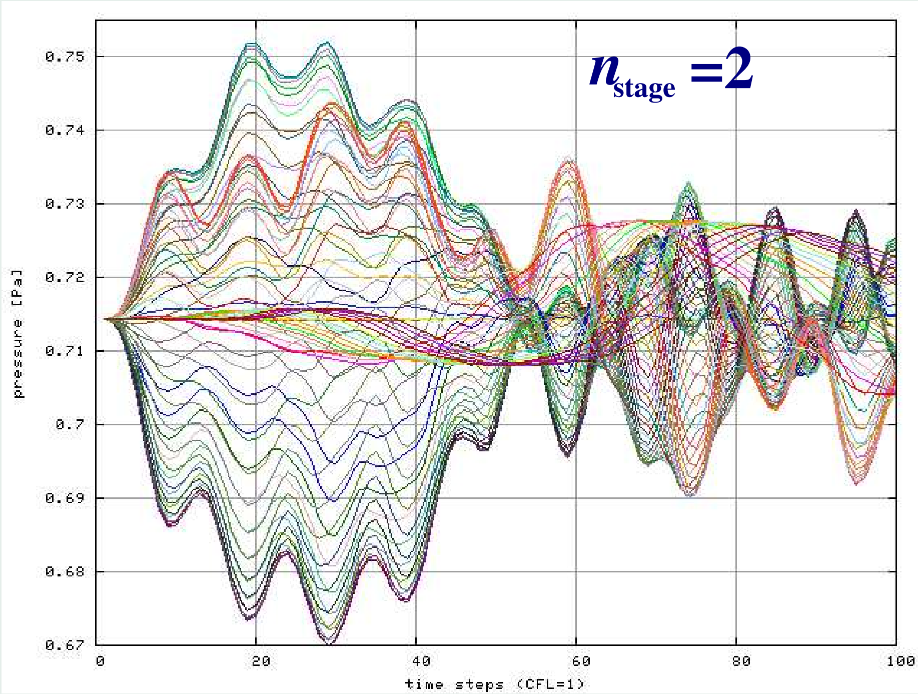
with $m = 0.00135$

$$\frac{N_M}{N_T M_\infty^2} = 12 < 200$$

(i.e., outside the flutter region)



11. Stability of the staged algorithm outside the flutter region (contd...)



12. Stability of the staged algorithm (flutter region)

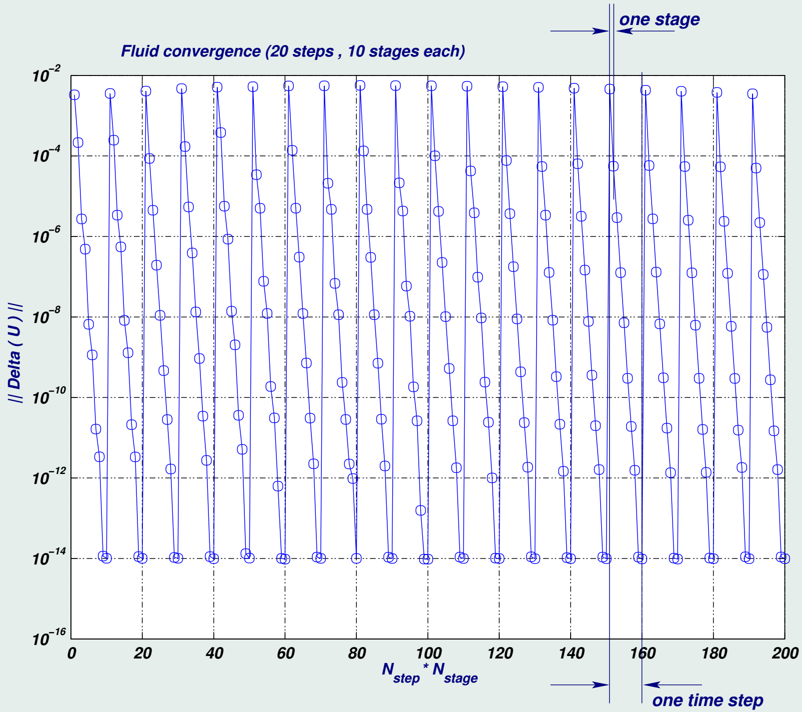


Figure 13: Convergence of fluid state in stage loop

13. Conclusions

- “Staged” strategy provides a smooth blending between weak coupling and strong coupling.

- Moderately coupled problems that can not be treated with the pure weak coupling approach, can be solved with the staged algorithm using few stages per time step.

- The elastic flat plate problem is geometrically simple, but gives physical insight in the flutter phenomena, and was very useful in testing the proposed algorithm in a wide range of non-dimensional parameters.

Acknowledgment

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- We made extensive use of freely distributed software such as GNU/Linux OS, MPI, PETSc, Newmat, Metis,Octave, the CGAL geometrical library, OpenDX and many others.