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SLUG FLOW MODELING BY A TWO-STATE MARKOV CHAIN FROM A RESISTIVE SENSOR MEASUREMENTS

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Abstract. Intermittent flows are common flow patterns in gas-liquid horizontal flow and attract attention and great research effort due to its importance for industrial and engineering applications. The slug flow is typically modelled based on a unit cell varying from an elongated air bubble with a liquid film in segregated flow pattern and an aerated liquid plug, the slug region, with remarkable stochastic characteristics of its alternating regions. In this paper, a two-state Markov chain model is proposed to represent the stochastic dynamics of developed slug flow in horizontal pipes. Each state represents either the liquid slug or the elongated bubble regions and the transition probabilities dictate the change of the given discrete time measurement to stay at a given state or change. This simple but insightful description of the phenomenon allows an analytical treatment of the statistics of Markov chain stochastic process. Measurement stations with two double wire resistive sensors are used to obtain the void fraction time series and a corresponding two-state representation. Each state is classified by a threshold defined by an unsupervised and non-parametric pattern recognition approach. The state transition matrix is then estimated for each corresponding experimental point. Thus random samples can be synthetically regenerated with the same statistical features of the corresponding measurements. It is shown that the Markov chain model can successfully represent second-order statistics of the measurement, such as the autocorrelation and power spectral density, given an appropriate choice of the chain order.

1 INTRODUCTION

Multiphase flows are of common occurrence in numerous natural and industrial processes. One of the most common is the gas-liquid flow, where the phases are distributed in different geometric arrangements, which are called flow patterns. These patterns depend on fluids properties (density, viscosity, surface tension) and flow conditions (flow rates, pipe diameter and slope, etc) (Shoham, 2006; Ishii and Hibiki, 2011). In gas-liquid horizontal flows the stratified, annular, bubbles and intermittent flows are the most common patterns, each one with its own characteristics. The correct definition of the flow pattern and its characteristics is of ultmost importance for industrial purposes, as the presence of an specific pattern may lead to severe problems. In the case of the oil industry, flow assurance problems are related to the occurrence of an specific flow pattern (Shippen and Bailey, 2012).

One of the most common and at same time complex gas-liquid flow pattern is the slug flow, as depicted in the schematic Fig. 1. The slug flow is typically modelled based on a unit cell varying from an elongated air bubble with a liquid film in segregated flow pattern (considered stratified flow) of length L_f, to a liquid slug with/without dispersed gas bubbles, of length L_s. Both patterns compose a whole periodic structure with length L_U called a unit cell (Taitel and Barnea, 1990; Fabre and Liné, 1992; Netto et al., 1999). One remarkable interesting aspect of the slug flow is its stochastic characteristic of alternating regions (Sarica et al., 2011; Soedarmo et al., 2019).

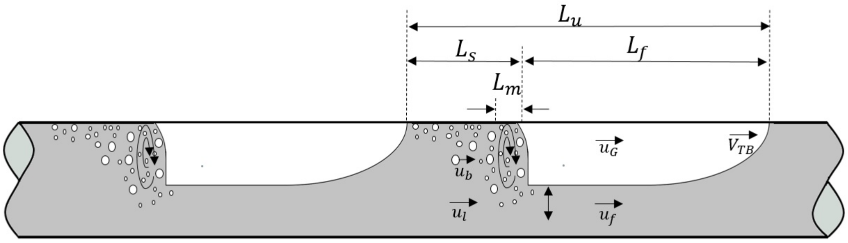

Figure 1: Schematic representation of the unit-cell model for the slug flow pattern.

In this paper, a two-state Markov chain model is proposed to represent the stochastic dynamics of developed slug flow in horizontal pipes. Each state represents either the liquid slug or the elongated bubble regions and the transition probabilities dictate the change of the given discrete time measurement to stay at a given state or change. This simple but intuitive description of the phenomenon allows an analytical treatment of the statistics of Markov chain stochastic process. Consequently, it can be used to investigate the physics of this rather complex flow dynamics and further comprehend several fundamental aspects of reducing the representation of this system by a simple stochastic process.

