Continuum Mechanics Nomenclature Sheet — run! save yourself!

Symbols RMB, Book	Name, description, Relevant equations	natural basis	Objectivity	SI units
$\underset{\sim}{\mathbf{a}}$	Material acceleration. $\underset{\sim}{\mathbf{a}} = \left(\frac{\partial \underset{\sim}{\mathbf{v}}}{\partial t}\right)_{\underset{\sim}{\mathbf{X}}} = \left(\frac{\partial^2 \underset{\sim}{\mathbf{u}}}{\partial t^2}\right)_{\underset{\sim}{\mathbf{X}}}$.	$\underset{\sim}{\mathbf{e}}$	$\ddot{\underset{\approx}{\mathbf{Q}}} \bullet \underset{\sim}{\mathbf{x}} + 2\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{v}}$ $+\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{a}} + \ddot{\underset{\sim}{\mathbf{c}}}$	m/s^2
$\underset{\sim}{\mathbf{b}}$	Body force per unit mass.	$\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{b}}$	m/s^2
$\underset{\approx}{\mathbf{B}}$, $\underset{\approx}{\mathbf{B}}$	Left Cauchy-Green tensor, $\underset{\approx}{\mathbf{B}} = \underset{\approx}{\mathbf{F}} \bullet \underset{\approx}{\mathbf{F}}^T = \underset{\approx}{\mathbf{V}}^2 = \sum_{k=1}^{3} \lambda_k^2 \underset{\sim}{\mathbf{q}}_k \underset{\sim}{\mathbf{q}}_k$	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{B}}$	1
$\underset{\approx}{\mathbf{C}}$, $\underset{\approx}{\mathbf{C}}$	Right Cauchy-Green tensor, $\underset{\approx}{\mathbf{C}} = \underset{\approx}{\mathbf{F}}^T \bullet \underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{U}}^2 = \sum_{k=1}^{3} \lambda_k^2 \underset{\sim}{\mathbf{P}}_k \underset{\sim}{\mathbf{P}}_k$ $\underset{\sim}{\mathbf{M}} \bullet \underset{\approx}{\mathbf{C}} \bullet \underset{\sim}{\mathbf{M}}$ is the square of the stretch, $(dl)^2/(dL)^2$ of a line element originally in the $\underset{\sim}{\mathbf{M}}$ direction.	$\underset{\sim}{\mathbf{E}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{C}}$	1
$\underset{\approx}{\mathbf{D}}$	The “rate” of deformation. AKA the “stretching”. Equal to the symmetric part of the velocity gradient. $\underset{\approx}{\mathbf{D}} = \frac{1}{2}(\underset{\approx}{\mathbf{L}} + \underset{\approx}{\mathbf{L}}^T)$. Not a true rate (except for infinitesimal deformations, when it is the strain rate).	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{D}} \bullet \underset{\approx}{\mathbf{Q}}^T$	$1/s$
$\underset{\approx}{\xi}$	Signorini strain. $\underset{\approx}{\xi} = \frac{1}{2}(\underset{\approx}{\mathbf{F}} \bullet \underset{\approx}{\mathbf{F}}^T - \underset{\approx}{\mathbf{I}}) = \frac{1}{2}(\underset{\approx}{\mathbf{B}} - \underset{\approx}{\mathbf{I}}) = \underset{\approx}{\mathbf{R}} \bullet \underset{\approx}{\mathbf{E}} \bullet \underset{\approx}{\mathbf{R}}^T$.	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\xi} \bullet \underset{\approx}{\mathbf{Q}}^T$	1
$\underset{\approx}{\mathbf{e}}$, $\underset{\approx}{\mathbf{e}}*$	Euler strain. $\underset{\approx}{\mathbf{e}} = \frac{1}{2}(\underset{\approx}{\mathbf{I}} - \underset{\approx}{\mathbf{B}}^{-1}) = \frac{1}{2}(\underset{\approx}{\mathbf{h}} + \underset{\approx}{\mathbf{h}}^T - \underset{\approx}{\mathbf{h}}^T \bullet \underset{\approx}{\mathbf{h}})$	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{e}} \bullet \underset{\approx}{\mathbf{Q}}^T$	1
