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TERMOVISCOELASTODINAMICA DE MEDIOS POROSOS MICROESTRUCTURADOS CON MICROTEMPERATURAS

THERMOVISCOELASTODINAMICS OF MICROSTRUCTURED POROUS MEDIA WITH MICROTEMPERATURES

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Palabras clave: Termoelasticidad, poro-termo-elasticidad, micro-temperaturas, teoría de segundo gradiente, Teoría de Green-Naghdi tipo II.

Resumen. En el presente trabajo se modela el fenómeno termo-elastodinámico en solidos porosos micro-estructurados con micro-temperaturas, para lo cual utilizamos, la formulación de Green Naghdi tipo II, y de micro-temperaturas (D. Ieşan, *Discrete and Continuous Dynamical Systems - B*, Volume 19, Number 7, 2169–2187, 2014), en relación con el acoplamiento al campo escalar de temperaturas, y la teoría de segundo gradiente para el cálculo de deformaciones. Las micro-estructuras consideradas son: inclusiones y dislocaciones genéricas. Se formula rigurosamente el problema de condiciones iniciales y de borde, luego se definen las funciones de Green asociadas a cada campo, para finalmente, utilizando el segundo y tercer teoremas de Green Lagrange, construir las representaciones integrales de las soluciones. Posteriormente utilizando el teorema de Picard, se muestra un método iterativo para calcular aproximaciones a todo orden

Keywords: Thermoelasticity, poro-thermoelasticity, microtemperatures, second gradient theory, Green-Naghdi type II theories.

Abstract. The present work models the thermo-elastodynamic phenomenon in micro-structured porous solids with micro-temperatures. For this purpose, we use the Green Naghdi type II formulation and micro-temperatures (D. Ieşan, *Discrete and Continuous Dynamical Systems - B*, Volume 19, Number 7, 2169–2187, 2014), in connection with the coupling to the scalar field of temperatures and the second gradient theory for strain calculation. The considered micro-structures are generic inclusions and dislocations. The problem of initial and boundary conditions is rigorously formulated, then the Green functions associated with each field are defined, and finally, using the second and third theorems of Green Lagrange, the integral representations of the solutions are constructed. Subsequently, by employing the Picard theorem, an iterative method is presented for calculating approximations to any order.

1. INTRODUCCIÓN

Se escriben las ecuaciones de gobierno de la poro-termo-visco-elastodinámica, con micro temperaturas en la formulación de segundo gradiente, sus variables descriptoras son

$$\Re = \{u_j, \hat{\nabla}^2 u_j, \dot{u}, \hat{\nabla}^2 u_j, w_j, p_a, p_b, w_j, T\} \tag{1}$$

Las ecuaciones de movimiento son las siguientes (Ieşan, 2007, 2014)

Distribución de campos de desplazamiento viscoelástico (cuerpo de Voigt) en la formulación de segundo gradiente

$$\begin{aligned}
&\rho\partial_t^2 u_j(\vec{x},t) - l_1^2(\partial_t^2 u_j(\vec{x},t))_{,kk} - \mu(1-l_1^2\hat{\nabla}^2)u_j(\vec{x},t) - (\mu+\lambda)\hat{\nabla}(\hat{\nabla}u_k(\vec{x},t)) - \\
&\quad - \bar{\mu}(1-l_1^2\hat{\nabla}^2)\dot{u}_j(\vec{x},t) - (\bar{\mu}+\bar{\lambda})\hat{\nabla}(\hat{\nabla}\dot{u}_k(\vec{x},t)) + \alpha_1 \boldsymbol{M}^a_{jk}(1-l_1^2\hat{\nabla}^2)p_{a,k}(\vec{x},t) + \\
&\quad + \alpha_2 \boldsymbol{M}^b_{jk}(1-l_1^2\hat{\nabla}^2)p_{b,k}(\vec{x},t) + \beta\boldsymbol{M}^E_{jk}(1-l_1^2\hat{\nabla}^2)(1+\tau\partial_t)T_{,k}(\vec{x},t) = \\
&\quad = -\boldsymbol{S}^{Es}_{jklm}\boldsymbol{\varepsilon}^*_{lm,k}(\vec{x},t) \quad \text{en } R_k
\end{aligned} \tag{2}$$

$$u_j(\vec{x},0) = u_j^0 \;/\; u_j^0 \in (\boldsymbol{H}_0^1(D_k))^3 \;;\; \partial_t u_j(\vec{x},0) = h_j^0 \;/\; h_j^0 \in (\boldsymbol{L}^2(D_k))^3 \tag{3}$$

$$l_1, \rho, \mu, \lambda, \bar{\mu}, \bar{\lambda}, \alpha_{1,2}, \beta, \tau \in R_0^+ \tag{4}$$

