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GENETIC ALGORITHMS AND MODAL SENSITIVITY FOR DAMAGE DETECTION ON PORTAL FRAMES

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Abstract. The use of natural frequency as a diagnostic parameter in structural damage detection and vibration monitoring, has been discussed in the last decade for many authors in several works. The large use of natural frequencies and mode shapes as sensitive indicators of structural integrity is due to the fact of the ease in measuring these modal parameters experimentally. Many types of methods are employed in the state of art in damage detection and location using low frequency data. There are two classes of methods that are investigated here: the first is based on frequency sensitivity to the damage and the second one is based on optimization techniques and parametric modeling. In this work a genetic algorithm and a modal sensitivity method are used to identify and evaluate damage cases in a numerical finite element model of a portal frame. The results of the identification and the evaluation of the damage in both methods were similar and could be compared.

1 INTRODUCTION

There are several ways to deal with the structural integrity evaluation. There is a consensus that it is necessary to establish inspection procedures which systematically evaluate the structural integrity. The main techniques may be separated as nondestructive and destructive testing. In particular, detection techniques based on non-destructive testing (NDT) has been preferable due to low cost and operational aspects related to the use of the analyzed structure. There are methods for damage detection based on sensitivity and statistical parameters. Some methods are based on dynamic characteristics of structures such as natural frequencies, dynamic mode shapes, and structural damping. These methods have take advantage of the present-day development of modal analysis techniques with accurate measurements of modal parameters. When damage event occurs, the structural dynamic characteristics are changed and may be used as indicator of damage.

2 BIBLIOGRAPHICAL REVIEW

A great number of nondestructive evaluation techniques have been developed based on the changes of the dynamic parameters. Cawley and Adams (1979) have used the changes in the natural frequency together with a Finite Element models to locate the damage.

Messina (1996) has proposed an uncertainty approach for damage detection that was later extended by Contursi and Messina (1998) to identify the damage extent in several sites. The data validation was accomplished through numerical tests free of noise. This approach, however, can involve a significant computational effort when dealing with large structures with many degrees of freedom.

Iturrioz et al. (1999) used a different index: The Coordinate Modal Assurance Criterion – COMAC. This index uses the numerical and experimental vibration modes to determinate the magnitude and the damage position. Two structures using a finite element model analysis were chosen to validate the method. The first structure analyzed was a simple supported beam where several damage sceneries and the damage was simulated by the reduction of the Young modulus. The second structure was a reinforced concrete part of a soccer stadium. For the first structure the COMAC index results were satisfactory determining the damage position. For the second structure the results were acceptable, if considered the few discretized elements and the small stiffness reduction.

Ostachowicz, Krawczuk and Cartmell (1996) performed a series of tests with genetic algorithms used as a maximization tool, to detect delaminatioin sites on a cantilever composite beam. The DLAC index was used as objective function and binary “gray code” with 33 bits, 11 of them for each variable to be optimized, damage location on two sites and the damaged layer depth. According to the authors the results are promising, particularly because the number of calculations needed for failure detection is much less than those required for classical search algorithms.

Sazonov, Klinkhachorn and Hatabe (2002) used the genetic algorithm to produce a sufficiently optimized amplitude characteristic filter to extract damage information from strain energy mode shapes. A finite element model was used to produce training data set with the known location. The filter amplitude characteristic was encoded as a genetic algorithm string where the pass coefficient for each harmonic of its discrete Fourier Transform representation was a number between 0 and 1 in an 8 bit “gray code” scheme. The genetic optimization was performed based on the minimization of the signal–to-distortion ratio. According to the authors the results obtained from the GA has confirmed the theoretical predictions and allowed improvements in the method’s sensitivity to damages of lower magnitude.

Some works shows the modals parameters behavior with damage increment only in an experimentally way. Çam et al. (2004) used a cantilever experimental beam to obtain information about the damage deep and localization. Two types of experimental tests were made: the first one varying, in the same place, only the damage deep and the second one varying, with the same deep, the damage site. A metal ball is dropped onto the beam from a constant height in order to excite vibrations. The artificial transversal damage increment was made in only one of the longitudinal sides of the cantilever beam. Further the Fast Fourier Transform of the vibration signals was calculated and the results show that the amplitude of vibration increases as the damage depth increases. For small damages the undamaged frequency amplitudes are bigger that the damages frequency amplitudes. With the damage increase the damage frequency amplitude gets bigger than the undamaged frequency amplitude.

3 BRIEF REVIEW OF THE USED DAMAGE DETECTION METHODS

3.1 Damage Detection by Modal Sensitivity Analysis

The dynamic behavior of linear elastic systems with n degrees of freedom can be described by equation 1:

$$\mathbf{M}\ddot{y}(t)+\mathbf{C}\dot{y}(t)+\mathbf{K}y(t)=\mathbf{F}(t) \tag{1}$$

where **M**, **K,** and **C** are the n x n mass, stiffness and damping matrix, $\mathbf{F}(t)$ means the external force vector, and $\ddot{y}, \dot{y}$ and y means the acceleration, velocity and displacements vectors respectively. When the system has small damping ratio, the frequencies and mode shapes may be obtained through an eigenvalue problem as described by equation 2:

$$\left(\mathbf{K}-\mathbf{\Omega M}\right)\mathbf{\Phi}=0 \tag{2}$$

where $\mathbf{\Omega}$ is a diagonal n x n matrix that contains the squares of natural frequencies ω_i^2 and $\mathbf{\Phi}$ is an n x n matrix that contains the respective vibration mode shapes, where the i-th column corresponds to the group of displacements for the i-th vibration mode shape $\mathbf{\Phi}_i$. If a small variation is applied to equation 2 one has equation 3, as follows:

$$(\mathbf{K}+\delta\mathbf{K})\ (\mathbf{\Phi}+\delta\mathbf{\Phi})=(\mathbf{\Omega}+\delta\,\mathbf{\Omega})(\mathbf{M}+\delta\mathbf{M})(\mathbf{\Phi}+\delta\mathbf{\Phi}) \tag{3}$$