$\underset{\approx}{\mathbf{E}}$	Lagrange strain, $\underset{\approx}{\mathbf{E}} = \frac{1}{2}(\underset{\approx}{\mathbf{F}}^T \bullet \underset{\approx}{\mathbf{F}} - \underset{\approx}{\mathbf{I}}) = \frac{1}{2}(\underset{\approx}{\mathbf{H}} + \underset{\approx}{\mathbf{H}}^T + \underset{\approx}{\mathbf{H}}^T \bullet \underset{\approx}{\mathbf{H}})$	$\underset{\sim}{\mathbf{E}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{E}}$	1
$\underset{\approx}{\mathbf{F}}$, $\underset{\approx}{\mathbf{F}}$	Deformation gradient. $\underset{\approx}{\mathbf{F}} = \left(\frac{\partial \underset{\sim}{\mathbf{x}}}{\partial \underset{\sim}{\mathbf{X}}}\right)_t$. $d\underset{\sim}{\mathbf{x}} = \underset{\approx}{\mathbf{F}} \bullet d\underset{\sim}{\mathbf{X}}$. Columns of $\underset{\approx}{\mathbf{F}}$ are $\mathbf{g}_i = \underset{\approx}{\mathbf{F}} \bullet \mathbf{E}_i$. Deformation of a reference unit line element $\underset{\sim}{\mathbf{M}}$ is $\underset{\approx}{\mathbf{F}} \bullet \underset{\sim}{\mathbf{M}}$.	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{F}}$	1
$\underset{\approx}{\mathbf{F}}^{-1}$, $\underset{\approx}{\mathbf{F}}^{-1}$	The inverse of $\underset{\approx}{\mathbf{F}}$. $\boxed{\underset{\approx}{\mathbf{F}}^{-1} = \left(\frac{\partial \underset{\sim}{\mathbf{X}}}{\partial \underset{\sim}{\mathbf{x}}}\right)_t}$. $\underset{\approx}{\mathbf{F}} \bullet \underset{\approx}{\mathbf{F}}^{-1} = \underset{\approx}{\mathbf{F}}^{-1} \bullet \underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{I}}$	$\underset{\sim}{\mathbf{E}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{F}}^{-1} \bullet \underset{\approx}{\mathbf{Q}}^T$	1

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$\underset{\approx}{\mathbf{F}}^T$, $\underset{\approx}{\mathbf{F}}^T$	The transpose of $\underset{\approx}{\mathbf{F}}$, $F^T_{ij} = F_{ji}$.	$\underset{\sim}{\mathbf{E}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{F}}^T \bullet \underset{\approx}{\mathbf{Q}}^T$	1
$\underset{\approx}{\mathbf{F}}^{-T}, (\underset{\approx}{\mathbf{F}}^{-1})^T$	The inverse of the transpose (or transpose of the inverse) of $\underset{\approx}{\mathbf{F}}$	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{F}}^{-T}$	1
$\underset{\approx}{\mathbf{h}}$	Spatial displacement gradient. $\underset{\approx}{\mathbf{h}} = \left(\frac{\partial \underset{\sim}{\mathbf{u}}}{\partial \underset{\sim}{\mathbf{x}}}\right)_t = \underset{\approx}{\mathbf{I}} - \underset{\approx}{\mathbf{F}}^{-1}$.	---	$\underset{\approx}{\mathbf{I}} - \underset{\approx}{\mathbf{Q}}^T + \underset{\approx}{\mathbf{h}} \bullet \underset{\approx}{\mathbf{Q}}^T$	1
$\underset{\approx}{\mathbf{H}}$	Reference displacement gradient. $\underset{\approx}{\mathbf{H}} = \left(\frac{\partial \underset{\sim}{\mathbf{u}}}{\partial \underset{\sim}{\mathbf{X}}}\right)_t = \underset{\approx}{\mathbf{F}} - \underset{\approx}{\mathbf{I}}$.	---	$\underset{\approx}{\mathbf{Q}} - \underset{\approx}{\mathbf{I}} + \underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{H}}$	1
J	Jacobian. $J = \det \underset{\approx}{\mathbf{F}}$. $dV = J\, dV_o$	1	J	1