Distribución de campos de micro-temperaturas (operador de Stokes)

$$\begin{aligned}
&b\partial_t w_j(\vec{x},t) - k_1\hat{\nabla}^2 w_j(\vec{x},t) - k_2\hat{\nabla}(\hat{\nabla}w_k(\vec{x},t)) + \beta\boldsymbol{M}^E_{jk}(1-l_1^2\hat{\nabla}^2)(1+\tau\partial_t)T_{,k}(\vec{x},t) + \\
&\quad + k_3 w_j(\vec{x},t) = 0 \quad \text{en } R_k
\end{aligned} \tag{5}$$

$$w_j(\vec{x},0) = w_j^0 \;/\; w_j^0 \in (\boldsymbol{H}_0^1(D_k))^3 \;;\; b, k_{1,2,3}, \beta, \tau \in R_0^+ \tag{6}$$

Distribución de presiones de poro

$$\begin{aligned}
&\theta_a\partial_t p_a(\vec{x},t) - (\boldsymbol{K}^a_{jk}p_{a,j}(\vec{x},t))_{,k} - (\tilde{\boldsymbol{K}}^a_{jk}\dot{p}_{a,j}(\vec{x},t))_{,k} + \bar{\alpha}_1\boldsymbol{M}^a_{jk}(1-l_1^2\hat{\nabla}^2)\dot{u}_{j,k}(\vec{x},t) + \\
&\quad + \bar{\theta}_a\partial_t\mathcal{H}(t,T,p_b) = 0 \quad \text{en } R_k
\end{aligned} \tag{7}$$

$$p_a(\vec{x},0) = p_a^0 \;/\; p_a^0 \in H_0^1(D_k) \tag{8}$$

$$\begin{aligned}
&\theta_b\partial_t p_b(\vec{x},t) - (\boldsymbol{K}^b_{jk}p_{b,j}(\vec{x},t))_{,k} - (\tilde{\boldsymbol{K}}^b_{jk}\dot{p}_{b,j}(\vec{x},t))_{,k} + \bar{\alpha}_2\boldsymbol{M}^b_{jk}(1-l_1^2\hat{\nabla}^2)\dot{u}_{j,k}(\vec{x},t) + \\
&\quad + \bar{\theta}_b\partial_t\mathcal{H}(t,T,p_a) = 0 \quad \text{en } R_k
\end{aligned} \tag{9}$$

$$p_b(\vec{x},0) = p_b^0 \;/\; p_b^0 \in H_0^1(D_k) \tag{10}$$

Distribución de temperaturas según Green-Naghdi tipo II

$$\begin{aligned}
&\rho c_v\tau\partial_t^2 T(\vec{x},t) - (\boldsymbol{K}^E_{jk}T_{,j}(\vec{x},t))_{,k} - (\tilde{\boldsymbol{K}}^E_{jk}\dot{T}_{b,j}(\vec{x},t))_{,k} + \rho c_v\partial_t T(\vec{x},t) - \\
&\quad - k_0 w_{j,j}(\vec{x},t) + T_0\beta\boldsymbol{M}^E_{jk}(1-l_1^2\hat{\nabla}^2)(1+\tau\partial_t)\dot{u}_{j,k}(x,t) + \xi_1\partial_t\mathcal{H}(t,p_b,p_a) = 0
\end{aligned} \tag{11}$$

$$T(\vec{x},0) = T_0 \;/\; T_0 \in H_0^1(D_k) \;;\; \partial_t T(\vec{x},0) = q_0 \;/\; q_0 \in L^2(D_k) \tag{12}$$