2 TWO-STATE MARKOV CHAIN MODEL

The slug flow is typically modelled as a unit cell varying from an elongated air bubble with a liquid film in segregated pattern, considered stratified flow, and a dispersed bubble region, the liquid slug. In this section, we propose that the complex dynamics of such phenomenon can be approximated by a stochastic process.

Assuming the void fraction measurements are made at a constant sampling rate Δ and the void fraction, i.e. $\alpha_n = \alpha(t = n\Delta)$, a very simple model for this process can be cast in the form of a two-state Markov chain (Norris, 1997; Soize, 2017), which follows closely the representation given by Fabre et al. (1989) for the time evolution of the flow structure. Each state represents either the elongated air bubble with liquid film or the liquid slug and the Markov chain (MC) is characterised by the probabilities of the sequence maintain, or changing, its state at the n-th sample given the previous sample, i.e. $P(X_n = x_n \mid X_{n-1} = x_{n-1})$ for a first order Markov chain and $x_n = 1$ for the air bubble and $x_n = 0$ for the liquid film. This classification is given from the time series α_n and it is discussed in the next section. This model is schematically represented in Figure 2.

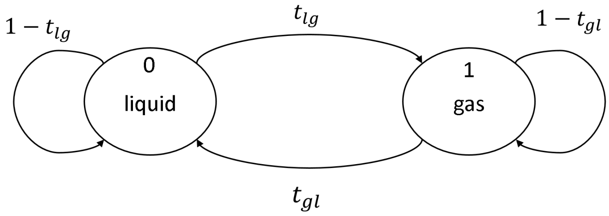

Figure 2: Two-state Markov chain model diagram for liquid slug ($X_n = 0$) and gas bubble ($X_n = 1$). State transition probability from liquid to gas t_α and from gas to liquid t_β.

The conditional probabilities of state transition can be defined as $t_{lg} = P(X_n = 1|X_{n-1} = 0)$ and $t_{gl} = P(X_n = 0|X_{n-1} = 1)$ with $0 \leq t_{lg,gl} \leq 1$. The discrete probability density function at the n-th sample $\boldsymbol{\pi}^n$ can be given from the marginal probabilities for each state, given as $P(X_n = 0) = P(X_n = 0 \mid X_{n-1} = 0)\, P(X_{n-1} = 0) + P(X_n = 0 \mid X_{n-1} = 1)\, P(X_{n-1} = 1)$ and $P(X_n = 1) = P(X_n = 1 \mid X_{n-1} = 0)\, P(X_{n-1} = 0) + P(X_n = 1 \mid X_{n-1} = 1)\, P(X_{n-1} = 1)$. Rearranging in matrix form, yields

$$\boldsymbol{\pi}^n = \boldsymbol{\pi}^{n-1}\mathbf{P}, \tag{1}$$

where the state transition matrix is given in terms of the conditional state transition probabilities.

The chain is stationary when $\boldsymbol{\pi}^n = \boldsymbol{\pi}^{n-1}$ which yields an eigenproblem $\boldsymbol{\pi}^n = \boldsymbol{\pi}^n\mathbf{P}^n$ with eigenvalue $\lambda_1 = 1$ and corresponding eigenvector $\boldsymbol{\phi}_1 = [t_{gl}\ t_{lg}]^T/(t_{lg} + t_{gl})$. Consequently, its steady-state distribution is

$$\boldsymbol{\pi} = \begin{bmatrix} \pi_0 \\ \pi_1 \end{bmatrix} = \begin{bmatrix} \frac{t_{gl}}{t_{lg}+t_{gl}} \\ \frac{t_{lg}}{t_{lg}+t_{gl}} \end{bmatrix} \tag{2}$$

