Inviscid/Viscous Hypersonic Flow considering anisotropic shock capturing and adaptive mesh refinement techniques

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1. Introduction.

- **Three tests proposed by ESA (ESTEC)/OPEN ENGINEERING to evaluate three CFD codes for the ‘strong *Fluid-Structure Interaction* project’ at hypersonic regime ($M \geq 5$):**
  - ▷ **AeroULG-3D code from Université de Liège, Belgium,**
  - ▷ **FINE/HEXA-3D code by Numeca (C. Hirsch), Belgium, and**
  - ▷ **PETSc-FEM code from CIMEC-INTEC, Argentina. (`http://www.cimec.org.ar/petscfem`).**

2. Introduction (contd...).

These tests involved different numerical techniques, namely:

- Parallel Solution of 'Inviscid/Viscous Hypersonic Flows' in 'Beowulf' clusters of PC's.
- The Galerkin/SUPG formulation with added 'isotropic/anisotropic shock capturing' operators.
- 'Adaptive mesh refinement' techniques.
- The use of 'non-reflecting' boundary conditions on fictitious (and subsonic) walls.
- Finally, a 'Domain Decomposition' preconditioner for an efficient solution of the linear system.

3. Physical Model

3.1. The compressible viscous/inviscid Navier-Stokes equations.

Conservative differential form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial(\mathcal{F}^a(\mathbf{U}))_i}{\partial x_i} = \frac{\partial(\mathcal{F}^d(\mathbf{U}))_i}{\partial x_i} + \mathcal{G} \quad \text{in} \quad \Omega \times (0, t_+]$$

$$\mathcal{F}^a(\mathbf{U}) = \begin{pmatrix} \rho u_i \\ \rho u_1 u_i + \delta_{i1} p \\ \rho u_2 u_i + \delta_{i2} p \\ \rho u_3 u_i + \delta_{i3} p \\ \rho \mathcal{H} u_i \end{pmatrix}, \quad \mathcal{F}^d(\mathbf{U}) = \begin{pmatrix} 0 \\ \tau_{i1} \\ \tau_{i2} \\ \tau_{i3} \\ \tau_{ik} u_k - q_i \end{pmatrix}$$

where $\mathbf{U} = (\rho, \rho \mathbf{u}, \rho E)^t$, $\quad \mathcal{G} = (0, \rho \mathbf{f_e}, W_f + q_H)$, $\quad W_f = \rho \mathbf{f_e} \cdot \mathbf{u}$.

$$\mathcal{H} = e + p/\rho + {}^1\!/_2|\mathbf{u}|^2 = E + p/\rho \qquad \text{total specific enthalpy,}$$

$$q_i = -\kappa\nabla T \qquad \text{heat flux,}$$

$$\overline{\overline{\tau}} = 2\mu\epsilon(\mathbf{u}) - 2/3\mu(\nabla\cdot\mathbf{u})\mathbf{I} \qquad \text{Newtonian viscous stress tensor,}$$

$$\epsilon(\mathbf{u}) = \frac{1}{2}(\partial_j u_i + \partial_i u_j) \qquad \text{the strain rate tensor.}$$

Viscosity and thermal conductivity given by the Sutherland formula (*standard atmosphere***),**

$$\mu = \mu_0\left(\frac{T}{T_0}\right)^{3/2}\frac{T_0+110}{T+110}, \qquad \kappa = \frac{\gamma R\mu}{(\gamma-1)Pr},$$

μ_0**:** **viscosity at the reference temperature** T_0**,**

$Pr = \nu/\iota$**:** **Prandtl number,**

ι**:** **thermal diffusivity coefficient).**

4. Variational formulation.

Interpolation and weighting function spaces:

$$\mathcal{S}^h = \{\mathbf{U}^h | \mathbf{U}^h \in [\mathbf{H}^{1h}(\Omega)]^{n_{\text{d.o.f.}}}, \mathbf{U}^h|_{\Omega^e} \in [P^1(\Omega^e)]^{n_{\text{d.o.f.}}}, \mathbf{U}^h = \mathbf{g} \ \textbf{on} \ \Gamma_g\}$$

$$\mathcal{V}^h = \{\mathbf{W}^h | \mathbf{W}^h \in [\mathbf{H}^{1h}(\Omega)]^{n_{\text{d.o.f.}}}, \mathbf{W}^h|_{\Omega^e} \in [P^1(\Omega^e)]^{n_{\text{d.o.f.}}}, \mathbf{W}^h = \mathbf{0} \ \textbf{on} \ \partial\Omega_g\}$$

