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AN ELECTROMECHANICAL SYSTEM WITH UNCERTAIN STICK-SLIP OSCILLATIONS

Rubens Sampaio and Roberta Lima

Mechanical Engineering Department, PUC-Rio, Rua Marquês de São Vicente 225, Gávea, Rio de Janeiro, Brazil, rsampaio@puc-rio.br, robertalima@puc-rio.br

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Abstract. This work analyzes the stochastic response of a multiphysics system with stick-slip oscillations. The system is composed of two interacting subsystems, a mechanical with Coulomb friction and an electromagnetic (a DC motor). An imposed source voltage in the DC motor stochastically excites the system. This excitation, combined with the dry-friction, induces in the mechanical subsystem stochastic stick-slip oscillations. The resulting motion of the mechanical subsystem can be characterized by a random sequence of two qualitatively different and alternate modes, the stick- and slip-modes, with a non-smooth transition between them. The duration of each stick-mode is uncertain and depends on electromagnetic and mechanical parameters and variables, specially the position of the mechanical subsystem during the stick-mode. Duration and position are dependent random variables and must be jointly analyzed. The objective of this paper is to characterize and quantify this stochastic dependence. The high amount of data required to perform the analysis and to construct joint histograms puts the problem into the class of big data problems.

1 INTRODUCTION

This paper deals with three research topics: stick-slip oscillations (caused by dry-friction), electromechanical coupling, and uncertainties. The literature dealing with these three topics is extensive. Several authors have published papers in these areas. However, this paper puts together the three topics in a same problem. It analyzes the sick-slip oscillations in an electromechanical system with uncertainties. To the best of our knowledge, the study of this kind of system (putting together the three topics) is a novelty in the literature since no references dealing with it were found after an extensive literature review. Only papers that study the topics separately were found. To explain how these three research topics integrate and the importance of combining them, the topics are hierarchically presented and assembled in the next subsections.

1.1 Stick-slip caused by dry-friction

The presence of dry-friction in mechanical systems may induce the occurrence of stick-slip oscillations, a type of motion with a non-smooth behavior (Anh, 2002; Cao and Léger, 2017; Glendinning and Jeffrey, 2019; Jeffrey, 2018; Lima and Sampaio, 2015). When stick-slip occurs, the system response is characterized by two qualitatively different modes, the stick- and slip-modes (Galvanetto and Bishop, 1999; Leine et al., 2019; Luo and Gegg, 2006). We call stick when the relative velocity between the bodies in contact is null in a time-interval of non-null duration and we call slip if the relative velocity is non-zero, or zero in isolated points. These two modes alternate with a non-smooth transition between them (Jeffrey, 2020; Bengisu and Akay, 1999; Braiman et al., 1997; Jeffrey, 2017; Léger and Pratt, 2016).

The literature dealing with sick-slip oscillations in mechanical systems is vast (Fidlin, 2006; Vande Vandre et al., 1999). However, this paper deals with this kind of dynamics in a context different from the one that is found in literature of the area. It analyzes the sick-slip oscillations in a multiphysics system, an electromechanical system.

1.2 Electromechanical system with stick-slip

The system analyzed in this paper is composed of two interacting subsystems, a mechanical and an electromagnetic (DC motor). In the mechanical subsystem there is dry-friction. By coupling between the two subsystems we mean mutual influence, i.e., the dynamics of the mechanical subsystem influences and is influenced by the dynamics of the electromagnetic subsystem (Lima and Sampaio, 2016; Lima et al., 2019; Manhães et al., 2018). Consequently, the dry-friction present in the mechanical subsystem affects and is affected by the interaction. Furthermore, the sick-slip oscillations affect and are affected by the electromechanical coupling. The system mode at each instant (stick or slip) depends on the imposed source voltage and on the state of the whole system, i.e., depends on mechanical and electromagnetic variables.