By increasing the order Markov Chain model, the state of m previous samples other than the immediately previous are also taken into account at the probabilities of the sequence, i.e., $P(X_n = x_n \mid X_{n-1} = x_{n-1}, X_{n-1} = x_{n-2}, \ldots X_{n-m} = x_{n-m})$ (Katz, 1981). This introduces a finite memory to the chain closely related to its order. Although it can, in principle, improve the Markov Chain model, it also significantly increases the number of parameters for estimation, with the increasing number of possible transition probabilities. It might require significantly longer measurements times which limits its practical uses and imposes a parsimonious approach for the order selection of the model. Ideally, a first order model should succeed in adequately representing a certain phenomenon even with longer time dependency (Rafteryt, 1985).

2.1 Statistical moments of the first order MC model

Some relevant statistical moments can be analytically derived for the proposed steady-state two-state first order MC model. The mean value of the series is given by

$$\mathbb{E}\left(X_n\right) = \sum_{n=0}^{1} x_n \pi_n = \frac{t_{lg}}{t_{lg} + t_{gl}}, \tag{3}$$

where $\mathbb{E}(\cdot)$ stands for the mathematical expectation. Note that this statistical moment is identical to the intermittent factor, i.e., β.

$$\mathbb{E}\left(X_n\right) = \beta, \tag{4}$$

The variance $Var\left(X_n\right)$ is given by

$$\mathrm{Var}\left(X_n\right) = \mathbb{E}\left(X_n^2\right) - \mathbb{E}\left(X_n\right)^2 = \sum_{n=0}^{1} x_n^2 \pi_n - \sum_{n=0}^{1} (x_n \pi_n)^2 = \frac{t_{lg} t_{gl}}{(t_{lg} + t_{gl})^2}, \tag{5}$$

The autocorrelation $\mathbb{E}\left(X_n X_{n+\tau}\right)$ can be evaluated as

$$\mathbb{E}\left(X_n X_{n+\tau}\right) = \sum_{n=0}^{1} x_n x_{n+\tau} P\left(X_{n+\tau} = x_{n+\tau} \cap X_n = x_n\right), \tag{6}$$

from which $P\left(X_{n+\tau} = x_{n+\tau} \cap X_n = x_n\right) = P\left(X_{n+\tau} = x_{n+\tau} \mid X_n = x_n\right) P\left(X_n = x_n\right)$. Bearing in mind that the only non-null terms of the sum are given when $X_n = 1$, thus $\mathbb{E}\left(X_n X_{n+\tau}\right) = P\left(X_n = 1 \mid X_{n+\tau} = 1\right) P\left(X_n = 1\right)$. Recalling that $P\left(X_n = 1\right) = \mathbb{E}\left(X_n\right)$, then recursively from the chain using Eq. 1, term $P\left(X_n = 1 \mid X_{n+\tau} = 1\right) = \mathbf{P}_{2,2}^{\tau}$, which is second row and second column term of the matrix $\mathbf{P}^{\tau}$. Finally, it yields

$$\mathrm{Cov}\left(X_n, X_{n+\tau}\right) = \mathrm{a}\mathrm{b}^{|\tau|}, \tag{7}$$

where $\mathrm{a} = t_{lg} t_{gl} \left(t_{lg} + t_{gl}\right)^{-2}$ and $\mathrm{b} = (1 - t_{lg} - t_{gl})$, and normalised by the variance gives $\mathrm{Cov}\left(X_n, X_{n+\tau}\right) / \mathrm{Var}\left(X_n\right) = \mathrm{b}^{|\tau|}$.