$\underset{\approx}{\mathbf{L}}$	Velocity (spatial) gradient. $\underset{\approx}{\mathbf{L}} = \left(\frac{\partial \underset{\sim}{\mathbf{v}}}{\partial \underset{\sim}{\mathbf{x}}}\right)_t = \underset{\sim}{\mathbf{v}}\overleftarrow{\nabla} = \left(\frac{\partial v_i}{\partial x_j}\right)_t \underset{\sim}{\mathbf{e}}_i \underset{\sim}{\mathbf{e}}_j$, $\underset{\approx}{\mathbf{L}} = \dot{\underset{\approx}{\mathbf{F}}} \bullet \underset{\approx}{\mathbf{F}}^{-1}$	---	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{L}} \bullet \underset{\approx}{\mathbf{Q}}^T + \dot{\underset{\approx}{\mathbf{Q}}} \bullet \underset{\approx}{\mathbf{Q}}^T$	$1/s$
$\mathcal{P}_s$	Stress power *per unit mass*. Represents the “rate” of work per unit mass going into distortion of the material. $\mathcal{P}_s = \frac{1}{\rho_o}\underset{\approx}{\sigma}:\underset{\approx}{\mathbf{D}} = \frac{1}{\rho_o}\underset{\approx}{\tau}:\dot{\underset{\approx}{\mathbf{E}}} = \frac{1}{\rho_o}\hat{\underset{\approx}{\tau}}:\dot{\underset{\approx}{\mathbf{F}}}$.	1	$\mathcal{P}_s$	$\frac{J}{kg \cdot s}$
$\underset{\sim}{\mathbf{p}}_k$	k^{th} eigenvector of right stretch $\underset{\approx}{\mathbf{U}}$.	$\underset{\sim}{\mathbf{E}}$	$\underset{\sim}{\mathbf{p}}_k$	1
$\underset{\sim}{\mathbf{q}}_k$	k^{th} eigenvector of left stretch $\underset{\approx}{\mathbf{V}}$.	$\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{q}}_k$	1
$\underset{\sim}{\mathbf{q}}$	Heat flux across the boundary ∂B of a spatial body.	$\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{q}}$	$(J/s)/m^2$
r	Externally supplied heating per unit mass (from, say, a microwave oven)	1	r	$(J/s)/kg$
$\underset{\approx}{\mathbf{R}}$, $\underset{\approx}{\mathbf{R}}$	Rotation tensor from polar decomposition, $\underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{R}} \bullet \underset{\approx}{\mathbf{U}}$	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{R}}$	1
$\underset{\approx}{\sigma}$, $\underset{\approx}{\mathbf{T}}$	Cauchy stress tensor, defined such that the Cauchy traction $\underset{\sim}{\mathbf{t}}$ (i.e., force per unit *current* area) is related to the normal of area by $\underset{\sim}{\mathbf{t}} = \underset{\approx}{\sigma} \bullet \underset{\sim}{\mathbf{n}}$. The Cauchy stress has the greatest physical significance, even though actual computations may utilize other stress tensors. The quantity $J\underset{\approx}{\sigma}$ is called the Kirchhoff stress.	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\sigma} \bullet \underset{\approx}{\mathbf{Q}}^T$	Pa
$\underset{\sim}{\mathbf{t}}$	Cauchy traction (force per unit area) applied to a surface. The traction varies linearly with the unit normal to the surface, with the linearity tensor being the Cauchy stress $\underset{\sim}{\mathbf{t}} = \underset{\approx}{\sigma} \bullet \underset{\sim}{\mathbf{n}}$.	$\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{t}}$	Pa

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$\hat{\underset{\approx}{\tau}}$, $\underset{\sim}{\mathbf{t}}_o$	First Piola-Kirchhoff stress tensor defined so that $\hat{\underset{\approx}{\tau}} \bullet \underset{\sim}{\mathbf{N}} dA_o = \underset{\approx}{\sigma} \bullet \underset{\sim}{\mathbf{n}} dA$ and therefore, by Nanson's relation, $\hat{\underset{\approx}{\tau}} = J\underset{\approx}{\sigma} \bullet \underset{\approx}{\mathbf{F}}^{-T}$. Balance of angular momentum requires $\hat{\underset{\approx}{\tau}} \bullet \underset{\approx}{\mathbf{F}}^{T} = \underset{\approx}{\mathbf{F}} \bullet \hat{\underset{\approx}{\tau}}^{T}$. Beware: many books use the transpose of the above definition.	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{Q}} \bullet \hat{\underset{\approx}{\tau}}$	Pa