Traditionally, for a purely mechanical system, stick-slip is analyzed as the interplay of a friction and an elastic force (Lima and Sampaio, 2015). A stick lasts as long as these two forces balance each other. In our case, there is no elastic force, it is replaced by a force generated by the motor. During the stick modes, the DC motor rules the system dynamics Lima and Sampaio (2020a,b, 2021). This dynamics is described by an initial value problem involving the current and the imposed voltage in the electric circuit of the motor.

1.3 Electromechanical system with stick-slip and uncertainties

To better characterize stick-slip oscillations, it is important to consider the uncertainties that affect the system behavior. These uncertainties influence, for example, the instants at which each mode starts, the position of the mechanical subsystem when the sticks begin and the duration of the modes. Due to the uncertainties, the sequence of stick- and slip-modes becomes stochastic and the variables that characterize it become random (Lima and Sampaio, 2017a,b). Deterministic predictions of these variables will probably fail. Predictions should be made with a stochastic approach. In particular, we are interested in the random variables, components of a random vector, describing the stick duration and position, which are dependent.

A common source of uncertainties in DC motors is the imposed source voltage. Usually, transmission lines are subjected to noises which perturb transmitted signals Hlalele and Du (2016); Sivanagaraju et al. (2014). These uncertainties affect the behavior of the systems powered by these lines and are critical in planning and modeling.

The analysis performed in this paper takes into account two sources of uncertainties: the imposed source voltage in the electromagnetic subsystem and the initial condition of the position of the mechanical subsystem. We aim to quantify the influence of these two sources of uncertainties in the system response. The focus is on the analysis of the stick duration and position. The objective is to characterize these variables with a stochastic approach.

1.4 Novelties of the paper

As explained in the beginning of the Introduction, this paper puts together three research topics (stick-slip oscillations caused by dry-friction, electromechanical coupling, and uncertainties) in a same problem. This is new.

In our previous work (Lima and Sampaio, 2020a), we dealt with a combination of two of these topics. We analyzed the same electromechanical system with dry-friction studied in this paper, but without uncertainties. Then, this paper is a continuation of our previous work (Lima and Sampaio, 2020a) to which we added randomness.

In (Lima and Sampaio, 2020a), we proposed an analytical approximation to the upper bound for the stick duration. With this analytical approximation, it was possible to discriminate the influence of the mechanical and the electromagnetic variables on the upper bound, as for example, the position of the mechanical subsystem. The paper (Lima and Sampaio, 2020a) is the first work in the literature that addresses the dependence between stick duration and position. However, it does the study from a deterministic point of view.

In this paper, we go far beyond. We do a complete mapping of the dependence, not only to the upper bound, and do it from a stochastic point of view. Due to the consideration of uncertainties, duration and position become dependent random variables and they should be jointly analyzed. Only marginal distributions are not enough to characterize them. The objective of this paper is to characterize and to quantify this stochastic dependence, a novelty in the literature. In the stochastic mapping, the uncertainty quantification is done by joint histograms (Lima and Sampaio, 2018). In these histograms, the upper bound found in (Lima and Sampaio, 2020a) appears as the support of the joint distribution of the duration and the position of a stick.

Histograms are the main tool used in this paper. The use of histograms in the characterization of stick-slip dynamics is recent and proposed by us (Lima and Sampaio, 2017a). However, in (Lima and Sampaio, 2017a) the histograms were applied to quantify the uncertainties of random variables related with stick-slip dynamics in purely mechanical systems. In this paper, we use them to quantify the uncertainties of random vectors in electromechanical systems.

With Monte Carlo simulations, scatter plots, marginal and normalized joint histograms of the stick duration and the position of the mechanical system during the stick were construct for stick duration for different values of friction coefficient. To construct the joint histograms with accuracy, several realizations of the system response were required. Since each realization was obtained by a numerical integration of the initial value problem that gives the dynamics of the system, the computational cost to perform the simulations was high. The developed analysis belongs to a class of big data problems.