Moreover, the power spectral density (PSD) can be obtained from the Fourier transform of Eq. 7 as

$$F\left(\omega\right) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{+\infty} \mathrm{a}\mathrm{b}^{|\tau|} e^{-j\omega\Delta\tau} = \frac{\mathrm{a}}{2\pi} \sum_{\tau=-\infty}^{+\infty} \mathrm{b}^{|\tau|} e^{-j\omega\Delta\tau} = \frac{\mathrm{a}}{2\pi} \left[1 + 2\sum_{\tau=1}^{+\infty} \mathrm{b}^{\tau} \cos(\omega\Delta\tau)\right]. \tag{8}$$

By definition, $|\mathrm{b}| < 1$ thus the infinite series converges to

$$F\left(\omega\right) = \frac{\mathrm{a}}{2\pi} \left[\frac{1 - \mathrm{b}^2}{1 - 2\mathrm{b}\cos(\Delta\omega) + \mathrm{b}^2}\right] = \frac{\mathrm{a}}{2\pi} \left[\frac{1 - \mathrm{b}^2}{1 + \mathrm{b}^2 - 2\mathrm{b}\cos(2\pi\omega/\omega_s)}\right]. \tag{9}$$

This is a periodic function in $-\omega_s/2 < \omega < \omega_s/2$, where $\Delta\omega = 2\pi\omega/\omega_s$, ω_s is the sampling frequency in rad/s. This periodicity is expected due to the discrete nature of the MC sequence. Finding the similar analytical expressions of the proposed higher order two-state Markov Chain is beyond the scope of this paper.

3 RESULTS

The experimental campaign was performed at the Multiphase Flow Research Center of the Federal University of Technology of Paraná (Brazil) and it is described in detail by Rodrigues et al. (2020). For estimation of the two-state Markov chain model, it is important to establish a threshold for classifying a measurement sample as either elongated bubble or a liquid slug.

This threshold is dependent on the experimental point thus a completely data-driven approach is proposed. The Otsu's approach (Otsu, 1979) is used to find the best threshold for every experimental point. It is a non-parametric and unsupervised method of automatic threshold selection.

3.1 Statistics of the Markov Chain model

In this section, the estimation of the first order Markov chain model and some statistical moments for each experimental point are investigated. The time series of each experimental point is used to estimate the state transition matrix. A Maximum Likelihood Estimator (MLE) is used (Billingsley, 1961; Teodorescu, 2009).

Figure 3 shows the autocorrelation estimated for all the 7 experimental points using the original measurement, Otsu threshold, a sample realisation of the first order Markov chain, the analytical first order MC model, Eq. 7 and the periodic model. It can be seen that the autocovariance from the Otsu threshold presents a very good agreement with the original measurements, which indicates that very little information is lost by the thresholding in a second-order sense. It indicates that the proposed approach captures the second-order statistical features of the original time series. The Markov chain model presents a good agreement only for a short lag, i.e. only short term variations of the time series are well represented by this model. The oscillations on the autocorrelation for higher lags matches those of the periodic model, which indicate the level of periodicity on the signals. Note that for every experimental point, different levels of oscillations are present. In contrast, the Markov chain analytical model does not present these oscillations, i.e. it does not capture periodic fluctuations of longer periods. This effect is emphasised on the analysis of the PSD.

Figure 4 presents the PSD estimate for all the experimental points, estimated by using a Welch's segment and average approach (Shin and K Hammond, 2008), with $N_b = 20$ segments and 1/3 overlap. It can be noticed a very good agreement between the results from the original measurements and the Otsu threshold while the MC model presents a good agreement only for higher frequencies. This is expected from the inspection on the autocorrelation results. Shorter correlation lags τ on the autocorrelation are represented by higher frequencies on the PSD. Similarly, longer correlation lags are associated to lower frequencies on the PSD. These results also show that reducing the signal to a two-state representation does not causes a great loss on its spectral content. However, the first order MC model does not capture the main peak, which typically represents the frequency of passage of the unit cell, f_U. The estimation of this parameter is discussed in detail in the following section of this paper. In other words, the first order MC model does not capture the fundamental component of periodic content of the signal. This effect is highlighted by the amplitude value of the coefficients of the Fourier series from the periodic representation of the slug flow, as presented by Vieira et al. (2021). The Fourier series presents a discrete spectrum due to its periodic nature. The amplitude of the coefficients are normalised such that they can be compared to a PSD. The fundamental component of the periodic representation matches that of the PSD peak because this is set as the fundamental period of the Fourier series. Notice that the higher frequency content decays following a different amplitude decay when compared to the Markov Chain model and the experimental results, both with a significant reduction on the power density compared to the periodic case. This result suggests that the actual stochastic process representing the slug flow is somewhere between these two representations. Following the principle of a parsimonious and physically interpretable model, ideally the stochastic process representing the slug flow has to be as simple as possible. However, the first order MC model clearly fails to capture

