Answers to Problems

CHAPTER 2

2A1. (a) 5 (b) 28 (c)23

2A2. (a) and (c)

2A5. (a)$T_{11} = T_{22} = T_{33} = 0, T_{12} = T_{21} = 0$

$T_{23} = -T_{32} = 1, T_{31} = -T_{13} = 2$

(b) $c_1 = 3,\ c_2 = 2,\ c_3 = 0$

2B5.

$$[\mathbf{mn}]_{\mathbf{e}_i} = \frac{1}{2}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{\mathbf{e}_i}$$

2B10. (b) $[\mathbf{S}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}_{\mathbf{e}_i}$ (c) $[\mathbf{T}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}_{\mathbf{e}_i}$

2B17. (a) $[\mathbf{Q}]_{\mathbf{e}_i} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\mathbf{e}_i}$ (b) $[\mathbf{a}] = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}_{\mathbf{e}_i'}$

2B19. $T_{11}' = \frac{4}{5};\ T_{12}' = -3\sqrt{5};\ T_{31}' = \frac{2}{5}$

2B20. (a) $[T_{ij}'] = \begin{bmatrix} 0 & -5 & 0 \\ -5 & 1 & 5 \\ 0 & 5 & 1 \end{bmatrix}$

(b) $T_{ii} = T_{ii}' = 2$; $\det[T_{ij}] = \det[T_{ij}'] = -25$

2B28. $[\mathbf{T}^S] = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{bmatrix}$, $[\mathbf{T}^A] = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$, (b) $[\mathbf{t}^A] = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

2B31. Eigenvector of $\mathbf{T}$ is $\mathbf{n}$, Eigenvector of $\mathbf{T}$ is $r_1 s_1$

2B34. (a) $\mathbf{n} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$ (b)$\mathbf{a}\cdot\mathbf{b} = a_i b_i$

2B35. (a) $[\mathbf{R}]_{\mathbf{e}_i} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b)$\mathbf{n} = \pm\frac{1}{\sqrt{3}}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)$

2B.36. (b) $[\mathbf{R}^S]_{\mathbf{e}_i'} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $[\mathbf{R}^A]_{\mathbf{e}_i'} = \begin{bmatrix} 0 & -\sin\theta & 0 \\ \sin\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\lambda_1 = \cos\theta$; $\mathbf{n}_1 = \alpha\mathbf{e}_1' + \beta\mathbf{e}_2'$, $\lambda_2 = \cos\theta$; $\mathbf{n}_2 = -\beta\mathbf{e}_1' + \alpha\mathbf{e}_2'$

$\lambda_3 = 1$; $\mathbf{n}_3 = \mathbf{e}_3'$, $\alpha^2 + \beta^2 = 1$

(d) $R_{ii}' = 2\cos\theta + 1$; (e) $\mathbf{t}^A = \sin\theta\mathbf{e}_3'$, (f) $\theta = \arccos(-1/2) = 120^o$

$\mathbf{e}_3' = \frac{1}{\sqrt{3}}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)$

2B41. (a) $I_1 = 3$; $I_2 = -16$; $I_3 = -48$

$\lambda_1 = 3$; $\mathbf{n}_1 = \mathbf{e}_1$, $\lambda_2 = 4$; $\mathbf{n}_2 = \frac{1}{\sqrt{2}}(\mathbf{e}_2 + \mathbf{e}_3)$, $\lambda_3 = -4$; $\mathbf{n}_3 = \frac{1}{\sqrt{2}}(-\mathbf{e}_2 + \mathbf{e}_3)$

(b) $[\mathbf{T}]_{\mathbf{n}_i} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -4 \end{bmatrix}_{\mathbf{n}_i}$

(c) No, the first invariants are not equal

2C2. (a) $\mathbf{n} = \mathbf{e}_3$ (b) $|\nabla\varphi| = 2$ (c) $\dfrac{d\varphi}{dr} = \sqrt{2}$

2C3. $\mathbf{n} = \left(\dfrac{x}{a^2}\mathbf{e}_1 + \dfrac{y}{b^2}\mathbf{e}_2 + \dfrac{z}{c^2}\mathbf{e}_3\right)\left(\dfrac{x^2}{a^4} + \dfrac{y^2}{b^4} + \dfrac{z^2}{c^4}\right)^{-1/2}$