Condiciones de borde mixtas

$$
\begin{aligned}
&(-\mu\hat{n}_k u_{j,k}(\vec{x},t) + l_1^2\hat{n}_k\hat{\nabla}^2 u_{j,k}(\vec{x},t))\Big|_{\partial\Gamma_1} - \\
&- (-\bar{\mu}\hat{n}_k\dot{u}_{j,k}(\vec{x},t) - \bar{\mu}l_1^2\hat{n}_k\hat{\nabla}^2\dot{u}_{j,k}(\vec{x},t))\Big|_{\partial\Gamma_1} - (\mu+\lambda)\hat{n}_j((1-l_1^2\hat{\nabla}^2)u_{k,k}(\vec{x},t))\Big|_{\partial\Gamma_1} - \\
&- (\bar{\mu}+\bar{\lambda})\hat{n}_j((1-l_1^2\hat{\nabla}^2)\dot{u}_{k,k}(\vec{x},t))\Big|_{\partial\Gamma_1} + \\
&+ (\alpha_1\boldsymbol{M}^a_{jk}(1-l_1^2\hat{\nabla}^2)p_a(\vec{x},t) + \alpha_2\boldsymbol{M}^b_{jk}(1-l_1^2\hat{\nabla}^2)p_b(\vec{x},t))\hat{n}_k\Big|_{\partial\Gamma_1} + \\
&+ \beta\boldsymbol{M}^E_{jk}(1-l_1^2\hat{\nabla}^2)(1+\partial_t)T(\vec{x},t)\hat{n}_k\Big|_{\partial\Gamma_1} = q^a_j\Big|_{\partial\Gamma_1} \;/\; q^a_j \in (\boldsymbol{L}^2(\partial\Gamma_1))
\end{aligned} \tag{13}
$$

$$
(-k_1 w_{j,k}(\vec{x},t) - k_2(\hat{\nabla}w_l(\vec{x},t)) + \beta\boldsymbol{M}^E_{jk}(1-l_1^2\hat{\nabla}^2)(1+\tau\partial_t)T(\vec{x},t))\hat{n}_k\Big|_{\partial\Gamma_1} = 0 \tag{14}
$$

$$
(-\boldsymbol{K}^a_{jk}p_{a,k}(\vec{x},t) - \tilde{\boldsymbol{K}}^a_{jk}\dot{p}_{b,k}(\vec{x},t) + \bar{\alpha}_1\boldsymbol{M}^a_{jk}(1-l_1^2\hat{\nabla}^2)\dot{u}_k(\vec{x},t))\hat{n}_j\Big|_{\partial\Gamma_1} = 0 \tag{15}
$$

$$
(-\boldsymbol{K}^b_{jk}p_{b,k}(\vec{x},t) - \tilde{\boldsymbol{K}}^b_{jk}\dot{p}_{b,k}(\vec{x},t) + \bar{\alpha}_2\boldsymbol{M}^b_{jk}(1-l_1^2\hat{\nabla}^2)\dot{u}_k(\vec{x},t))\hat{n}_j\Big|_{\partial\Gamma_1} = 0 \tag{16}
$$

$$
(-\boldsymbol{K}^E_{jk}T_{,j}(\vec{x},t) + T_0\bar{\beta}\boldsymbol{M}^E_{jk}(1-l_1^2\hat{\nabla}^2)(1+\tau\partial_t)\dot{u}_j(\vec{x},t))\hat{n}_k\Big|_{\partial\Gamma_1} - k_0 w_k(\vec{x},t)\hat{n}_k\Big|_{\partial\Gamma_1} = 0 \tag{17}
$$

$$
u_j(\vec{x},t)\Big|_{\partial\Gamma_2} = w_j(\vec{x},t)\Big|_{\partial\Gamma_2} = 0\ ;\ p_a(\vec{x},t)\Big|_{\partial\Gamma_2} = p_b(\vec{x},t)\Big|_{\partial\Gamma_2} = 0\ ;\ T(\vec{x},t)\Big|_{\partial\Gamma_2} = 0 \tag{18}
$$

Las constantes que intervienen en el modelo son todas reales positivas, las matrices asociadas a los tensores de segundo orden son simétricas y sus formas cuadráticas propias son definidas positivas. Las funciones $\mathcal{H}$ se definen como funciones de tipo histeretico esencialmente de tipo Preisach (Cacciola et al., 2009) (Mörée y Leijon, 2023)

$\boldsymbol{S}^{Es}_{jklm}$ es el tensor de Eshelby-Mura, que satisface

$$
\boldsymbol{S}^{Es}_{jklm} = \boldsymbol{S}^{Es}_{kjlm} = \boldsymbol{S}^{Es}_{jkml}
$$

$\varepsilon_{lm,k} \in (\boldsymbol{L}^2(R_k))^{3\times3}$: Deformaciones residuales.