The problem is: find $\mathbf{U}^h \in \mathcal{S}^h$ such that $\forall\, \mathbf{W}^h \in \mathcal{V}^h$

$$\int_{\Omega} \mathbf{W}^h \left(\frac{\partial \mathbf{U}^h}{\partial t} + \mathbf{A}_i^h \frac{\partial \mathbf{U}^h}{\partial x_i} - \mathcal{G} \right) d\Omega + \int_{\Omega} \frac{\partial \mathbf{W}^h}{\partial x_i} \mathbf{K}_{ij}^h \frac{\partial \mathbf{U}^h}{\partial x_j} d\Omega - \int_{\Gamma_h} \mathbf{W}^h H^h d\Gamma +$$

$$+ \sum_{e=1}^{n_{\text{el}}} \int_{\Omega^e} \tau (\mathbf{A}_k^h)^T \frac{\partial \mathbf{W}^h}{\partial x_k} \left\{ \frac{\partial \mathbf{U}^h}{\partial t} + \mathbf{A}_i^h \frac{\partial \mathbf{U}^h}{\partial x_i} - \frac{\partial}{\partial x_i} \left(\mathbf{K}_{ij}^h \frac{\partial \mathbf{U}^h}{\partial x_j} \right) - \mathcal{G} \right\} d\Omega +$$

$$+ \sum_{e=1}^{n_{\text{el}}} \int_{\Omega^e} \delta_{shc} \frac{\partial \mathbf{W}^h}{\partial x_i} \frac{\partial \mathbf{U}^h}{\partial x_i} d\Omega = \mathbf{0}$$

4.1. Stabilization and Shock Capturing operators.

- **SUPG intrinsic time** τ **(by Aliabadi and Tezduyar):** $\tau = \max[\mathbf{0}, \tau_a - \tau_d - \tau_\delta]$

$$\tau_a = \frac{h}{2(c + |\mathbf{u}|)}\mathbf{I}, \qquad \tau_d = \frac{\sum_{j=1}^{n_{\mathrm{sd}}} \beta_j^2 \,\mathrm{diag}\,(\mathbf{K}_{jj})}{(c + |\mathbf{u}|)^2}\mathbf{I}, \qquad \tau_\delta = \frac{\delta_{shc}}{(c + |\mathbf{u}|)^2}\mathbf{I},$$

$$h = 2\left(\sum_{a=1}^{n_{\mathrm{en}}} |\mathbf{u} \cdot \nabla N_a| \right)^{-1}$$ **: the element length in the streamline direction.**

- **Shock Capturing term**
  - ▷ **isotropic operator**

$$\delta_{shc} = \frac{h_{JGN}}{2} u_{char} \left(\frac{|\nabla \rho^h| h_{JGN}}{\rho_{ref}} \right)^{\beta},$$

$$h_{JGN} = 2 \left(\sum_{a=1}^{n_{\mathrm{en}}} |\mathbf{j} \cdot \nabla N_a| \right)^{-1}$$ **: a characteristic length,**

$$\mathbf{j} = \nabla \rho^h / |\nabla \rho^h|$$ **: direction along density gradient.**

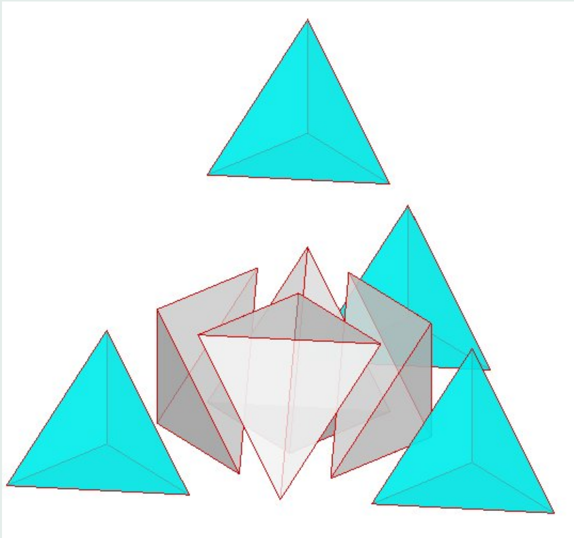
4.2. Stabilization and Shock Capturing operators (contd...).