$\underset{\approx}{\tau}$,	Second Piola-Kirchhoff stress tensor defined such that $\underset{\approx}{\tau} = \underset{\approx}{\mathbf{F}}^{-1} \bullet J\underset{\approx}{\sigma} \bullet \underset{\approx}{\mathbf{F}}^{-T}$. Balance of angular momentum requires $\underset{\approx}{\tau} = \underset{\approx}{\tau}^{T}$.	$\underset{\sim}{\mathbf{E}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\tau}$	Pa
$\underset{\approx}{\mathbf{U}}$, $\underset{\approx}{\mathbf{U}}$	Right stretch from the polar decomposition, $\underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{R}} \bullet \underset{\approx}{\mathbf{U}}$. Thus $\underset{\approx}{\mathbf{U}} = \underset{\approx}{\mathbf{C}}^{1/2} = \sum_{k=1}^{3} \lambda_k \underset{\sim}{\mathbf{p}}_k \underset{\sim}{\mathbf{p}}_k$	$\underset{\sim}{\mathbf{E}}\underset{\sim}{\mathbf{E}}$	$\underset{\approx}{\mathbf{U}}$	1
$\underset{\sim}{\mathbf{u}}$	Material displacement. $\underset{\sim}{\mathbf{u}} = \underset{\sim}{\mathbf{x}} - \underset{\sim}{\mathbf{X}} + \underset{\sim}{\mathbf{c}}$, where $\underset{\sim}{\mathbf{c}}$ is the vector connecting the origin for $\underset{\sim}{\mathbf{X}}$ to the origin for $\underset{\sim}{\mathbf{x}}$. In other parts of this nomenclature list, $\dot{\underset{\sim}{\mathbf{c}}}$ is taken to be zero.	$\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{x}} - \underset{\sim}{\mathbf{X}} + \underset{\sim}{\mathbf{c}}$	m/s
$\underset{\sim}{\mathbf{v}}$	Material velocity. $\underset{\sim}{\mathbf{v}} = \left(\frac{\partial \underset{\sim}{\mathbf{u}}}{\partial t}\right)_{\underset{\sim}{\mathbf{X}}}$.	$\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\sim}{\mathbf{v}} + \dot{\underset{\approx}{\mathbf{Q}}} \bullet \underset{\sim}{\mathbf{x}}$ $+ \dot{\underset{\sim}{\mathbf{c}}}$	m/s
$\underset{\approx}{\mathbf{V}}$, $\underset{\approx}{\mathbf{V}}$	Left stretch from polar decomposition, $\underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{V}} \bullet \underset{\approx}{\mathbf{R}}$. Thus $\underset{\approx}{\mathbf{V}} = \underset{\approx}{\mathbf{B}}^{1/2} = \sum_{k=1}^{3} \lambda_k \underset{\sim}{\mathbf{q}}_k \underset{\sim}{\mathbf{q}}_k$. The magnitude of $\underset{\approx}{\mathbf{V}} \bullet \underset{\sim}{\mathbf{M}}$ equals the stretch, $(dl)/(dL)$ of a unit line element originally in the $\underset{\sim}{\mathbf{M}}$ direction.	$\underset{\sim}{\mathbf{e}}\underset{\sim}{\mathbf{e}}$	$\underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{V}} \bullet \underset{\approx}{\mathbf{Q}}^{T}$	1
$\underset{\approx}{\mathbf{W}}$	The vorticity tensor. Equal to the skew-symmetric part of the velocity gradient. $\underset{\approx}{\mathbf{W}} = \frac{1}{2}(\underset{\approx}{\mathbf{L}} - \underset{\approx}{\mathbf{L}}^{T})$. The vorticity vector $\underset{\sim}{\mathbf{w}}$ is the dual vector associated with $\underset{\approx}{\mathbf{W}}$.	--	$\dot{\underset{\approx}{\mathbf{Q}}} \bullet \underset{\approx}{\mathbf{Q}}^{T}$ $+ \underset{\approx}{\mathbf{Q}} \bullet \underset{\approx}{\mathbf{W}} \bullet \underset{\approx}{\mathbf{Q}}^{T}$	$1/s$
ρ	Mass density	1	ρ	kg/m^3
ρ_o	Initial mass density	1	ρ_o	kg/m^3