Frequently, the damage occurrence generates on one side significant reduction on stiffness and on the other a small mass reduction. In the following equation (see equation 4) the effect of the mass reduction is ignored as well as second order variational terms (δ^2):

$$(\mathbf{K}-\mathbf{\Omega M})\ \mathbf{\Phi}+\delta\mathbf{K\Phi}-\delta\,\mathbf{\Omega M\Phi}+(\mathbf{K}-\mathbf{\Omega M})\ \delta\mathbf{\Phi}=0 \tag{4}$$

Hence, after some algebraic operations, yields (see equation 5):

$$\delta\mathbf{\Omega}=\frac{\mathbf{\Phi}^T\delta\mathbf{K\Phi}}{\mathbf{\Phi}^T\mathbf{M\Phi}} \tag{5}$$

and particularly, for a single mode shape $\mathbf{\Phi}_i$, yields (see equation 6):

$$\delta\omega_i^2=\frac{\mathbf{\Phi}_i^{\ T}\delta\mathbf{K\Phi}_i}{\mathbf{\Phi}_i^{\ T}\mathbf{M\Phi}_i} \tag{6}$$

which represents the changes in the i-th natural frequency as consequence of a small variation of the global stiffness matrix. Through the adoption of a finite element model that represents the structural system it is possible to obtain a relationship between the damage at an individual element and the variations in the global natural frequencies. Thus, the global stiffness **K** matrix and the mode shape vector $\mathbf{\Phi}_i$ can be decomposed as (see equation 7):

$$\mathbf{\Phi}_i^T \mathbf{K} \mathbf{\Phi}_i = \sum_{e=1}^{n} u(\mathbf{\Phi}_i)_e^T \mathbf{k}_e u(\mathbf{\Phi}_i)_e \tag{7}$$

where $\mathbf{k}_e$ and $u(\mathbf{\Phi}_i)_e$ are the element's stiffness matrix and the corresponding displacements of the i-th mode shape of the e-th element. For example, the above mentioned matrix and vectors for a planar bar element with six degrees of freedom (see Figure 1) are indicated by equation 8:

$$u(\mathbf{\Phi}_i)_e = \{u_1, u_2, \ldots, u_6\}^T \qquad \mathbf{k}_e = \begin{bmatrix} k_{11} \ldots k_{16} \\ \vdots \quad \ddots \quad \vdots \\ k_{61} \ldots k_{66} \end{bmatrix} \tag{8}$$

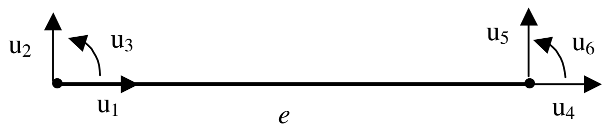

Figure 1: Beam Finite Element with six degrees of freedom.

Following the same procedure for $\delta \mathbf{K}$, one obtains (see equation 9):

$$\mathbf{\Phi}_i^T \delta \mathbf{K} \mathbf{\Phi}_i = \sum_{e=1}^{n} u(\mathbf{\Phi}_i)_e^T \delta \mathbf{k}_e u(\mathbf{\Phi}_i)_e \tag{9}$$

and substituting this equation in the equation 6, yields (see equation 10):

$$\delta \omega_i^2 = \frac{\sum_{e=1}^{n} u(\mathbf{\Phi}_i)_e^T \delta \mathbf{k}_e u(\mathbf{\Phi}_i)_e}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} \tag{10}$$

which represents the changes in the i-th natural frequency as consequence of a small variation of the element local stiffness. Particularizing the damage for an element m (see equation 11):

$$\delta \omega_{m,i}^2 = \frac{u(\mathbf{\Phi}_i)_m^T \delta \mathbf{k}_m u(\mathbf{\Phi}_i)_m}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} \tag{11}$$

this represents the changes in the i-th natural frequency as consequence of a small variation of the local stiffness of the element m. Assuming that there is a direct relationship between the variation of the element stiffness and the damage extent, yields (see equation 12):

$$\delta \mathbf{k}_m = \delta D_m \mathbf{k}_m \tag{12}$$

where δD_m is a real number representing the damage extent. Hence, we can substitute this last equation on the equation 11 and obtain (see equation 13):

$$\delta\omega_{m,i}^2 = \frac{\delta D_m \; u(\mathbf{\Phi}_i)_m^T \mathbf{k}_m u(\mathbf{\Phi}_i)_m}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} \tag{13}$$

which represents the variation of natural frequencies of the structure as function of the location and damage extent. Normalizing this last equation with regard to the largest element local frequencies variations in the structure (at least for the factor $\delta D_m / \delta D_n$ since in this point we are not concerned on the absolute value of the damage but just on its location), one obtains (see equation 14):

$$\delta\omega_{m,i}^2 / \delta\omega_{n,j}^2 = \frac{u(\mathbf{\Phi}_i)_m^T \mathbf{k}_m u(\mathbf{\Phi}_i)_m}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} / \frac{u(\mathbf{\Phi}_j)_n^T \mathbf{k}_n u(\mathbf{\Phi}_j)_n}{\mathbf{\Phi}_j^T \mathbf{M} \mathbf{\Phi}_j} \tag{14}$$

which is used to evaluate the location of the damage through the evaluation of the damage index location (*Jm*). This index estimates, for all elements, the inverse of the standard deviation ($1/\sigma_m$) of the differences between numerical and experimental values of the changes in frequency, as it indicates by equation 15. Index values close to 1 will indicate the matching of the numerical and experimental patterns and therefore the presence of the damage on those elements.