- **anisotropic operator**

$$\sum_{e=1}^{n_{\mathrm{el}}} \int_{\Omega^e} \frac{\partial \mathbf{W}^h}{\partial x_i} \mathbf{j}_{\mathbf{i}} \, \delta_{shc} \, \mathbf{j}_{\mathbf{k}} \frac{\partial \mathbf{U}^h}{\partial x_k} d\Omega.$$

5. Adaptive Refinement [Reference G Rios MECOM 2005].

- Reduce the error in solution near walls, discontinuities, shock waves, etc,
- Error estimation metric needed.
- Homogeneous scheme,
- The strategy is similar for triangles, quads and hexas in 3D. Special cases of partitions for tetras.



6. Domain Decomposition Methods (SD/SCMI).

Re-ordering the state variables vector u and the forces vector f as $u = (u_L, u_I)^T$ and $f = (f_L, f_I)^T$,

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{LL} & \mathbf{A}_{LI} \\ \mathbf{A}_{IL} & \mathbf{A}_{II} \end{bmatrix},$$

The numerical solution of $\mathbf{A}u = f$ is equivalent to solving

$$\mathbf{S}u_I = g \quad \text{on interfaces } \Gamma,$$

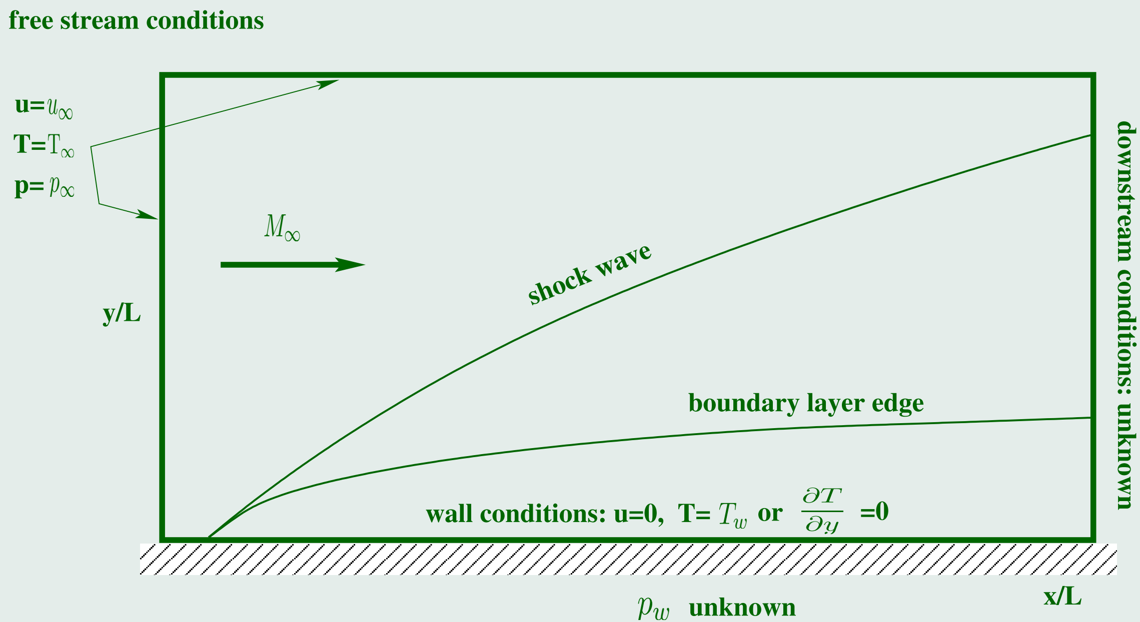
$$\mathbf{A}_{LL}u_L = f_L - \mathbf{A}_{LI}u_I \quad \text{on } \Omega_i$$

$$\mathbf{S} = \mathbf{A}_{II} - \sum_{i=1}^{n} \mathbf{A}_{IL}\mathbf{A}_{LL}^{-1}\mathbf{A}_{LI},$$

where $\mathbf{S}$ is the well-known Schur complement matrix.