2C4. (a) $\mathbf{q} = -3k(\mathbf{e}_1 + \mathbf{e}_2)$ (b) $\mathbf{q} = -3k(\mathbf{e}_1 + 2\mathbf{e}_2)$

2C5. (a) $\mathbf{E} = -\nabla\varphi = -\alpha(\cos\theta\mathbf{e}_1 + \sin\theta\mathbf{e}_2)$ (b) $\mathbf{D} = -\varepsilon_1\alpha\cos\theta\mathbf{e}_1 - \varepsilon_2\alpha\sin\theta\mathbf{e}_2$

(c) $|\mathbf{D}|^2 = \alpha^2(\varepsilon_1^2\cos^2\theta + \varepsilon_2^2\sin^2\theta)$; $|\mathbf{D}|_{\max}$ for $\theta = 0; \dfrac{\pi}{2}; \dfrac{3\pi}{2}$.....

2C7. (a) $[\nabla \mathbf{v}] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ (b) $[(\nabla \mathbf{v})\mathbf{v}] = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

(c) $\operatorname{div} \mathbf{v} = 2$ (d) $\operatorname{curl} \mathbf{v} = 2\mathbf{e}_1$ (e) $d\mathbf{v} = 2ds(\mathbf{e}_1 + \mathbf{e}_3)$

CHAPTER 3

3.1. (a)$\mathbf{v} = k\mathbf{e}_1$, $\mathbf{a} = \mathbf{0}$ (b) $\dfrac{D\theta}{Dt} = Ak$ (c) $\dfrac{D\theta}{Dt} = 0$

3.2. (b)$\mathbf{v} = 2\,k\,X_1^2\,t\,\mathbf{e}_2$, $\mathbf{a} = 2\,k\,X_1^2\mathbf{e}_2$, (c)Use $X_1 = x_1$

3.3. (b)$v_1 = \dfrac{2\,k\,t\,x_2^2}{(1 + k\,t)^2}$ $v_2 = \dfrac{k\,x_2}{1 + k\,t}$ $v_3 = 0$ (c)$a_1 = \dfrac{2\,k\,x_2^2}{(1 + k\,t)^2}$, $a_2 = 0$, $a_3 = 0$

3.4. (b)$\mathbf{v} = k\mathbf{e}_1$, $\mathbf{a} = \mathbf{0}$ (c)$\mathbf{v} = \left(\dfrac{k}{1 + t}\right)\mathbf{e}_1$, $\mathbf{a} = \mathbf{0}$

3.7. (b)$\mathbf{v} = \left(\dfrac{x_1}{1 + t}\right)\mathbf{e}_1$

3.15. (b)$\mathbf{a} = -\dfrac{1}{4}(\mathbf{e}_1 + \mathbf{e}_2)$, $\dfrac{D\theta}{Dt} = 2k$

3.19. (b) $E'_{11} = k/2$

3.20. (a) Unit elongation: $5k$, $2k$. Decrease in angle : k

3.21. (a) $k/3$

3.22. (a)$\dfrac{58}{9} \times 10^{-4}$ (b)$\dfrac{32}{3\sqrt{5}} \times 10^{-4}$

3.25. $\left(\dfrac{\Delta l}{l}\right)_{\max} = 3 \times 10^{-6}$

3.31. $E_{11} = a$, $E_{22} = c$, $E_{12} = b - \dfrac{1}{2}(a + c)$

3.34. $E_{11} = a$, $E_{22} = \dfrac{1}{3}(2b + 2c - a)$, $E_{12} = \dfrac{b - c}{\sqrt{3}}$

3.37. (b) $3k$

3.38. $\dfrac{1}{ds_1}\dfrac{Dds_1}{Dt} = -(k + 1)$, $\dfrac{1}{ds_2}\dfrac{Dds_2}{Dt} = -\dfrac{1}{2}(k + 1)$