$$\sigma_m = (1/n)\sum_{i=1}^{s}\left[(\Delta\omega_i^2/\Delta\omega_f^2) - (\delta\omega_{m,i}^2/\delta\omega_{m,j}^2)\right]^2 \qquad \text{and} \qquad J_m = 1/\sigma_m.[\sum_{e=1}^{n} 1/\sigma_i] \tag{15}$$

A simple approach to evaluate the damage extent without using the structural mode shapes is to use the previously evaluated damage location index. Once the elements with possible damage are located, the index can be used jointly with an inverse analysis (for example, with the singular value decomposition method), since a fewer number of frequencies can be measured. This way, the contributions for the square variations of natural frequencies of an element ${}_m\delta\varpi_i^2$ are beforehand weighted by the damage location index as indicated by equation 16.

$$\begin{Bmatrix} \Delta\omega_1^2 \\ \Delta\omega_2^2 \\ \vdots \\ \Delta\omega_i^2 \end{Bmatrix} = \begin{bmatrix} \delta\omega_{1,1}^2 & \delta\omega_{2,1}^2 & \cdots & \delta\omega_{n,1}^2 \\ \delta\omega_{1,2}^2 & \delta\omega_{2,2}^2 & \cdots & \delta\omega_{n,2}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \delta\omega_{1,i}^2 & \delta\omega_{2,i}^2 & \cdots & \delta\omega_{n,i}^2 \end{bmatrix} . \begin{Bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_n \end{Bmatrix} . \begin{Bmatrix} \delta D_1 \\ \delta D_2 \\ \vdots \\ \delta D_n \end{Bmatrix} \tag{16}$$

3.2 Damage Detection by Genetic Agorithm

The Genetic Algorithms (GA) are optimization techniques based on the Darwin's Theory of evolution and survival of the fittest. The Darwin's Theory of Natural Selection (1859) says that "… any being, if it varies slightly in any manner profitable to itself, will have better chance of surviving…" GA simulates the evolutionary process numerically. They represent the parameters in a given problem by encoding them into a string. As in genetics, genes are constituted by chromosomes. Similarly, in simple GA, encoded strings are composed of bits. A string of bits can be decoded to the respective problem parameter value and the total evaluation of the string of bits for and individual may be weighted following some fitness function representing the phenotype to that string of bits.

A simple genetic algorithm consists of three basic operations, these being reproduction, crossover and mutation. The algorithm begins with a population of individuals each of them representing a possible solution of the problem. The individuals, as in nature, perform the three basic operations and evolve in generations where prevails the Darwin's Theory, or in other words, a population of individuals more adapted emerges as natural selection.

At the reproduction level, the evaluation of the objective (fitness) function indicates which individuals will have more chances to procreate and generates a larger offspring.

In the genetic operations the genes of pair of individuals are exchanged and as in nature this may be performed by several ways being called by crossover.

The basic differences between conventional techniques and the genetic algorithm (GA) can be summarized as follows:

- GA operates on a coded form of the task parameters instead of the parameters itself;
- The GA works with a population which represents numerical values of a particular variable;
- Differently of most of optimization algorithms which requires objective functions evaluations and gradients, GA only requires the use of the objective function;
- Only probabilistic rules of natural selection are used with GA.

The binary representation has a historical importance due to first uses by Holland (1975). When working with binary coded genetic algorithms, each of the real parameters b_i to be optimized is translated to binary codes by the following equation (see equation 17):

$$s = bin_n\{round(2^n - 1)\frac{[b_i(k) - P_{\min}(k)]}{[P_{\max}(k) - P_{\min}(k)]}\} \tag{17}$$

where bin_n indicates a binary translation to a string s of n bits, n means the number of bits, $P(k)$ means the range of maximum and minimum values allowed for each variable.

To transform the binary codes to real values the following equation (see equation 18) is used in the sequence:

$$b_i(k) = P(k)_{\min} + bin^{-1}(s)\frac{P(k)_{\max} - P(k)_{\min}}{2^n - 1} \tag{18}$$

where $\mathrm{bin}^{-1}(\mathrm{s})$ means the translation of the binary coded values to respective real ones.

It could be noted that with this formulation it is implicit that the mapping has a resolution of $[P(k)_{\max} - P(k)_{\min}]/(2^n - 1)$. This restricts the search space of the real parameters to discrete values which could induce to local maxima/minima.

This could be outlined by using real coded genetic algorithms. This approach assumes real values to each variable. The main differences are found on the crossover operator. There are several methods to deal with the real coded genetic algorithms crossover such as flat crossover, simple crossover, arithmetical crossover, Wright's crossover, linear BGA crossover, etc. In this paper the BLX-α is used because it uses an initial exploration of the parameters field followed by an exploitation phase to improve resolution. It may be described by (see equation 19):

$$\begin{aligned} &\Delta = max[b_i(k), b_{i+1}(k)] - min[b_i(k), b_{i+1}(k)] \\ &b(k) = random\{min[b_i(k), b_{i+1}(k)] - \alpha\Delta,\ max[b_i(k), b_{i+1}(k)] + \alpha\Delta\} \end{aligned} \tag{19}$$