7. TEST 1: viscous flow over a flat plate at M=5.

$$M_\infty = 5$$
$$T_\infty = 80\text{K}$$
$$p_\infty = 10^5\text{Pa}$$
$$Re_\infty = 10^4$$
$$R = 287\frac{\text{J}}{\text{kgK}}$$
$$T_{wall} = 288\text{K}$$
$$\gamma = 1.4$$
$$Pr = 0.72$$
$$\mu = 2.5 \cdot 10^{-5}\,\frac{\text{Kg}}{\text{m sec}}$$
$$\kappa = 3.47 \cdot 10^{-5}\frac{\text{W}}{\text{mK}}.$$



7.1. Results: Mach number distribution.

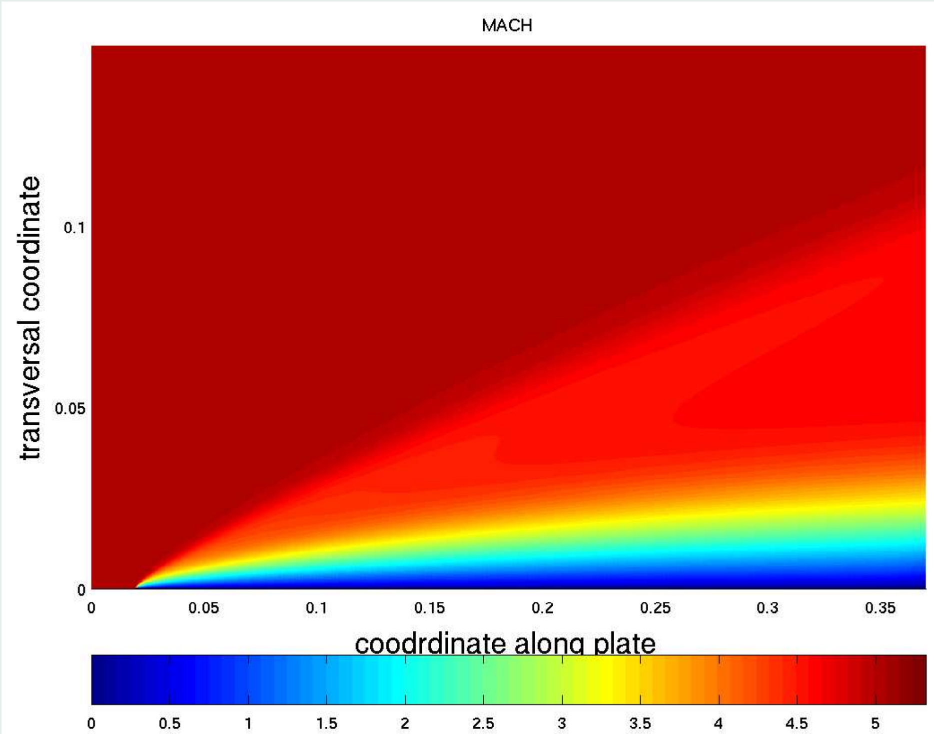
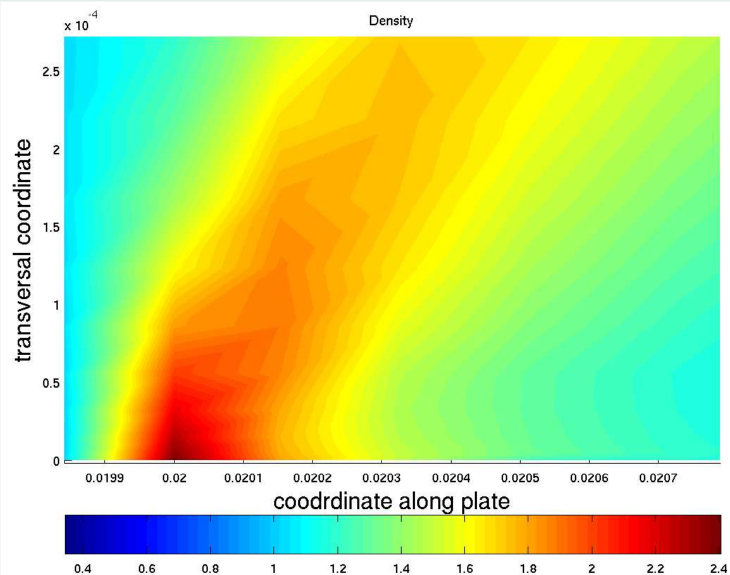
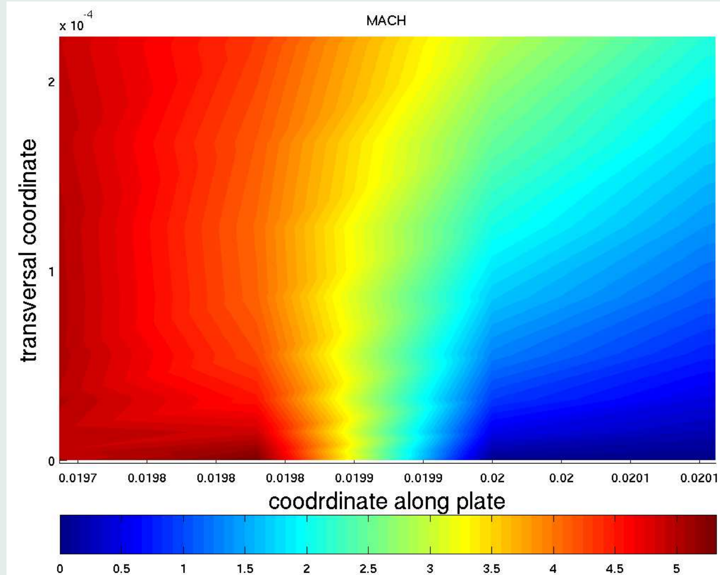