3.48. $k = 1$

3.52. $v_1 = v_1(x_2, x_3)$, $v_2 = v_3 = 0$

3.53. (a) $\rho = \dfrac{\rho_o}{(1+t)}$ (b) $\rho = \dfrac{\alpha}{x_1}$

3.54. $\rho = \rho_o e^{-t^2}$

CHAPTER 4

4.1. (a) On $\mathbf{e}_2$, $T_n = 4$ MPa (b) On $\mathbf{e}_3$, $T_s = 5.83$ MPa

4.2. (a) $\mathbf{t} = \frac{1}{3}(5\mathbf{e}_1 + 6\mathbf{e}_2 + 5\mathbf{e}_3)$ MPa (b) $T_n = 3$ MPa $T_s = \frac{\sqrt{5}}{3}$ MPa

4.4. $\mathbf{t} = \frac{100}{4}(\sqrt{3}\mathbf{e}_1 + \mathbf{e}_2 - \sqrt{3}\mathbf{e}_3)$

4.6. $T'_{11} = -6.43$ MPa, $T'_{13} = 18.6$ MPa

4.7. $\mathbf{F} = 4\beta\,\mathbf{e}_2$, $\mathbf{M} = -\frac{4\alpha}{3}\mathbf{e}_3$

4.9. $\mathbf{F} = 4\alpha\,\mathbf{e}_1$, $\mathbf{M} = -\frac{4}{3}\alpha\mathbf{e}_1$

4.10 (a) $\mathbf{t} = \mathbf{0}$, $\mathbf{t} = \alpha x_3\mathbf{e}_2 - \alpha x_2\mathbf{e}_3$, $\mathbf{t} = -\alpha x_3\mathbf{e}_2 + \alpha x_2\mathbf{e}_3$

(b) $\mathbf{F} = \mathbf{0}$, $\mathbf{M} = 8\pi a\,\mathbf{e}_1$

4.22. $T_{11} = 1$ MPa $T_{33} = 1$ MPa

4.25. (a) $T_1 = \tau$, $T_2 = 0$, $T_3 = -\tau$, $\mathbf{n}_1 = \frac{1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2)$, $\mathbf{n}_2 = \mathbf{e}_3$, $\mathbf{n}_3 = \frac{1}{\sqrt{2}}(\mathbf{e}_1 - \mathbf{e}_2)$

(b) $T_s]_{\max} = \tau$, $\mathbf{n} = \mathbf{e}_1$ and $\mathbf{e}_2$

4.26. $T_s]_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3)$, $\mathbf{n} = \frac{\sqrt{2}}{2}(\mathbf{e}_1 \pm \mathbf{e}_3)$

4.28. $T_{12} = 2x_1 - x_2 + 3$

4.29. $T_{33} = (1 + \rho g/\alpha)x_3 + f(x_1, x_2)$

4.31. Yes

CHAPTER 5

5.4. $E_Y = 207$ GPa$(30\times10^6$psi$)$, $\nu = 0.30$, $k = 172$GPa$(25\times10^6$psi$)$

5.6. $\nu = 0.27$, $\lambda = 91$GPa$(13.2\times10^6$psi$)$, $k = 141$GPa$(20.5\times10^6$psi$)$

5.8. (a) $[\mathbf{T}] = \begin{bmatrix} 6.6 & -2.3 & 0 \\ -2.3 & 3.2 & 0 \\ 0 & 0 & 8.9 \end{bmatrix} \times 10^3 \text{ psi} = \begin{bmatrix} 45.5 & -15.9 & 0 \\ -15.9 & 22.1 & 0 \\ 0 & 0 & 61.4 \end{bmatrix}$ MPa

5.10. (a) $[\mathbf{E}] = \begin{bmatrix} 3.33 & 1.26 & 0 \\ 1.26 & -2.32 & 0 \\ 0 & 0 & -0.43 \end{bmatrix} \times 10^{-5}$ $(b)\Delta V = 3.04 \times 10^{-3}\text{cm}^3$

5.14. (a) $[\mathbf{T}] = k\mu \begin{bmatrix} 0 & 2X_3 & 2X_1 + X_2 \\ 2X_3 & 0 & X_1 - 2X_2 \\ 2X_1 + X_2 & X_1 - 2X_2 & 0 \end{bmatrix}$ (b)Yes

