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Introduction

1.1 CONTINUUM THEORY

Matter is formed of molecules which in turn consist of atoms and sub-atomic particles. Thus matter is not continuous. However, there are many aspects of everyday experience regarding the behaviors of materials, such as the deflection of a structure under loads, the rate of discharge of water in a pipe under a pressure gradient or the drag force experienced by a body moving in the air etc., which can be described and predicted with theories that pay no attention to the molecular structure of materials. The theory which aims at describing relationships between gross phenomena, neglecting the structure of material on a smaller scale, is known as continuum theory. The continuum theory regards matter as indefinitely divisible. Thus, within this theory, one accepts the idea of an infinitesimal volume of materials referred to as a particle in the continuum, and in every neighborhood of a particle there are always neighbor particles. Whether the continuum theory is justified or not depends on the given situation; for example, while the continuum approach adequately describes the behavior of real materials in many circumstances, it does not yield results that are in accord with experimental observations in the propagation of waves of extremely small wavelength. On the other hand, a rarefied gas may be adequately described by a continuum in certain circumstances. At any case, it is misleading to justify the continuum approach on the basis of the number of molecules in a given volume. After all, an infinitesimal volume in the limit contains no molecules at all. Neither is it necessary to infer that quantities occurring in continuum theory must be interpreted as certain particular statistical averages. In fact, it has been known that the same continuum equation can be arrived at by different hypothesis about the molecular structure and definitions of gross variables. While molecular-statistical theory, whenever available, does enhance the understanding of the continuum theory, the point to be made is simply that whether the continuum theory is justified in a given situation is a matter of experimental test, not of philosophy. Suffice it to say that more than a hundred years of experience have justified such a theory in a wide variety of situations.

1.2 Contents of Continuum Mechanics

Continuum mechanics studies the response of materials to different loading conditions. Its subject matter can be divided into two main parts: (1) general principles common to all media,

and (2) constitutive equations defining idealized materials. The general principles are axioms considered to be self-evident from our experience with the physical world, such as conservation of mass, balance of linear momentum, of moment of momentum, of energy, and the entropy inequality law. Mathematically, there are two equivalent forms of the general principles: (1) the integral form, formulated for a finite volume of material in the continuum, and (2) the field equations for differential volume of material (particle) at every point of the field of interest. Field equations are often derived from the integral form. They can also be derived directly from the free body of a differential volume. The latter approach seems to suit beginners. In this text both approaches are presented, with the integral form given toward the end of the text. Field equations are important wherever the variations of the variables in the field are either of interest by itself or are needed to get the desired information. On the other hand, the integral forms of conservation laws lend themselves readily to certain approximate solutions.

The second major part of the theory of continuum mechanics concerns the "constitutive equations" which are used to define idealized material. Idealized materials represent certain aspects of the mechanical behavior of the natural materials. For example, for many materials under restricted conditions, the deformation caused by the application of loads disappears with the removal of the loads. This aspect of the material behavior is represented by the constitutive equation of an elastic body. Under even more restricted conditions, the state of stress at a point depends linearly on the changes of lengths and mutual angle suffered by elements at the point measured from the state where the external and internal forces vanish. The above expression defines the linearly elastic solid. Another example is supplied by the classical definition of viscosity which is based on the assumption that the state of stress depends linearly on the instantaneous rates of change of length and mutual angle. Such a constitutive equation defines the linearly viscous fluid. The mechanical behavior of real materials varies not only from material to material but also with different loading conditions for a given material. This leads to the formulation of many constitutive equations defining the many different aspects of material behavior. In this text, we shall present four idealized models and study the behavior they represent by means of some solutions of simple boundary-value problems. The idealized materials chosen are (1) the linear isotropic and anisotropic elastic solid (2) the incompressible nonlinear isotropic elastic solid (3) the linearly viscous fluid including the inviscid fluid, and (4) the Non-Newtonian incompressible fluid.

One important requirement which must be satisfied by all quantities used in the formulation of a physical law is that they be coordinate-invariant. In the following chapter, we discuss such quantities.