Figure 1: Mach number distribution.

7.2. Results: stagnation point





7.3. Results: skin friction coefficient.

Definition: $C_f = \tau_{wall}/(0.5\rho_\infty ||u_\infty||^2)$.

Theoretical: $C_f = 0.664 \left(\frac{T_{wall}}{T_\infty}\right)^{-0.5(1-\omega)} \left(\frac{p_{wall}}{p_\infty Re_x}\right)^{0.5}, \quad \omega = 0.75.$

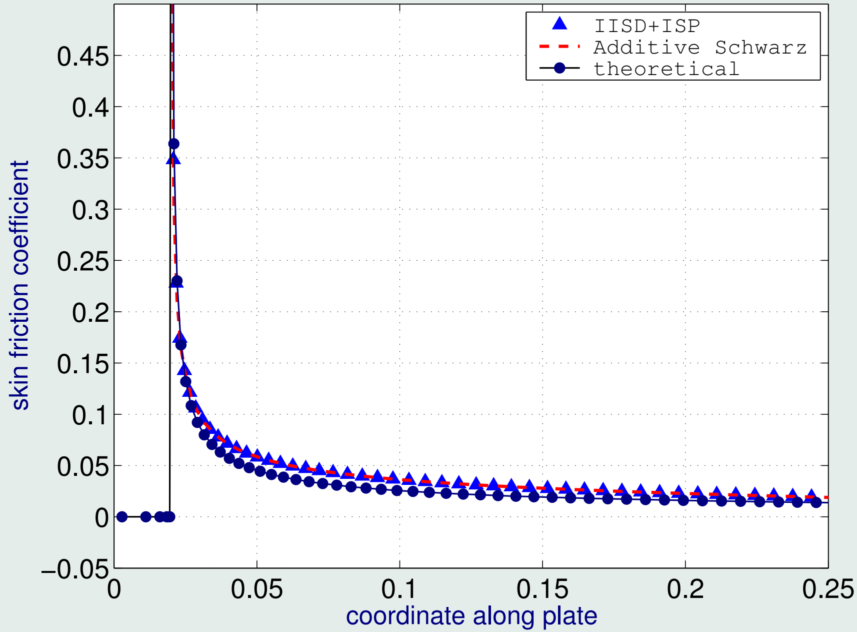


Figure 2: Skin friction coefficient.