1.5 Organization of the paper

This paper is organized as follows. Section 2 presents the dynamics of an electromechanical system with dry-friction, i.e., the initial value problem (IVP) that describes the dynamics of the coupled system. The dry-friction force model, and the necessary conditions for the occurrence of the stick- and slip-modes are defined in Section 3. In Section 4, an analytical approximation to the upper bound for the stick duration is presented. This result, despite not being new since it appears in Lima and Sampaio (2020a), is presented for completeness. The influence of mechanical and electromagnetic variables and parameters values in this upper bound is discussed in Section 5. With the analytical approximation, it is possible to determine the influence of the position of the mechanical system during the stick in the stick duration. The upper bound will define the support of the joint distributions of interest in the paper. The construction of the probabilistic model of the uncertain source voltage is given in Section 6. Scatter plots and joint histograms constructed for stick duration and position of the mechanical system during the stick are presented in Section 7.

2 DYNAMICS OF THE ELECTROMECHANICAL SYSTEM WITH DRY-FRICTION

The system analyzed in this paper is composed by a cart-disk whose motion is driven by a DC motor. The motor is coupled to the cart by a scotch-yoke mechanism that transforms the motor rotational motion into horizontal cart motion over a rail, as shown in Fig. 1. The initial

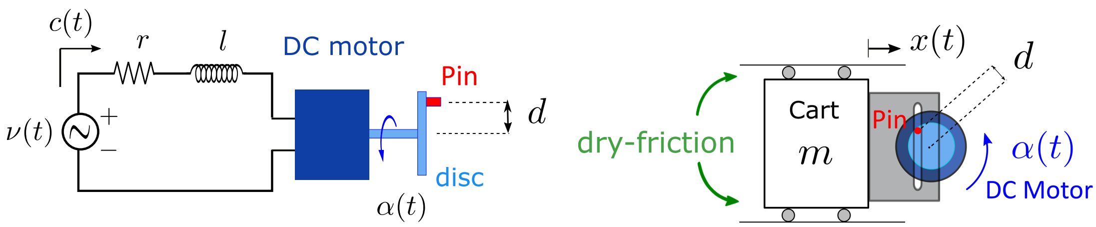


Figure 1: Electromechanical system with dry-friction between the cart and the rail.

value problem that describes the system dynamics is given in Eqs. (1) and (2). Find $(\alpha,\ c)$ such that, for all $t > 0$,

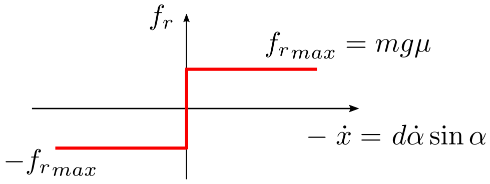
$$l\dot{c}(t) + r\,c(t) + k_e\dot{\alpha}(t) = \nu\,, \tag{1}$$

$$\begin{aligned}\ddot{\alpha}(t)[j_m + m\,d^2\,(\sin{(\alpha(t))})^2] + \dot{\alpha}(t)[b_m + m\,d^2\,\sin{(\alpha(t))}\cos{(\alpha(t))}]\\ -k_e c(t) = -f_r(t)\,d\,\sin{(\alpha(t))}\,,\end{aligned} \tag{2}$$

with the initial conditions $\dot{\alpha}(0) = \beta$, $\alpha(0) = \alpha_0$ and $c(0) = c_0$. In Eq. (1) and (2) t is the time, v is the source voltage, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the motor moment of inertia, b_m is the damping ratio in the transmission of the torque generated by the motor, k_e is the motor electromagnetic force constant, r is the electrical resistance, m is the mass of the cart, d is the eccentricity of the scotch-yoke mechanism and f_r is the dry-friction force between the cart and the rail. The source voltage ν is considered to be $\nu(t) = \nu_0 + \nu_1 \sin{(\omega_v\,t + \theta)}$, i.e., the source voltage oscillates around ν_0 with amplitude ν_1 and phase θ. Please note that the system state is given by three variables, two of them mechanical (angular velocity and position of the motor) and one of them electromagnetic (current). The dynamics of the electromechanical system analyzed is given by an initial value problem comprising a set of two coupled differential equations.