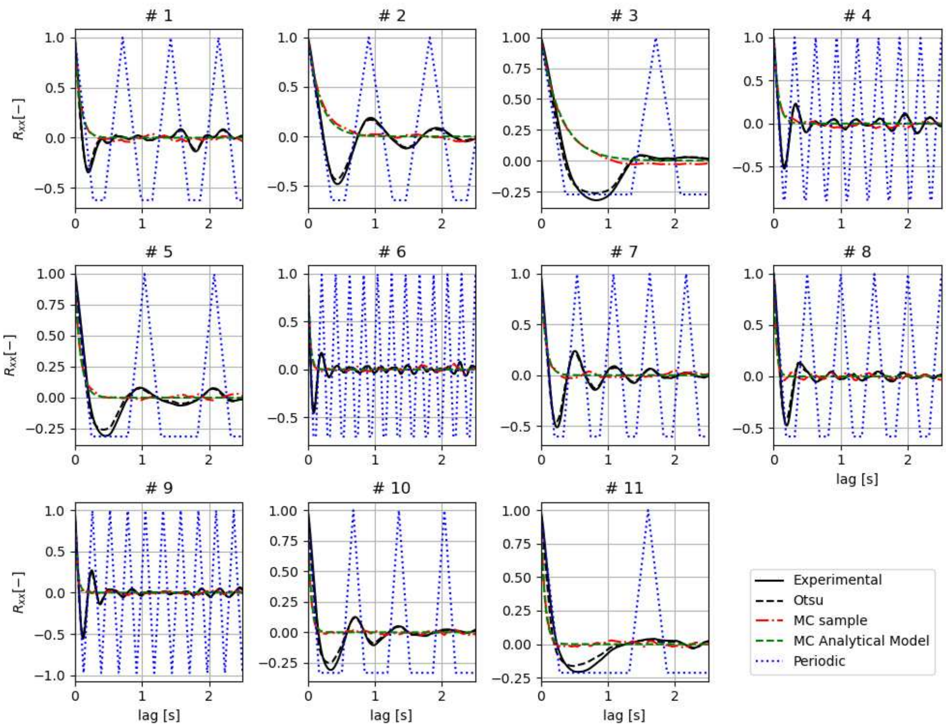

Figure 3: Normalised autocorrelation from each original measurement point (black full line), time series from Otsu threshold (black dashed line), a MC sample (red dash-dotted line), the MC analytical model (green dashed line) and the periodic model (blue dotted line).

the long term behaviour indicating the need of increasing the order of the MC model. In the next section the appropriate choice of the order of the MC model is investigated aiming at the simplest stochastic representation for the features of interest.

The Akaike Information Criterion (AIC) (Akaike, 1974), originally proposed as a means of selecting competing models, can be used to determine the order m of the Markov Chain (Tong, 1975) that best suits the data by minimises the function (Rafteryt, 1985) $AIC(m) = -2L_L + 2m$, where $L_L = \sum_i n_{ti} \log t_i$ is the log-likelihood function of the transition probabilities and n_{ti} is the number of transitions occurring in a sequence and t_i is the corresponding transition probability. Similarly, the Bayesian Information Criterion (BIC) also establishes a metric for model selection (Schwarz, 1978) and has been proposed as a consistent estimator (Katz, 1981), unlike the AIC. The selected order m is such that it minimises (Rafteryt, 1985) $BIC(m) = 2L_L + m \log N_T$, where N_T is the sample size.