7.4. Results: Stanton number.

Definition: $\mathrm{St} = \dfrac{q_{\mathrm{wall}}}{\rho_\infty U_\infty C_p |T_{\mathrm{wall}} - T_\infty^0|}$.

Theoretical: $\mathrm{St} = 0.5 Pr^{-2/3} \mathrm{C_f}$.

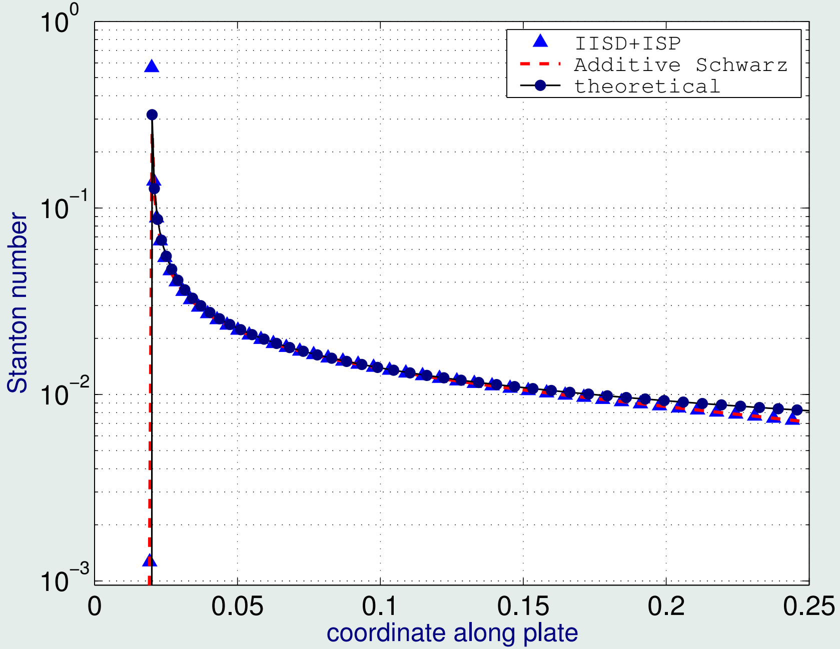


Figure 3: Stanton number.

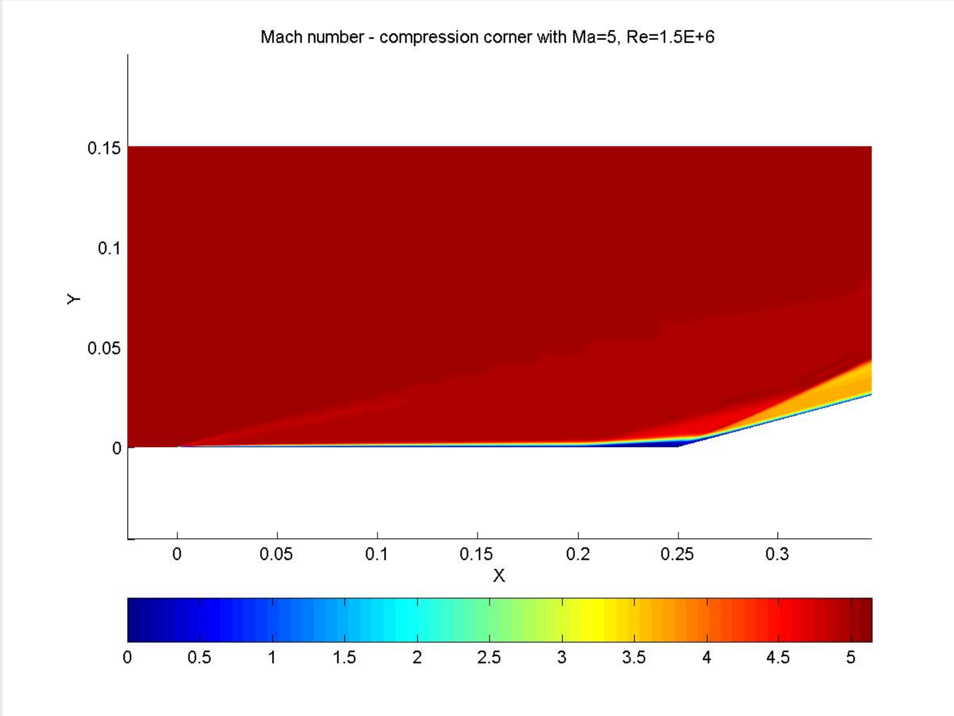
8. TEST 2: compression corner viscous flow at M=5