5.17. $\dfrac{c_L}{c_T} = 2;\ 7.14;\ 22.4$

5.19. (a)Transverse wave in $\mathbf{e}_3$ direction. (b) $c = c_T$ (c)$\alpha = -1$ (d)$\beta = \dfrac{n\pi}{l},\ n = 1,2,3\ldots$

5.20. (c) $\alpha = +1$ (d) $\beta = \dfrac{n\pi}{2l,}\ n = 1,3,5\ldots$

5.21 (c) $\alpha = +1$ (d) $\beta = \dfrac{n\pi}{l,}\ n = 1,2,3\ldots$

5.22. (a) Longitudinal wave in $\mathbf{e}_3$ direction (b)$c = c_L$ (c) $\alpha = -1$ (d) $\beta = \dfrac{n\pi}{l}, n = 1,2,3\ldots$

5.27. (a) $\alpha_1 = \alpha_2 = \alpha_3 = 0,\ \ \varepsilon_2 = \varepsilon_1\ \ \varepsilon_3 = 0$ (b) $\alpha_3 = 31.2^o,\ \dfrac{\varepsilon_2}{\varepsilon_1} = 0.74,\ \dfrac{\varepsilon_3}{\varepsilon_1} = 0.50$

5.28. $\alpha_1 = 30^o$

5.34. (a) $u_1(x_1, t) = \alpha \cos \omega t \left[\cos \dfrac{\omega x_1}{c_L} + \left(\tan \dfrac{\omega l}{c_L}\right) \sin \dfrac{\omega x_1}{c_L}\right]$, (b) $\omega = \dfrac{n\pi c_L}{2l},\ \ n = 1,3,5\ldots$

5.39. (a) $T_n]_{\max} = 11{,}300\text{psi}(78\text{MPa})$ (b)$\Delta l = 3.62 \times 10^{-2}\text{inch}\ (9.2 \times 10^{-2}\text{cm})$

$T_s]_{\max} = 5650\text{psi}\ (39\text{MPa})$.

5.40. $\delta = 0.72$ cm

5.45. $P_1 = \dfrac{P}{12},\ P_2 = \dfrac{P}{3,}\ P_3 = \dfrac{7P}{12}$

5.48. $d = 5.8$cm

5.49. $d = 3.38$inch (8.59cm)

5.51. $M_1 = \dfrac{M_t}{\dfrac{l_1}{l_2}\left(\dfrac{a_2}{a_1}\right)^4 + 1},\qquad M_2 = \dfrac{M_t}{1 + \dfrac{l_2}{l_1}\left(\dfrac{a_1}{a_2}\right)^4}$

5.53. $M_L = M\dfrac{b + 2c}{(a + b + c)}$

5.54. $(b) T_n]_{\max} = 21.4\text{ksi } (147\text{MPa}), \quad T_s]_{\max} = 16.6\text{ksi } (114\text{MPa})$

5.62. $d = 10.5\text{cm}$

5.63. $b = 10\text{inch } (25.4\text{cm})$

5.69. (a) Yes (b) $T_{11} = 0, \; T_{12} = -2\alpha x_1, \; T_{22} = 2\alpha, \; T_{33} = 2\nu\alpha$

(c) On $x_1 = a, \mathbf{t} = -2\alpha a\, \mathbf{e}_2$, on $x_1 = b, \mathbf{t} = -2\alpha x_1 \mathbf{e}_1 + 2\alpha \mathbf{e}_2$

5.70. (a) If $\alpha = -\beta$

5.71. (a) Yes (b) $T_{11} = 2\alpha x_1, \quad T_{22} = 0, \quad T_{12} = -2\alpha x_2 - 3\beta x_2^2$

CHAPTER 6

6.1. $B = 5.10 \times 10^4 \text{N}$

6.5. $p_A = p_o + \rho(g + a)h$

6.8. $z = \dfrac{\omega^2 r^2}{2g}$

6.10. $p = p_o e^{-\rho_o g(z - z_o)/p_o}$ for $n = 1$

6.11. (b) $T_n = -p + \mu, \; T_s = 0$

6.12. (a) -4μ $(b) 8\mu$

6.23. $v_1 = \dfrac{\rho g \sin\theta d^2}{\mu}\left[\left(\dfrac{x_2}{d}\right) - \dfrac{1}{2}\left(\dfrac{x_2}{d}\right)^2\right]$