3 DRY-FRICTION MODEL AND NECESSARY CONDITIONS FOR THE OCCURRENCE OF THE STICK- AND SLIP-MODES

The friction is modeled as Coulomb's and as simple as possible, Fig. 2(a).



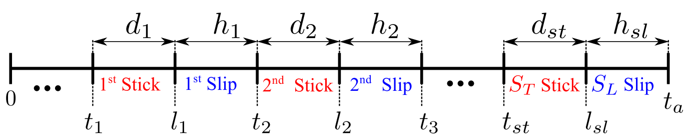


Figure 2: (a) Coulomb dry-friction and (b) Sequence of stick- and slip-modes in a electromechanical system response.

When the cart velocity is not zero, $\dot{x} \neq 0$, the dry-friction force can only assume the values $-f_{rmax}$ and f_{rmax}. However, when $\dot{x} = 0$, it can assume any value in the interval $[-f_{rmax}, f_{rmax}]$. It is considered $f_{rmax} = \mu m\, g$, where g is the gravity and μ is the friction coefficient between the cart and the rail. Note that the dry-friction force is not a function of the cart velocity. For one value of cart velocity, $\dot{x} = 0$, the friction can take infinite values, but it is confined in an interval, i.e., its magnitude is bounded. Depending on the values of the system parameters, the response of the system may be composed of a sequence alternating stick- and slip-modes, as illustrated in Fig. 2(b). During a stick-mode, the disk-cart does not move, so that the angle, describing the angular position of the disk, is constant. The frictional force and the current, however, can vary. Hence, stick means only no motion of the disk-cart, the mechanical subsystem. The electromagnetic subsystem continues to change its state until it gathers enough power to move the disk-cart again. The stick-mode occurs when $\dot{\alpha} = 0$ in an time interval and the frictional force is the interval $[-f_{rmax}, f_{rmax}]$. To better understand the effect of the stick-mode in the dynamics of the electromechanical system, let us investigate how it influences the IVP that describes the system dynamics. Considering $\dot{\alpha} = 0$ and $\ddot{\alpha} = 0$ in Eqs. (1) and (2), the following equations are obtained

$$l\dot{c}(t) + r\, c(t) = v_0 + v_1 \sin(\omega_v\, t)\,, \tag{3}$$

$$k_e\, c(t) = f_r(t)\, d\, \sin(\alpha(t))\,. \tag{4}$$

Observe that the first differential equation of the IVP becomes a differential equation (Eq. (3)) which just depends on the current, an electromagnetic variable. The second differential equation of the IVP becomes an algebraic equation (Eqs. (4)). During the stick-mode, the coupling between the electromagnetic and mechanical subsystems is made by Eq. (4). The value of the friction force, during the stick-mode, can vary and follows Eqs. (4), i.e., it depends on an electromagnetic variable and on the angular position of the motor, a mechanical variable. Remark that during the stick-mode, the sum of the forces that act over the cart is zero (it does not move). The horizontal coupling force between the DC motor and the cart, f, is balanced by the dry-friction force, f_r. This balance lasts until the frictional force, given in Eq. (4), reaches its maximum value, $f_{r_{max}}$. Recall that when $\dot{x}(t) = 0$, the magnitude of the frictional force is bounded. During the stick-mode, the dynamics of the system is governed only by the dynamics of the electrical circuit of the motor, the electromagnetic subsystem. During the slip-mode, the dry-friction force is

$$f_r(t) = -m\, g\mu\, \text{sgn}(\dot{x}(t)) = -m\, g\mu\, \text{sgn}(-\dot{\alpha}(t)\, d\, \sin(\alpha(t)))\,. \tag{5}$$