1.1. Definición de las funciones de Green

$$
-\partial_t g_a(\Delta\vec{x},\Delta t) - (\boldsymbol{K}^a_{jk}g_{a,k}(\Delta\vec{x},\Delta t))_{,j} - (\tilde{\boldsymbol{K}}^a_{jk}\dot{g}_{a,k}(\Delta\vec{x},\Delta t))_{,j} = \delta(\Delta\vec{x},\Delta t) \tag{19}
$$

$$
-\partial_t g_b(\Delta\vec{x},\Delta t) - (\boldsymbol{K}^b_{jk}g_{b,k}(\Delta\vec{x},\Delta t))_{,j} - (\tilde{\boldsymbol{K}}^b_{jk}\dot{g}_{b,k}(\Delta\vec{x},\Delta t))_{,j} = \delta(\Delta\vec{x},\Delta t) \tag{20}
$$

$$
\rho c_v\tau\partial_t^2 g_T(\Delta\vec{x},\Delta t) - (\boldsymbol{K}^E_{jk}g_T(\Delta\vec{x},\Delta t)_{,j})_{,k} + \rho c_v\tau\partial_t g_T(\Delta\vec{x},\Delta t) = \delta(\Delta\vec{x},\Delta t) \tag{21}
$$

$$
g_T(\Delta\vec{x},0) = \partial_t g_T(\Delta\vec{x},0) \tag{22}
$$

$$
\begin{aligned}
&\rho\partial_t^2 g_{jk}(\Delta\vec{x},\Delta t) - \mu(1-l_1^2\hat{\nabla}^2)\hat{\nabla}^2 g_{jk}(\Delta\vec{x},\Delta t) - \\
&- (\mu+\lambda)((1-l_1^2\hat{\nabla}^2)g_{jn,n}(\Delta\vec{x},\Delta t))_{,k} - \bar{\mu}(1-l_1^2\hat{\nabla}^2)\hat{\nabla}^2\dot{g}_{jk}(\Delta\vec{x},\Delta t) - \\
&- (\bar{\mu}+\bar{\lambda})((1-l_1^2\hat{\nabla}^2)\dot{g}_{jn,n}(\Delta\vec{x},\Delta t))_{,k} = \delta_{jk}\delta(\Delta\vec{x},\Delta t)
\end{aligned} \tag{23}
$$

$$
b\partial_t G_{jk}(\Delta\vec{x},\Delta t) - k_1\hat{\nabla}^2 G_{jk}(\Delta\vec{x},\Delta t) - k_2(G_{kn,n}(\Delta\vec{x},\Delta t))_{,j} + k_3 G_{jk}(\Delta\vec{x},\Delta t) = 0 \tag{24}
$$

$$
g_a(\Delta\vec{x},-t_f) = g_b(\Delta\vec{x},-t_f) = 0\ ;\ G_{jk}(\Delta\vec{x},-t_f) = g_{jk}(\Delta\vec{x},0) = \partial_t g_{jk}(\Delta\vec{x},0) = 0 \tag{25}
$$

Condiciones de borde

$$
\begin{aligned}
&(-\mu\hat{n}_l g_{jk,l}(\Delta\vec{x},\Delta t)+l_1^2\hat{n}_l\hat{\nabla}^2 g_{jk,l}(\Delta\vec{x},\Delta t))\Big|_{\partial\Gamma_1}-\\
&\quad-(\mu+\lambda)\hat{n}_j((1-l_1^2\hat{\nabla}^2)g_{kl,l}(\Delta\vec{x},\Delta t))\Big|_{\partial\Gamma_1}+\\
&\quad+(-\bar{\mu}\hat{n}_l\dot{g}_{jk,l}(\Delta\vec{x},\Delta t)+\bar{\mu}l_1^2\hat{n}_l\hat{\nabla}^2\dot{g}_{jk,l}(\Delta\vec{x},\Delta t))\Big|_{\partial\Gamma_1}-\\
&\quad-(\bar{\mu}+\bar{\lambda})\hat{n}_j((1-l_1^2\hat{\nabla}^2)\dot{g}_{kl,l}(\Delta\vec{x},\Delta t))\Big|_{\partial\Gamma_1}=0
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&(-\boldsymbol{K}^a_{jk}g_{a,k}(\Delta\vec{x},\Delta t)-\tilde{\boldsymbol{K}}^a_{jk}\dot{g}_{a,k}(\Delta\vec{x},\Delta t))\hat{n}_k\Big|_{\partial\Gamma_1}=\\
&\quad=(-\boldsymbol{K}^b_{jk}g_{b,k}(\Delta\vec{x},\Delta t)-\tilde{\boldsymbol{K}}^b_{jk}\dot{g}_{b,k}(\Delta\vec{x},\Delta t))\hat{n}_k\Big|_{\partial\Gamma_1}=0
\end{aligned}
\tag{27}
$$