(“Hypersonic flows for re-entry problems”, by J-A Désidéri, R. Glowinski and J. Périaux.)

$$\alpha = 15^\circ$$
$$M_\infty = 5$$
$$T_\infty = 80\text{K}$$
$$p_\infty = 10^5\text{Pa}$$
$$Re_\infty = 1.5\,10^6$$
$$R = 287\frac{\text{J}}{\text{kgK}}$$
$$T_{wall} = 288\text{K}$$
$$\gamma = 1.4$$
$$Pr = 0.72$$

μ **Sutherland form.**

κ **Sutherland form.**



9. Results: Stanton number.

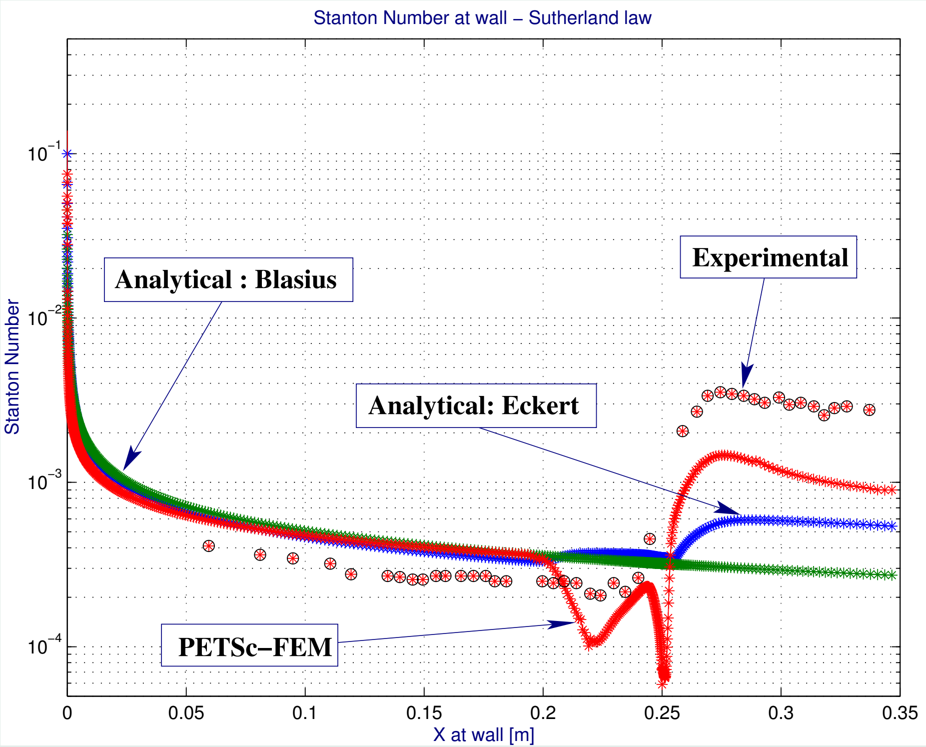


Figure 4: Stanton number (refined mesh).