4 APPROXIMATION TO THE UPPER BOUND FOR THE STICK DURATION

To obtain an analytical approximation to the upper bound for the stick duration, the starting point is the initial-value problem that describes the dynamics of the coupled motor-disk-cart system with dry-friction during the stick-phase given by Eq. (3). Calling the instant of beginning of a stick as t_b; the instant of end of a stick as t_e; the angle of the disk in the beginning of a stick, the angle where the disk is stuck, as $\alpha(t_b) = \alpha^\star$; and the current in the beginning of a stick as $c(t_b) = c_0$. The stick-mode ($t_b \leq t \leq t_e$) is governed by the following IVP:

$$\begin{aligned} &l\dot{c}(t) + \ r\ c(t) = \nu_0 + \nu_1 \sin\left(\omega_v\, t\right). \\ &c(t_b) = c_0 \end{aligned} \tag{6}$$

The analytical solution of this IVP ($t_b \leq t \leq t_e$) is given by

$$c(t) = \left[\frac{c_0 - \dfrac{\nu_0}{r} - \dfrac{\nu_1\sqrt{r^2+l^2}}{l^2\omega_v^2 + r^2}\sin\left(\omega_v t_b + \phi\right)}{e^{-r/l\; t_b}}\right] e^{-r/l\; t} + \frac{\nu_1\sqrt{r^2+l^2}}{l^2\omega_v^2 + r^2}\sin\left(\omega_v t + \phi\right) + \frac{\nu_0}{r}, \tag{7}$$

where $\phi = \arctan\dfrac{-l}{r}$. Since, the variable of interest is the upper bound for the stick duration, the influence of the initial condition (c_0) can be neglected. Thus, the solution of the IVP during the the stick-mode ($t_b \leq t \leq t_e$) can be approximated by:

$$c(t) \approx \frac{\nu_1\sqrt{r^2+l^2}}{l^2\omega_v^2 + r^2}\sin\left(\omega_v t + \phi\right) + \frac{\nu_0}{r}. \tag{8}$$

Recalling that during the stick-mode $f_r \in [-f_{r_{max}}, f_{r_{max}}]$ and $k_e\ c(t) = f_r(t)\, d\, \sin\alpha(t)$. The stick ends when $c(t_e) = \dfrac{f_{r_{max}}\, d\, \sin\alpha^\star}{k_e}$. The upper bound for the stick duration $t_e - t_b$ is the maximal interval during which the current, given by Eq. (8), is lower than the value $\dfrac{f_{r_{max}}\, d\, \sin\alpha^\star}{k_e}$. Please observe that if c is always under this value, the stick can last forever. If c is always above, the stick does not happen.

5 INFLUENCE OF MECHANICAL AND ELECTROMAGNETIC VARIABLES AND PARAMETERS IN THE APPROXIMATION TO THE UPPER BOUND FOR THE STICK DURATION

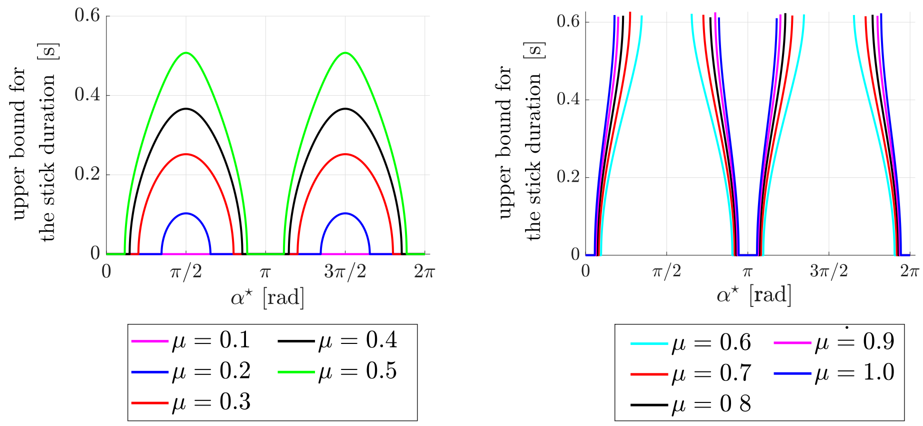
The upper bound for the stick duration depends on mechanical and electrical variables and geometry of the stick. Among all variables, two of them can be highlighted. They are the angle of the disk in the beginning of a stick, $\alpha(t_1) = \alpha^\star$, and the value of the friction coefficient, μ. To better visualize the influence of these two variables, the approximation to the upper bound