$$
k_1 G_{jk,k}(\Delta\vec{x},\Delta t)\hat{n}_k-k_2(G_{kn,n}(\Delta\vec{x},\Delta t))\hat{n}_j\Big|_{\partial\Gamma_1}=-\boldsymbol{K}^E_{jk}g_T(\Delta\vec{x},\Delta t)_{,k}\hat{n}_k\Big|_{\partial\Gamma_1}=0 \tag{28}
$$

$$
g_a(\Delta\vec{x},\Delta t)\Big|_{\partial\Gamma_2}=g_b(\Delta\vec{x},\Delta t)\Big|_{\partial\Gamma_2}=0 \tag{29}
$$

$$
G_{jk}(\Delta\vec{x},\Delta t)\Big|_{\partial\Gamma_2}=g_{jk}(\Delta\vec{x},\Delta t)\Big|_{\partial\Gamma_2}=g_T(\Delta\vec{x},\Delta t)\Big|_{\partial\Gamma_2}=0 \tag{30}
$$

1.2. Construcción de las soluciones semi-analíticas

Utilizando el segundo y el tercer teorema de representación de Green-Lagrange podemos construir las representaciones integrales de las soluciones en la forma siguiente:

$$
\begin{aligned}
u_j(\vec{x},t)=&\iiint_{V_k}d^3x'\{g_{jk}(\vec{x}-\vec{x}',t)\partial_t u_k(\vec{x}',0)-u_k(\vec{x}',0)\partial_t g_{jk}(\vec{x}-\vec{x}',t)\}+\\
&+\int_0^t dt'\left\{\iiint_{V_k}d^3x' g_{jk}(\vec{x}-\vec{x}',t-t')\Xi_k(\vec{x}',t',\wp(u_j,p_a,p_b,T))\right\}+\\
&+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1}dS\, g_{jk}(\vec{x}-\vec{x}',t-t')\left\{q^a_j(\vec{x}',t')\Big|_{\partial\Gamma_1}+\wp^s_j(u_j,p_a,p_b,T)\Big|_{\partial\Gamma_1}\right\}\right\}
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
p_a(\vec{x},t)=&\iiint_{V_k}d^3x'\{g_a(\vec{x}-\vec{x}',t)p_a(\vec{x}',0)\}+\\
&+\int_0^t dt'\left\{\iiint_{V_k}d^3x' g_a(\vec{x}-\vec{x}',t-t')\Xi_a(\vec{x}',t',\wp(u_j,p_a,p_b,T))\right\}+\\
&+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1}dS\left\{g_a(\vec{x}-\vec{x}',t-t')\wp^s_a(u_j,p_a,p_b,T)\Big|_{\partial\Gamma_1}\right\}\right\}
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
p_b(\vec{x},t)=&\iiint_{V_k}d^3x'\{g_b(\vec{x}-\vec{x}',t)p_b(\vec{x}',0)\}+\\
&+\int_0^t dt'\left\{\iiint_{V_k}d^3x' g_b(\vec{x}-\vec{x}',t-t')\Xi_b(\vec{x}',t',\wp(u_j,p_a,p_b,T))\right\}+\\
&+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1}dS\left\{g_b(\vec{x}-\vec{x}',t-t')\wp^s_b(u_j,p_a,p_b,T)\Big|_{\partial\Gamma_1}\right\}\right\}
\end{aligned}
\tag{33}
$$

$$T(\vec{x},t) = \iiint_{V_k} d^3x'\{g_T(\vec{x}-\vec{x}',t)\partial_t T(\vec{x}',0) - T(\vec{x}',0)\partial_t g_T(\vec{x}-\vec{x}',t)\}+$$
$$+\int_0^t dt'\left\{\iiint_{V_k} d^3x' g_T(\vec{x}-\vec{x}',t-t')\Xi_T(\vec{x}',t',\wp_T(u_j,p_a,p_b,T))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS\left\{g_T(\vec{x}-\vec{x}',t-t')\wp_T^s(u_j,p_a,p_b,T)\Big|_{\partial\Gamma_1}\right\}\right\} \quad (34)$$