6.28. (a) $v = A\ln\dfrac{r}{a} + v_a, \quad A = \dfrac{v_b - v_a}{\ln(b/a)}$

(b) $v = \dfrac{\beta}{4}(r^2 - a^2) + A\ln\dfrac{r}{a}, \quad \beta = \dfrac{1}{\mu}\dfrac{dp}{dx}, \quad A = \dfrac{\beta(a^2 - b^2)}{4\ln(b/a)}$

6.30. $A = \dfrac{\beta a^2 b^2}{2(a^2 + b^2)}, \quad B = -A$

6.31. $A = -\dfrac{\beta\sqrt{3}}{6b} \quad B = 0$

6.37. $\theta = \left(\dfrac{\partial p}{\partial x}\right)^2 \dfrac{(b^4 - x_2^4)}{12\kappa\mu} + \dfrac{(\theta_2 - \theta_1)x_2}{2b} + \dfrac{\theta_1 + \theta_2}{2}$

6.40. (b) $T_{11} = -p + 2\mu k, \quad T_{22} = -p - 2\mu k, \quad T_{33} = -p,$ remaining $T_{ij} = 0$

(c) $\mathbf{a} = k^2(x_1\mathbf{e}_1 + x_2\mathbf{e}_2)$ (d) and (e) $p = -\dfrac{\rho k^2}{2}(x_1^2 + x_2^2) + p_o$

(f) $\Phi_{inc} = 4\,k^2\,\mu$

6.41. (b) $T_{11} = -p + 4\,\mu\,k\,x_1,\quad T_{22} = -p - 4\,\mu\,k\,x_1,\quad T_{12} = -4\,\mu\,k\,x_2$

$T_{33} = -p,\quad T_{13} = T_{23} = 0$

(c) $a_1 = 2\,k^2\,(x_1^3 + x_1\,x_2^2),\quad a_2 = 2\,k^2\,(x_2^3 + x_1^2\,x_2)$

6.46. $Q = 0.0762\ \mathrm{m}^3/\mathrm{sec}$

6.28. $Q = 0.113\mathrm{m}^3/\mathrm{sec}$

CHAPTER 7

7.7. (b)$3A\,\rho_o\,e^{-t},\quad -3A\,\rho$ (c)$-3\,A\,\rho$

7.8. (b) $m = A\,\rho_o$ (c) $\mathbf{p} = \frac{9}{2}\rho_o\,A\,e^{t-t_o}\mathbf{e}_1$

7.11. (a) $\dfrac{4A\rho_o}{(1+t)^2},\quad -\dfrac{8A\rho_o}{(1+t)^3}$ (b) $\dfrac{8A\,\rho_o}{(1+t)^3}$ (c) 0

(d) $\dfrac{13}{3}\dfrac{\rho_o}{(1+t)^3},\quad -\dfrac{13\rho_o}{(1+t)^4}$ (e) $\dfrac{13\rho_o A}{(1+t)^4}$

7.12. (a) $\frac{9}{2}\rho_o\,A\,e^{-t}\mathbf{e}_1,\quad -\frac{9}{2}\rho_o\,A\,e^{-t}\mathbf{e}_1$ (b) $9\rho_o\,A\,e^{-t}\mathbf{e}_1$ (c) $\frac{9}{2}\rho_o\,A\,e^{-t}\mathbf{e}_1$

7.14. $\mathbf{F} = 8\,\rho\,(\mathbf{e}_1 + \mathbf{e}_2)\qquad \mathbf{M} = 8\,\rho\,(-\mathbf{e}_1 + \mathbf{e}_2)$

7.18. $283\left(\frac{1}{2}\,\mathbf{e}_1 - \frac{\sqrt{3}}{2}\mathbf{e}_2\right)\mathrm{N}$

7.20. $p\,v_o^2\,A$

7.21. $y_2 = -\dfrac{y_1}{2} + \left[\left(\dfrac{y_1}{2}\right)^2 + \dfrac{2Q^2}{gy_1}\right]^{1/2}$

7.23. 612 rad/sec