$l = 1.880 \times 10^{-4}$ H	$k_e = 5.330 \times 10^{-2}$ V/(rad/s)
$j_m = 1.210 \times 10^{-4}$ kg m^2	$r = 0.307\ \Omega$
$b_m = 1.545 \times 10^{-4}$ Nm/(rad/s)	$m = 5.000$ kg
$\nu_0 = 1.000$ V	$\nu_1 = 0.500$ V
$\omega_v = 10.000$ rad/s	$d = 0.010$ m

Table 1: Parameter values used in simulations.

for the stick duration was computed for different combinations of $\alpha^\star$ and μ. The values to all the others parameters are given in Table 1.

The results are shown in Figs. 3(a) and 3(b). In Fig. 3(a), the values of friction coefficient are $0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5$. It can be observed that with $\mu = 0.1$, the approximation to the upper bound for the stick duration is 0 for all $\alpha^\star$, which means that there is no stick. When $\mu = 0.2$, that is possible to have stick. The longest stick happens when $\alpha^\star = \pi/2$ or $\alpha^\star = 3\pi/2$. In the region that $\alpha^\star$ is around 0 or π, there is no stick. As the friction coefficient increases, the region where there is no stick shrinks.



(a) (b)

Figure 3: Approximation to the upper bound for the stick duration as function of the angle of the disk in the beginning of a stick for different values of the friction coefficient.

In Fig. 3(b), the values of friction coefficient are $0.6,\ 0.7,\ 0.8,\ 0.9,\ 1.0$. There are regions of $\alpha^\star$ around $\pi/2$ and $3\pi/2$ that the stick can last forever. This means that if a stick happens when $\alpha^\star$ is in a certain region, the stick will last forever.

6 CONSTRUCTION OF THE PROBABILISTIC MODELS

We consider the source voltage and the initial condition of the angular position of the disk as sources of uncertainties in the stick-slip oscillator problem. We model the phase of the source voltage and the initial condition of the angular position of the disk as uniform random variables over $[0,\ 2\pi]$. Due to the consideration of uncertainties, the equations of motion of the system, became a stochastic differential equations. The response of the stochastic stick-slip oscillator becomes stochastic and involves two random processes: the angle of the disk and the current. The angle of the disk presents a sequence alternating stick and slip-modes. We are interested in the stochastic characterization of this sequence. Defined a time interval for analysis, the variables of interest are the number of time intervals in which stick or slip occur, the instants at which they start, and their duration. These variables are modeled as stochastic objects in order to allow the stochastic characterization the dynamics of the oscillator. In this paper, we focus on the position of the mechanical system (disk) and duration of the first stick. The objective is to determine the joint distribution of these variables.

7 SCATTER PLOTS AND JOINT HISTOGRAMS FOR STICK DURATION

. With Monte Carlo simulations, scatter plots and joint histograms of the duration and the position of the disk during the first stick were constructed for two different values of friction coefficient, $\mu = 0.2$ and $\mu = 0.5$. For each value of μ, the initial value problem that characterizes the system dynamics were numerically integrated $10,000$ times, totalizing $20,000$ numerical integrations. To perform Monte Carlo simulations in a system with stick-slip and uncertainties belong to a class of big data problems. The combination of dry friction and uncertainties makes simulations costly since both involve convergence analysis. In the Monte Carlo simulations, a large number of realizations of the system response was necessary in order to construct the histograms with accuracy (Lima and Sampaio, 2012; Souza de Cursi and Sampaio, 2015). It is important to remark that the number of realizations required to construct a joint histogram with accuracy is much higher than the number of realizations required to construct a marginal one. This number should be determined after a convergence study in a two dimension surface. Besides that, stick-slip is a type of dynamics with non-smooth behavior. Thus, when performing the numerical integration of the initial value problem that characterizes the system dynamics, at each time-step, ís necessary to determine in which mode the system is (stick or slip). To accurately predict the switching instants between modes, a small time-step must be used. In this paper, each realization in the Monte Carlo simulations was integrated with a time-step of 10^{-4} s. The computational and temporal cost to perform the simulations was high. The CPU time was approximately 30 hours and the amount of data generated was around 20 GB. It is worth noting that in the studied problem, we can generate a database as large as we wish just increasing the number of simulations and reducing the time-step. In our problem, big data can be as big as we want. The limiting factors are processing speed and storage capacity. The parameter values used in all simulations are listed in Section 5. Figures 4 and 5 show the scatter plots and normalized joint histograms of the duration (d_1) and the position $(\alpha_1^\star)$ of the mechanical subsystem during the first stick for the different values of friction coefficient. In the scatter plots, it is also shown the approximation to the upper bound for the stick duration as function of the position of the disk during the stick. Please remark that the upper bounds defines the supports of the joint distributions of the duration and the position of the disk during a stick. The non-square support of the joint distributions highlight the stochastic dependence between the two random variables.