It is not presented here, but both AIC and BIC criteria fails to give a clear consistent minima for all of the experimental points, which indicates that both information criteria might not be suited for this particular problem. The main objective of order identification is to include the long term effects of the chain and, consequently, to represent the behaviour of the passage of the unit cell. From the previous section, it was discussed that this is closely related to the zero-crossing of the autocorrelation function shown in Figure 3. Consequently, it can be argued that the order of the Markov Chain must be such that it can capture the lags at the first autocorrelation zero-crossing. Following this rational, the order of the chain is chosen that it is twice the number of lags until the first zero-crossing, summarised in Table 1.

In the following subsections, the characteristics and physical interpretation of the slug flow represented by the proposed two-state MC model is further investigated.

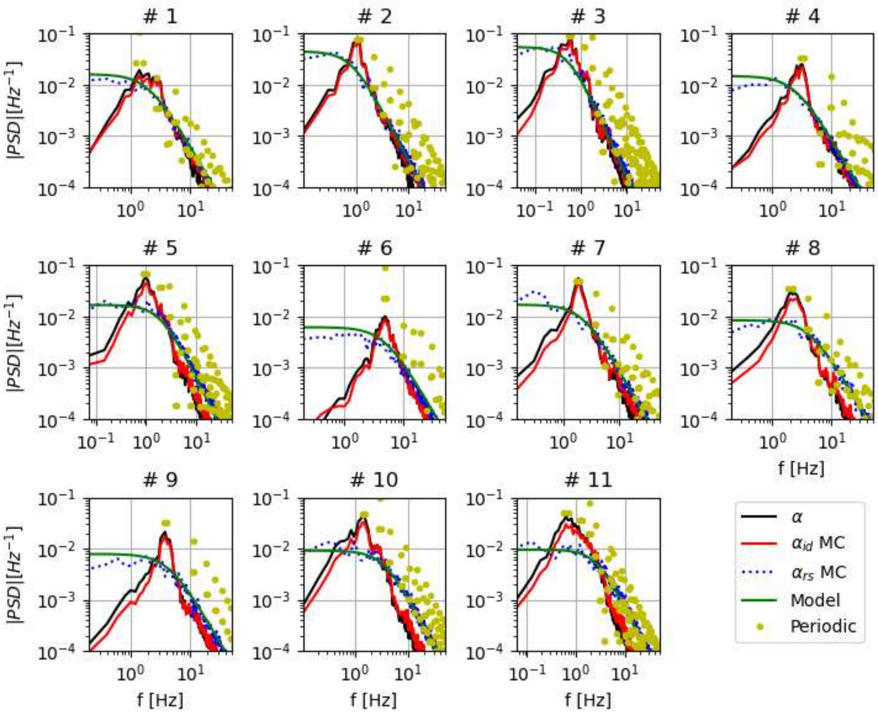

Figure 4: Power spectral density from each original measurement point (black full line), time series from Otsu threshold (red dashed line), a first order MC sample (blue dotted line), the first order MC analytical model (green full line) and the periodic case (yellow dot).

Table 1: Estimated Markov chain order for every experimental point.

Experimental point	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11
Markov chain order	20	46	68	16	36	8	22	18	12	28	38

3.2 Model validation

In this section, slug flow features are calculated for validation and discussion of the proposed approach. The intermittent factor β and the unit cell frequency f_u are calculated from the two-state time series generated from the experimental measurements, classified by the Otsu's threshold, and from the from the higher order Markov chain random sample, generated from the corresponding estimated transition matrix. The transition between two consecutive states is used to calculate the time $t_{Xn=0}$ at $X_n = 0$, the liquid slug, and the time $t_{Xn=1}$ at state $X_n = 1$, the gas bubble. Assuming the the same velocity for the unit cell, the intermittent factor is then estimated by $\beta = t_{Xn=1}/(t_{Xn=0} + t_{Xn=1})$. In addition, the frequency of passage of the unit cell can be estimated by $f_u = 1/(t_{Xn=0} + t_{Xn=1})$. Note that this approach estimates both β and f_u for each unit cell, thus providing a probability distribution for both variables as a consequence of the stochastic assumption about the nature of the void fraction α.