$$w_j(\vec{x},t) = \iiint_{V_k} d^3x' G_{jk}(\vec{x}-\vec{x}',t)w_k^0(\vec{x}',0)+$$
$$+\int_0^t dt'\left\{\iiint_{V_k} d^3x' G_{jk}(\vec{x}-\vec{x}',t-t')\Xi_k(\vec{x}',t',\wp_T(w_j,T))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS\, G_{jk}(\vec{x}-\vec{x}',t-t')\wp_j^s(w_j,T)\Big|_{\partial\Gamma_1}\right\} \quad (35)$$

1.3. Construcción de los aproximantes de Picard

Los aproximantes de Picard serán:

$$u_j^{(n+1)}(\vec{x},t) \cong K_j^0(u_j^0,w_j^0)+$$
$$+\int_0^t dt'\left\{\iiint_{V_k} d^3x' g_{jk}(\vec{x}-\vec{x}',t-t')\Xi_k^{(n)}(\vec{x}',t',\wp(u_j^{(n)},p_b^{(n)},p_a^{(n)},T_j^{(n)}))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS g_{jk}(\vec{x}-\vec{x}',t-t')\left\{q_j^a(\vec{x}',t')\Big|_{\partial\Gamma_1} + \wp_j^s(u_j,p_a,p_b,T)\Big|_{\partial\Gamma_1}\right\}\right\} \quad (36)$$

$$p_a^{(n+1)}(\vec{x},t) \cong K_a^0(t,p_a^0)+$$
$$+\int_0^t dt'\left\{\iiint_{V_k} d^3x' g_a(\vec{x}-\vec{x}',t-t')\Xi_a(\vec{x}',t',\wp(u_j^{(n)},p_a^{(n)},p_b^{(n)},T^{(n)}))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS\left\{g_a(\vec{x}-\vec{x}',t-t')\wp_a^s(u_j^{(n)},p_a^{(n)},p_b^{(n)},T^{(n)})\Big|_{\partial\Gamma_1}\right\}\right\} \quad (37)$$

$$p_b^{(n+1)}(\vec{x},t) \cong K_b^0(t,p_b^0)+$$
$$+\int_0^t dt'\left\{\iiint_{V_k} d^3x' g_b(\vec{x}-\vec{x}',t-t')\Xi_b(\vec{x}',t',\wp(u_j^{(n)},p_a^{(n)},p_b^{(n)},T^{(n)}))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS\left\{g_b(\vec{x}-\vec{x}',t-t')\wp_b^s(u_j^{(n)},p_a^{(n)},p_b^{(n)},T^{(n)})\Big|_{\partial\Gamma_1}\right\}\right\} \quad (38)$$

$$T^{(n+1)}(\vec{x},t) \cong K_j^0(T_0,q_0)+$$
$$+\int_0^t dt'\left\{\iiint_{V_k} d^3x' g_T(\vec{x}-\vec{x}',t-t')\Xi_T^{(n)}(\vec{x}',t',\wp(u_j^{(n)},p_a^{(n)},p_b^{(n)},T^{(n)},w_j^{(n)}))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS\left\{g_T(\vec{x}-\vec{x}',t-t')\wp_T^s(u_j^{(n)},p_a^{(n)},p_b^{(n)},T^{(n)},w_j^{(n)})\Big|_{\partial\Gamma_1}\right\}\right\} \quad (39)$$

$$w_j^{(n+1)}(\vec{x},t) \cong K_j^0(w_j^0)+$$
$$\int_0^t dt'\left\{\iiint_{V_k} d^3x' G_{jk}(\vec{x}-\vec{x}',t-t')\Xi_k^{(n)}(\vec{x}',t',\wp(w_j^{(n)},T_j^{(n)}))\right\}+$$
$$+\int_0^t dt'\left\{\oiint_{\partial\Gamma_1} dS\, G_{jk}(\vec{x}-\vec{x}',t-t')\wp_j^s(w_j^{(n)},T^{(n)})\Big|_{\partial\Gamma_1}\right\} \quad (40)$$

Los operadores definidos genéricamente como $\wp(u_j^{(n)}, T^{(n)}), \wp_j^s(u_j^{(n)}, T^{(n)}) \ldots$ etc.deben interpretarse en el sentido que son los términos acoplados que han sido trasladados al miembro de la derecha de cada ecuación y que actúan como forzados externos, todos ellos son acotados, compactos y están definidos en la clausura de los espacios utilizados habitualmente.