8 CONCLUSIONS

This article analyzes the dynamics of an electromechanical system with dry-friction. The source of uncertainties of the system are the imposed source voltage in the DC motor and the initial condition of α. The excitation induces in the mechanical subsystem stochastic stick-slip oscillations. Mechanical and electromagnetic parameters influence this stochastic sequence. The objective of this paper is to analyze the joint influence of these parameters in the statistical model of the system response. Scatter plots and joint histograms of stick duration and the position of the disk during the stick were constructed for different values of friction coefficient.

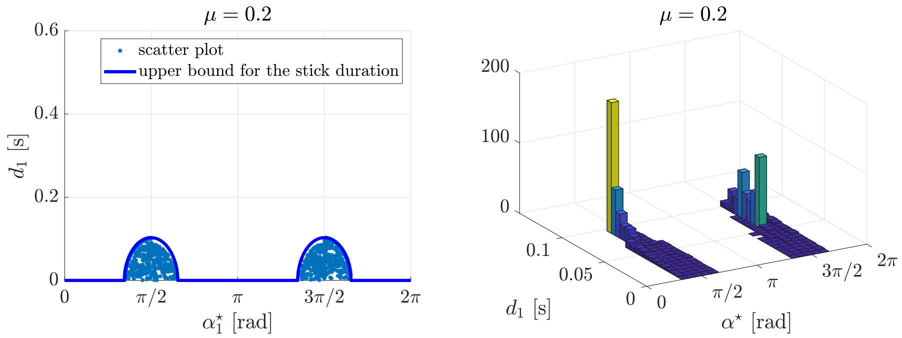


Figure 4: Scatter plots and normalized joint histograms of the duration and the position of the mechanical system of the first stick for $\mu = 0.2$.

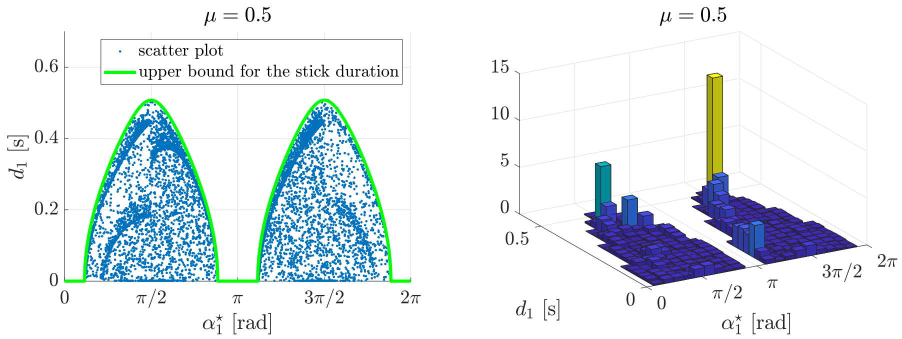


Figure 5: Scatter plots and normalized joint histograms of the duration and the position of the mechanical system of the first stick for $\mu = 0.5$.

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