where, i and i+1 are referred to two parents' chromosomes, α means a decreasing exploration parameter and *random* means a random number in the respective interval. Figure 2 summarizes the main steps followed by a real coded basic genetic algorithm to maximize functions.

```
Initialize Time t=0
Initialize Population size "m", Probability of Mutation "Pm", Probability of Crossover
"Pc", Number of Individual cromossomes "nc", allowed limits for each chromosome,
"Pmax(nc), Pmin(nc)".
Generate Initial Population B_0 = (b_{1,0}, b_{2,0}, ..., b_{m,0})
While Stopping Condition is not fullfiled
        "Proportional Selection"
        Loop i=1 to m
            x=random(0,1)
            k=1
            While k<m and x< sum_{j=1}^{k} f(b_{j,t}) / sum_{j=1}^{m} f(b_{j,t})
                k=k+1
                b_{i,t+1} = b_{k,t}
            End While
        End Loop
        "One Point CrossOver"
        Loop i=1 to m-1 step 2
           If random (0,1)<Pc then
                alpha = 0.5
                Delta = max[b_{i,t}(k), b_{i+1,t}(k)] - min[b_{i,t}(k), b_{i+1,t}(k)]
                b_{i,t+1}(k) = random{min[b_{i,t}(k), b_{i+1,t}(k)] - alpha Delta, max[b_{i,t}(k), b_{i+1,t}(k)] + alpha Delta}
                b_{i+1,t+1}(k) = random{min[b_{i,t}(k), b_{i+1,t}(k)] - alpha Delta, max[b_{i,t}(k), b_{i+1,t}(k)] + alpha Delta}
           End If
        End Loop
        "Mutation of Offsprings"
        Loop i=1 to m
           If random (0,1)<Pm then
                k=random(0,1)*nc
                b_{i,t+1}(k) = random{P_max(k), P_min(k)}
           End if
        End Loop
```

Figure 2: Real coded genetic algorithm used.

The index used as the objective function to be maximized in the optimization process carried out by genetic algorithms is demonstrated in equation 20. The same index were used with others examples in the work of Silva (2006).

$$f(\delta \mathbf{D}) = \frac{1}{1 + \sum_{i=1}^{N} \left(\frac{\delta\omega_i(\delta \mathbf{D})}{\max(\delta\omega_i)} - \frac{\Delta\omega_i}{\max(\Delta\omega_i)} \right)^2} \tag{20}$$

where N is the number of frequencies used, $\delta\omega$ is the theoretical variation of eigenvalues of a parametric model, $\Delta\omega$ is the experimentally variations and $\delta\mathbf{D}$ is the vector of multiple damages in the parametric model.

4 PORTAL FRAME MODEL EXAMPLE

The same portal frame model that is proposed in the work of Veizaga (1993) is also used in this work to show the robustness of the both proposed methods. The portal has a rectangular cross sectional area with height h=0,24m, width b=0,14m and lengths of L=2,4m and H=1,6m. The material has a Young Modulus of E=2,5x10^{10} N/m^2 and a material density of ρ= 2,5x10^3 kg/m^3. Figure 3 shows a sketch of the structural dimensions and the numbering of the discretized elements used in the finite element analysis. It was used 3D beam elements with six degrees of freedom per node, restrained in the plane of the structure, resulting in three degrees of freedom per node (two translational and one rotational).

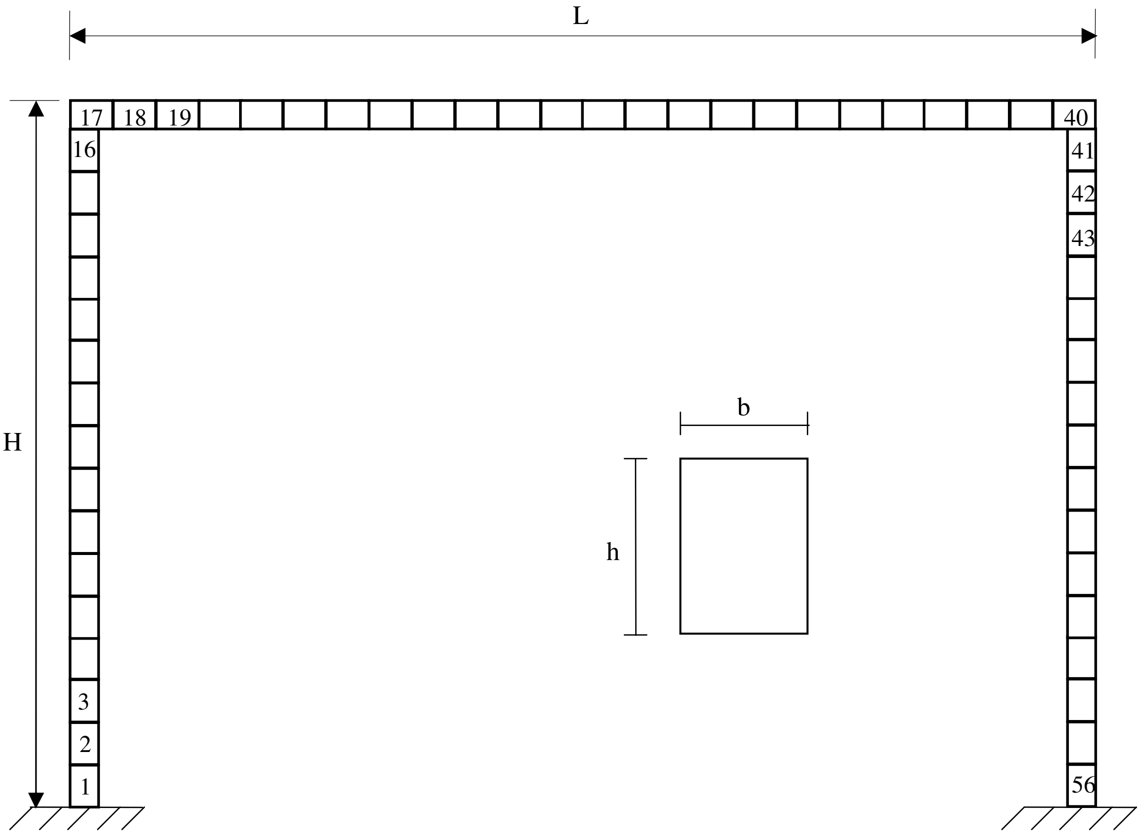

Figure 3: Portal frame model with cross sectional area and dimensions.