En general puede probarse que, para todas las variables descriptoras utilizadas en ambos modelos, dado que $\{u_j\}_{n=1}^{\infty}$ es de Cauchy; entonces

$$\lim_{n\to\infty}\left\{\sup_{u\in R_k}\left\|\sum_{n}^{\infty} q_j\{u_j\}_{n=1}^{\infty} - u_j\right\|\right\} = 0 \Longrightarrow u_j^{(n)} \to u \text{ uniformemente } ; \; q_j \in R_0^+$$

La estrategia computacional utilizada será la de wavelets, descriptos según la siguiente estructura

Base de Haar y formula de cuadratura asociada

$$\{t \to \psi_{n,k}(t) = \psi(2^n t - k)\,;\; n \in \mathbb{N}, 0 \le k < 2^n\} \tag{41}$$

$$\int_{-\infty}^{\infty} dt\, 2^m \psi(2^m t - n)\psi(2^{m_1} t - n_1) = \delta_{m,m_1}\delta_{n,n_1} \tag{42}$$

$$S_N(f) = \frac{(b-a)(d-c)(h-e)}{8N^3} \sum_{k=1}^{2N}\sum_{j=1}^{2N}\sum_{i=1}^{2N} f\{a + (\delta_x/2)(2i-1), c + (\delta_y/2)(2j+1), e + (\delta_z/2)(2k+1)\} \tag{43}$$

Finalmente puede probarse que el error cometido tiene la estructura siguiente

$$d\{\|u_j^* - u_j^{(m)}\|\} \le \frac{\sigma^{m+1}}{1-\sigma} M_0 \quad ; \quad M_0 = \sup_{\vec{x},t\in R_k}\{\|\boldsymbol{F}(\vec{x}, t, u_j^*)\|\} \tag{44}$$

Las representaciones integrales se expresarían en la forma siguiente

$$\begin{aligned}
\bar{u}_j^{(m)}(\vec{x}, t) = \hat{\bar{u}}_j^{0}(\vec{x}, t) + \\
+ \delta_x\delta_y\delta_z \sum_{k=1}^{2N}\sum_{j=1}^{2N}\sum_{i=1}^{2N}\Bigg\{\int_0^t dt' H_{jk}\{x, y, z, t-t', a + (\delta_x/2)(2i-1), c + (\delta_y/2)(2j+1),\\
e + (\delta_z/2)(2k+1)\}\, \psi_k\, \{u_j^{(m+1)}\{a + (\delta_x/2)(2i-1), c + (\delta_y/2)(2j+1),\\
e + (\delta_z/2)(2k+1)\}\}\Bigg\} + \\
+ \delta_x\delta_y\delta_z \sum_{k=1}^{2N}\sum_{j=1}^{2N}\sum_{i=1}^{2N}\Bigg\{\int_0^t dt' \bar{H}_{jk}\{x, y, 0, t-t', a + (\delta_x/2)(2i-1), c + (\delta_y/2)(2j+1),\\
e + (\delta_z/2)(2k+1)\}\, \bar{\psi}_k\, \{u_j^{(m+1)}\{a + (\delta_x/2)(2i-1), c + (\delta_y/2)(2j+1),\\
e + (\delta_z/2)(2k+1)\}\}\Bigg\}
\end{aligned} \tag{45}$$

2. CONCLUSIONES

En el presente trabajo se analizó y resolvió semi-analíticamente, un modelo de tipo poro-termo-visco-elastodinámico con micro-temperaturas desde la perspectiva de D. Iesan, y R.

Quintanilla, se construyeron las soluciones semi-analíticas y los aproximantes de Picard, se definieron funciones de Green asociadas a cada operador, y luego utilizando los teoremas de Green Lagrange, se construyeron las ya mencionadas representaciones integrales de las soluciones, el sistema de ecuaciones integrales puede resolverse por múltiples, vías siendo una de las exploradas por este grupo la de los wavelets.