10. Results: Skin friction coefficient.

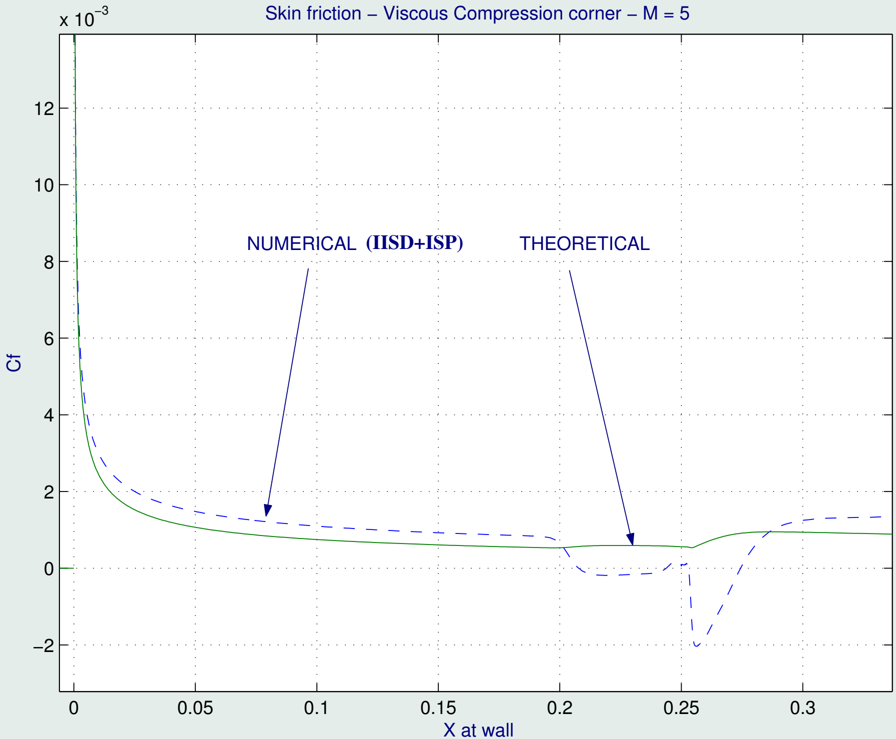


Figure 5: Skin friction coefficient (refined mesh).

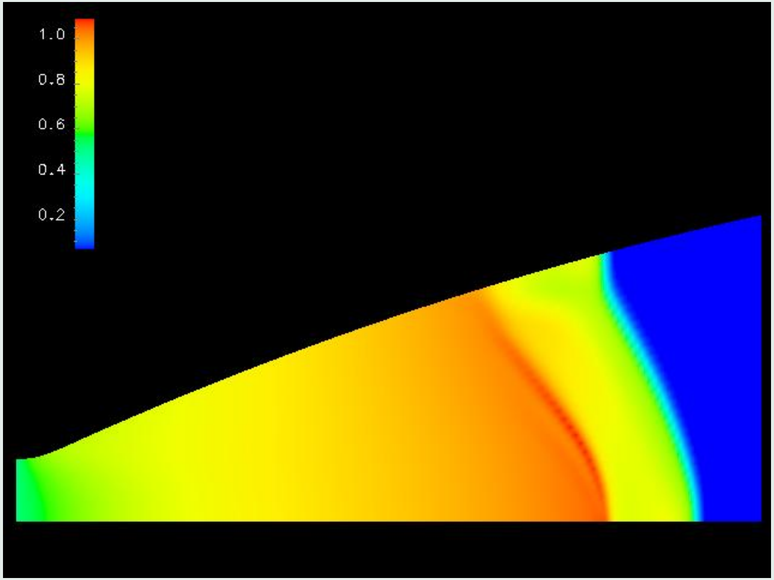
11. Inviscid Case: Mach=10. Results.

	p/p_∞	T/T_∞	Mach after the shock	Angle of the shock
Present Result	13.482	3.166	5.305	20.37°
Theoretical	13.404	3.194	5.279	19.941°
Relative error	0.6%	0.9%	0.5%	2.1%

Table 1: comparison between the analytical reference results and the computed results.

12. Shock wave propagation in a Nozzle.

$$\begin{aligned}
\mathbf{u}_{t<0,\forall x} &= 0\text{m}/\text{s}\\
p_{t<0,\forall x} &= 143\text{Pa}\\
T_{t<0,\forall x} &= 262\text{K}\\
p_{t=0,x=0} &= 600000\text{Pa}\\
T_{t=0,x=0} &= 4170\text{K}\\
\gamma &= 1.17.
\end{aligned}$$



Measured wave velocity: 2600-2700m/s

Computed wave velocity: 2620.5m/s**.**

13. Future Work

- ‘Loose coupling fluid-structure interaction’ inside PETSc-FEM for the ESA project.
- Strong coupling.

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