In order to investigate the methods some scenarios were numerically created with the inertial property Iz reduced in several quantities. In both methods were used five natural frequencies as set up parameter. The eight numerical simulated cases are shown in Table 1.

Some preliminary tests were made to decide the genetic algorithm set up parameters. The initial chosen parameters were based on others authors work. The final set up parameters used in this work are shown in Table 2.

Damage Element	Damage Amount (%)
24	5
24	10
24	20
24	30
24	40
7	10
44	10
10, 28 e 52	10

Table 1: Simulated numerical cases for the portal frame model.

Parameters	Value
Population Size	500
Number of Generation	2000
Probability of Crossover	1
Probability of Mutation	0,01
Inferior Limit of Damage in Element	0
Superior Limit of Damage in Element	0,6
Elitism Percentile	0,1
Natural Frequencies Used	5

Table 2: Genetic algorithm set up parameters.

The obtained results for the two damage detection proposed methods are demonstrated from Figure 4 to Figure 15. Only the results for Modal Sensitivity Analysis were placed in two distinct figures: one for the identification of the scenarios and other to the evaluation. Each figure contains two of the eight numerically created scenarios.

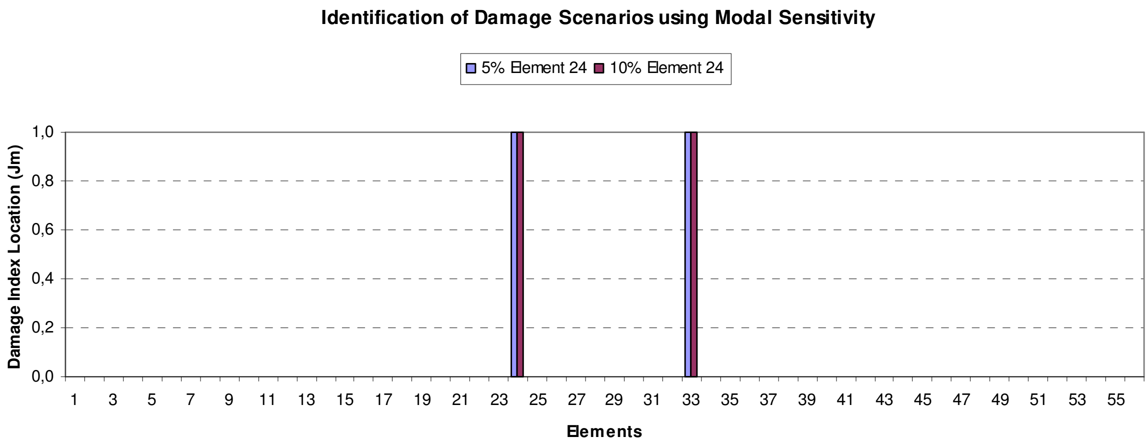

Figure 4: Damage identification with Modal Sensitivity Analysis.

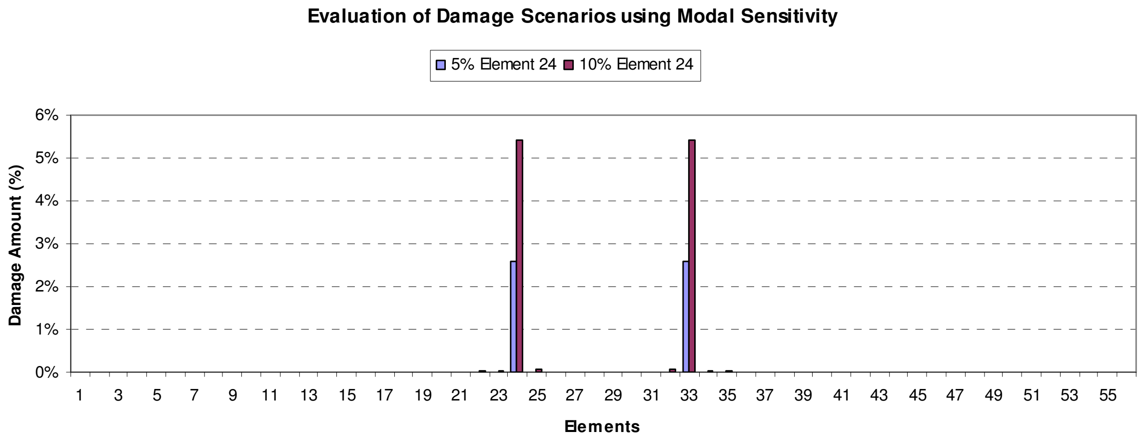

Figure 5: Damage evaluation predicted by Modal Sensitivity Analysis.

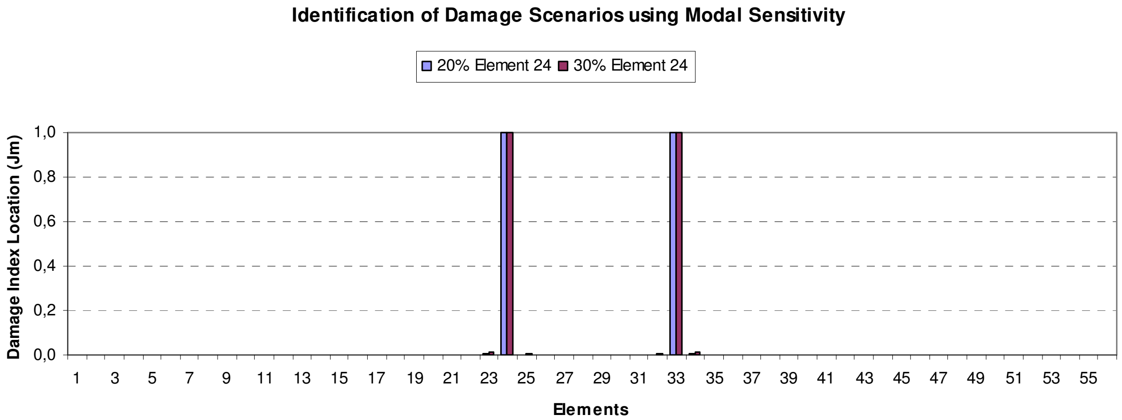

Figure 6: Damage identification with Modal Sensitivity Analysis.