REFERENCIAS

Aouadi M., Passarella F., y Tibullo V. Exponential stability in Mindlin's Form II gradient thermoelasticity with microtemperatures of type III. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 476(2241):20200459, 2020.

Cacciola P., Biondi G., y Cascone E. Site Response Analysis using the Preisach Formalism. 2009.

Casas P.S. y Quintanilla R. Exponential stability in thermoelasticity with microtemperatures. *International Journal of Engineering Science*, 43:33–47, 2005.

Ciarletta M. y Ieşan D. *Non-Classical Elastic Solids*. Longman Scientific & Technical, 1993.

Eringen A.C. *Microcontinuum Field Theories*. Springer New York, 1999.

Eringen A.C. y Suhubi E.S. Nonlinear theory of simple micro-elastic solids—I. *International Journal of Engineering Science*, 2(2):189–203, 1964. ISSN 0020-7225.

Forest S. y Aifantis E.C. Some links between recent gradient thermo-elasto-plasticity theories and the thermomechanics of generalized continua. *International Journal of Solids and Structures*, 47(25):3367–3376, 2010.

Green A.E. Micro-materials and multipolar continuum mechanics. *International Journal of Engineering Science*, 3(5):533–537, 1965. ISSN 0020-7225.

Green A.E. y Rivlin R.S. Multipolar continuum mechanics. *Arch. Rational Mech. Anal.*, 17(2):113–147, 1964.

Grot R.A. Thermodynamics of a continuum with microstructure. *International Journal of Engineering Science*, 7(8):801–814, 1969. ISSN 0020-7225.

Ieşan D. *Thermoelastic Models of Continua*. Solid Mechanics and Its Applications. Springer Dordrecht, 2004.

Ieşan D. On a Theory of Micromorphic Elastic Solids with Microtempetatures. *Journal of Thermal Stresses*, 24, 2001.

Ieşan D. Thermoelasticity of bodies with microstructure and microtemperatures. *International Journal of Solids and Structures*, 44(25):8648–8662, 2007. ISSN 0020-7683.

Ieşan D. Strain gradient theory of porous solids with initial stresses and initial heat flux. *Discrete and Continuous Dynamical Systems - B*, 19(7):2169–2187, 2014.

Ieşan D. y Quintanilla R. On a Theory of Thermoelastic Materials with a Double Porosity Structure. *Journal of Thermal Stresses*, 37(9):1017–1036, 2014.

Jou D. y Cimmelli V. Constitutive equations for heat conduction in nanosystems and nonequilibrium processes: an overview. *Communications in Applied and Industrial Mathematics*, 7, 2016.

Kunin I.A. *Elastic Media with Microstructure*, volumen I,II de *Springer Series in Solid-State Sciences*. Springer Berlin, Heidelberg, 1983.

Liu W., Saanouni K., Forest S., y Hu P. The Micromorphic Approach to Generalized Heat Equations. *Journal of Non Equilibrium Thermodynamics*, 42:327–357, 2017.

Mindlin R.D. Micro-structure in linear elasticity. *Arch. Rational Mech. Anal.*, 16(1):51–78, 1964.

Mörée G. y Leijon M. Review of Play and Preisach Models for Hysteresis in Magnetic Materials. *Materials*, 16:2422, 2023.

Nika G. A gradient system for a higher-gradient generalization of Fourier's law of heat conduction. *Modern Physics Letters B*, 37(11):2350011, 2023.

Pamplona P.X., Muñoz Rivera J.E., y Quintanilla R. Analyticity in porous-thermoelasticity with microtemperatures. *Journal of Mathematical Analysis and Applications*, 394(2):645–655, 2012. ISSN 0022-247X.

Pamplona P.X., Rivera J.E.M., y Quintanilla R. On uniqueness and analyticity in thermoviscoelastic solids with voids. *Journal of Applied Analysis & Computation*, 1(2):251–266, 2011.

Quintanilla R. Uniqueness in thermoelasticity of porous media with microtemperatures. *Archives of Mechanics*, 61(5):371–382, 2009.

Scalia A. y Svanadze M. On the Representations of Solutions of the Theory of Thermoelasticity with Microtemperatures. *Journal of Thermal Stresses*, 29(9):849–863, 2006.

Říha P. On the microcontinuum model of heat conduction in materials with inner structure. *International Journal of Engineering Science*, 14(6):529–535, 1976.