Figure 5 presents the histogram of the intermittent factor with 100 bins obtained from the measured time series and classified by the Otsu's threshold (Experimental - MC) and also from synthesised time series generated by a random sample of the Markov Chain (Sample - MC). Notice that for most experimental points, a good agreement between the estimates from experimental and simulated is found. This indicates the proposed two-state Markov chain model is representing well the statistics of the slug flow. The histogram of the experimental data can present a single dominant mode in the middle of the domain. In this cases, the mean values have a very good agreement. On the other hand, when the distribution presents some spikes of more than a single dominant mode outside the regions close to 0 and 1, the mean values present a greater divergence. But, in general, a very good representation is obtained for all cases.

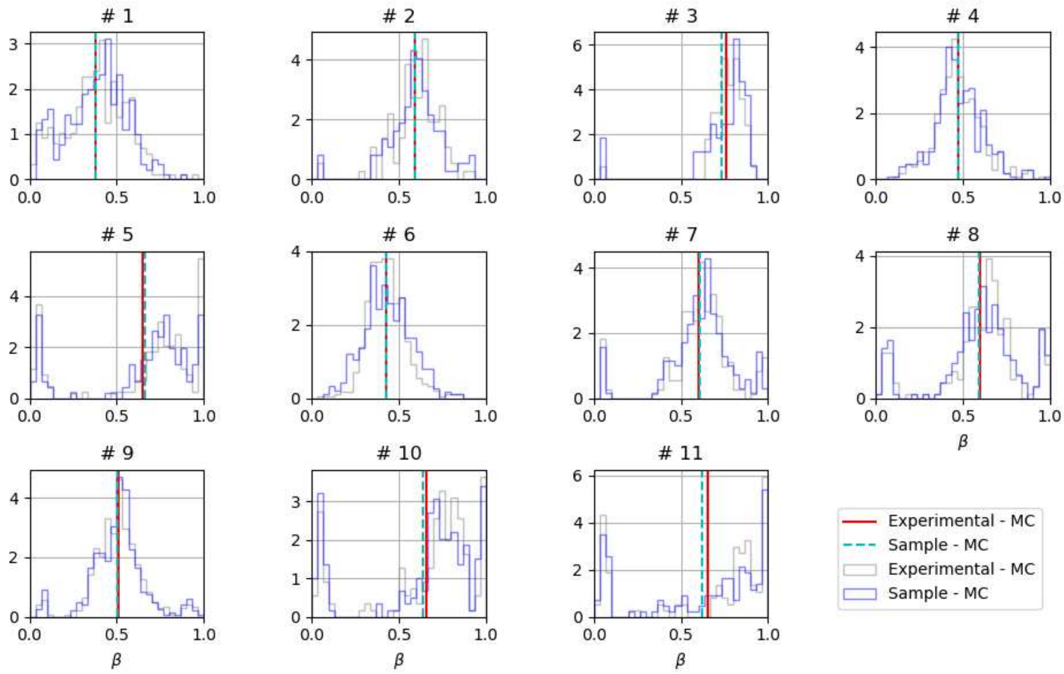

Figure 5: Histogram of the intermittent factor from the experimental data (grey) and from the Markov Chain model (blue). Vertical lines show the mean value for the experimental data (red full vertical line, the Markov Chain model (cyan dashed vertical line) and the slug fraction (black dotted vertical line) based on the local mean of the Otsu threshold.