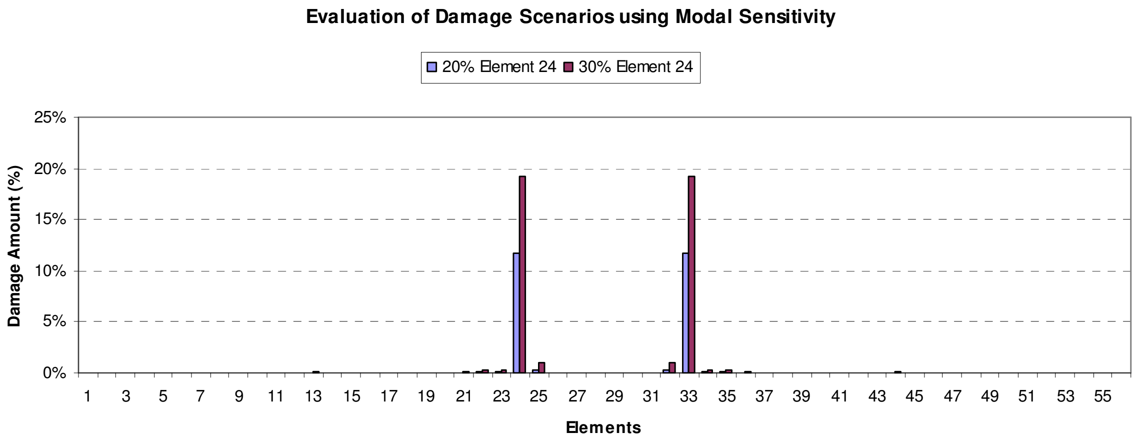

Figure 7: Damage evaluation predicted by Modal Sensitivity Analysis.

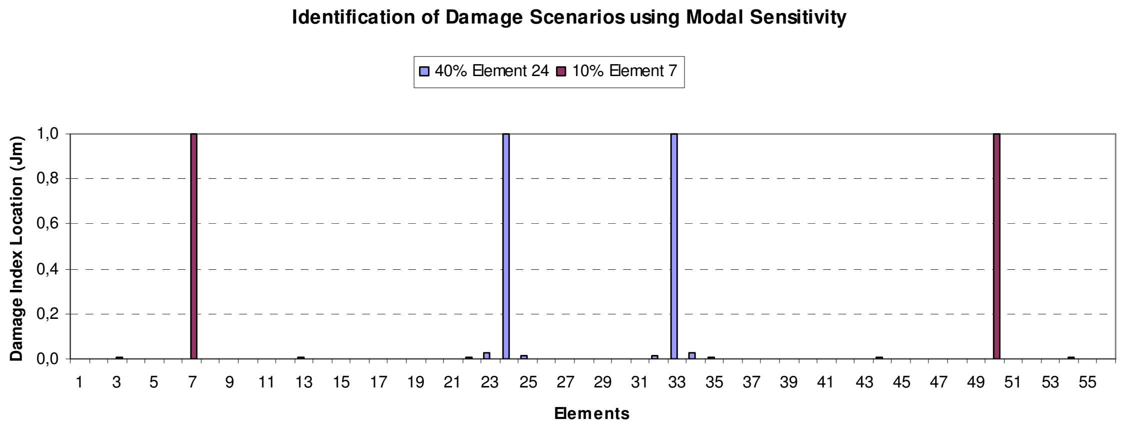

Figure 8: Damage identification with Modal Sensitivity Analysis.

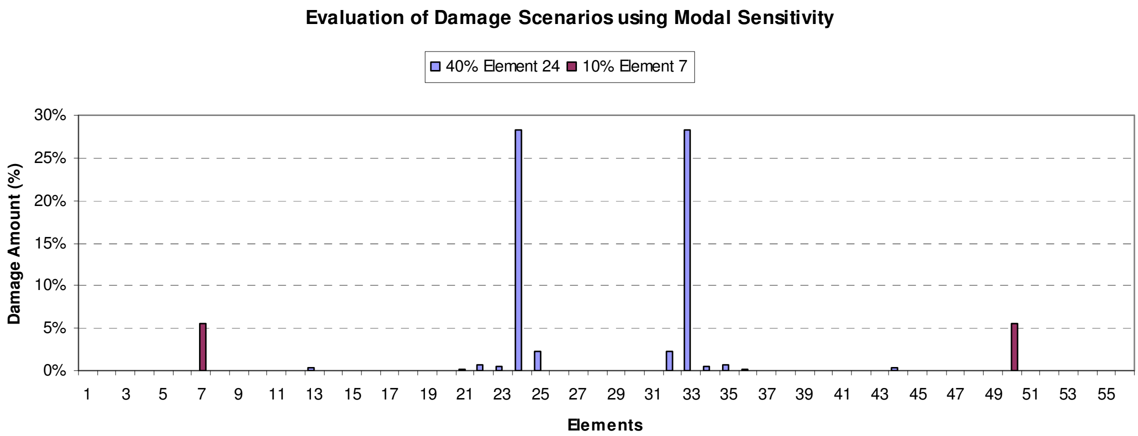

Figure 9: Damage evaluation predicted by Modal Sensitivity Analysis.

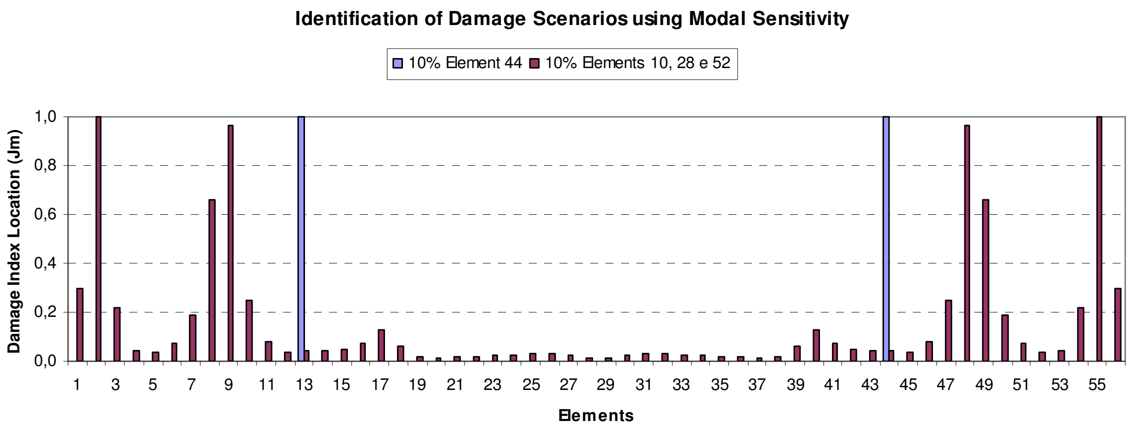

Figure 10: Damage identification with Modal Sensitivity Analysis.

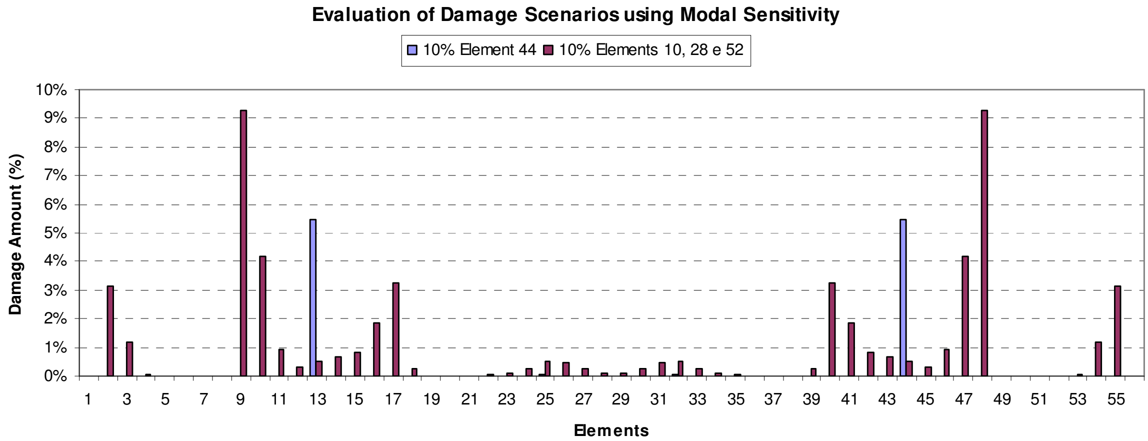

Figure 11: Damage evaluation predicted by Modal Sensitivity Analysis.

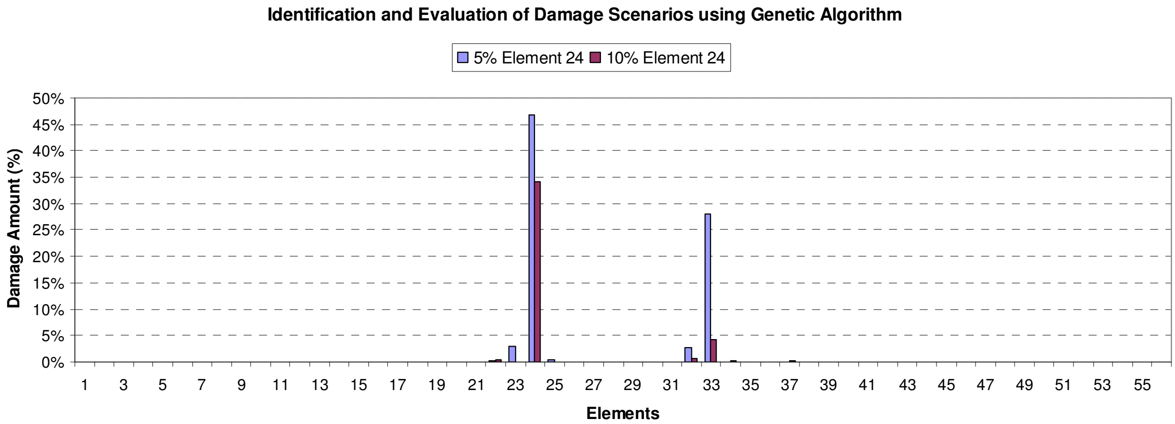

Figure 12: Damage identification and evaluation predicted with genetic algorithm.

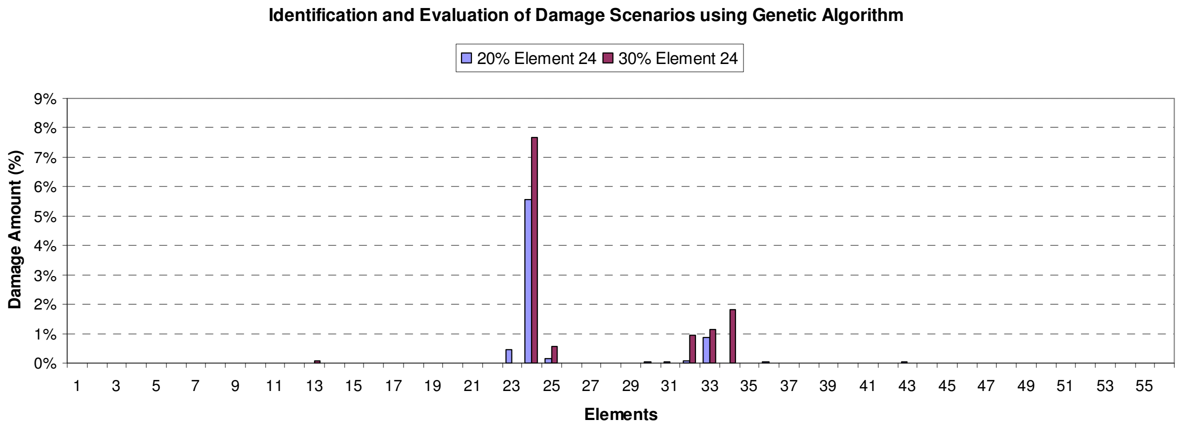

Figure 13: Damage identification and evaluation predicted with genetic algorithm.

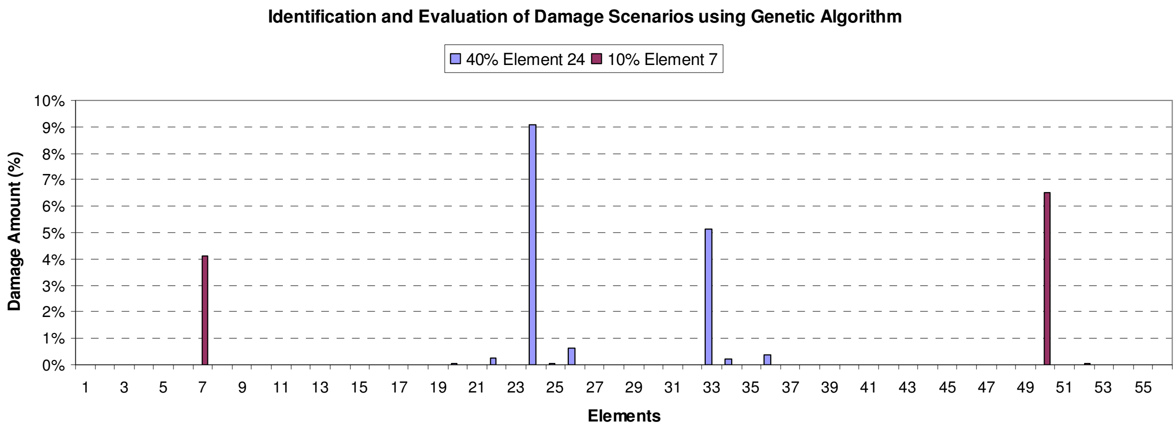

Figure 14: Damage identification and evaluation predicted with genetic algorithm.

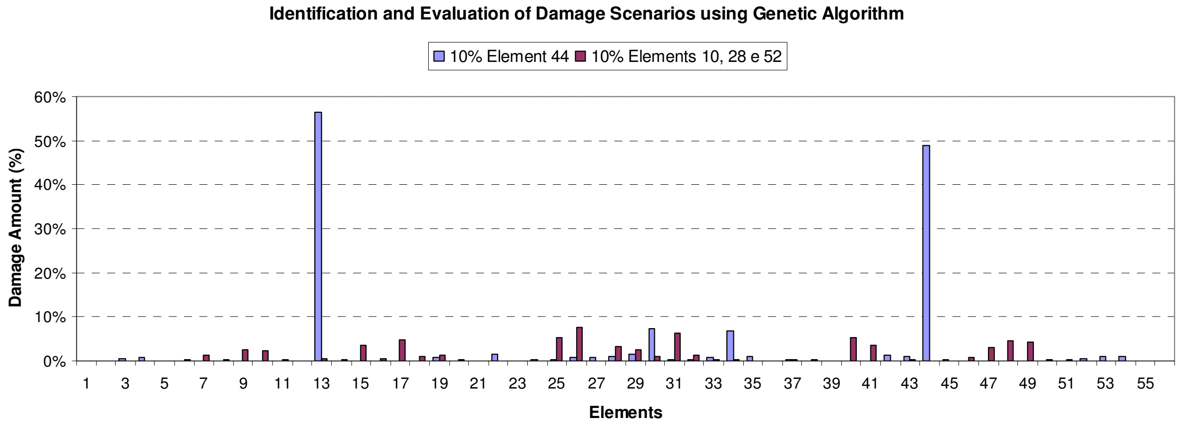

Figure 15: Damage identification and evaluation predicted with genetic algorithm.

5 CONCLUSION

It was noticed with this example that always the symmetrical element was identified together with the actual defective element as was expected. Therefore the both methods showed robustness in the identification of the eight numerically created scenarios. The results for identification using the Modal Sensitivity Analysis were similar as the work of Veizaga (1993) and the evaluation on one site was successfully found using only first five natural frequencies. The evaluation results for the genetic algorithm on one site were worse than the Modal Sensitivity Analysis. Damage occurring on more than one site wasn't so successfully for the both methods. The genetic algorithm here proposed only uses numerically created scenarios for the identification and evaluation of the natural damaged and undamaged frequencies.

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