Figure 6 presents the histogram of f_u with 100 bins obtained from the measured time series and classified by the Otsu's threshold (Experimental - MC) and also from synthesised time series generated by a random sample of the Markov Chain (Sample - MC). Similar to the previous case, the histograms in both cases present a very good agreement. The mean value for each case is also shown in the figure. In addition, the frequency of the peak value from the corresponding experimental PSDs, as shown in Figure 4, correspond to the dominant peak of histogram. Some experimental points present histograms well distributed around a prominent peak, e.g. #1, #2, #3, #4 and #7, while others present a flatter distribution, such as #5, #6, #8, #9, #10 and #11. In addition, the f_u distribution is clearly not unimodal for all of the cases. The peak frequency value of the PSD typically matches the peak of the f_u distribution, i.e., the most frequent case, rather than the mean value, as given for both the experimental and sampled two-state time series.

4 CONCLUSIONS

In this paper, a two-state Markov chain model was proposed to represent the stochastic dynamics of developed slug flow in horizontal pipes aiming at a simple but intuitive description of the phenomenon. Consequently, analytical expressions of the mean value, autocorrelation and power spectral densities were derived.

It is shown that this two-state representation is a reduced order representation that is suitable

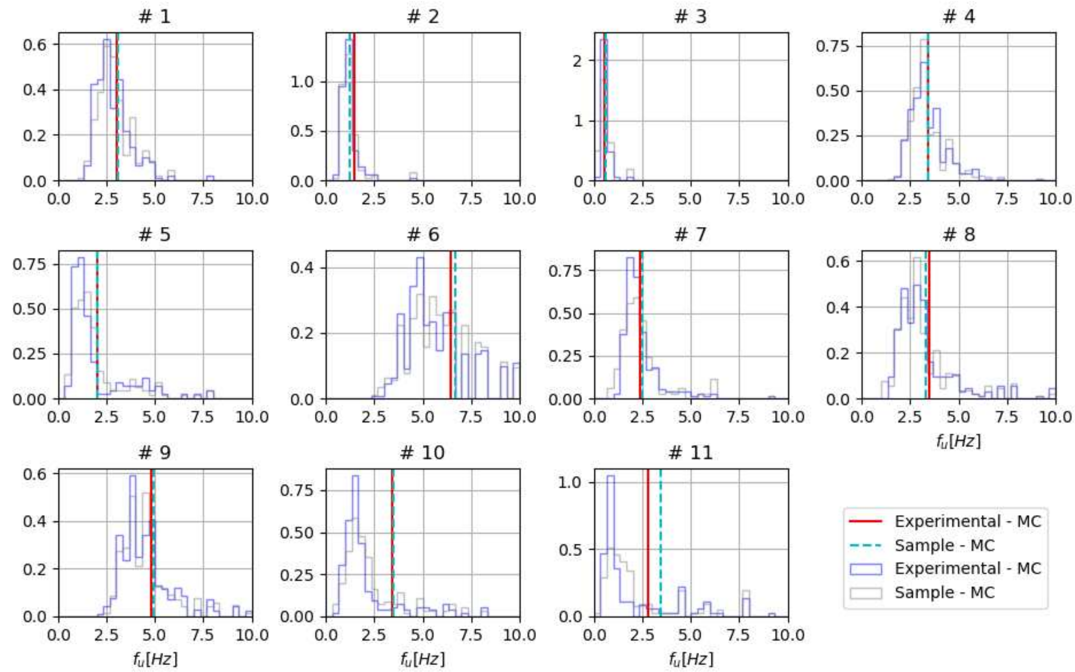

Figure 6: Histogram of the unit cell frequency estimated from the experimental data (grey) em from the Markov Chain sample (blue). Vertical lines show the median value of the experimental data (red full vertical line), the Markov Chain sample (cyan dashed vertical line) and the maximum PSD value.

to describe second-order statistics of the two-phase flow. Moreover, the proposed model leads to the representation of the intermittent factor and unit cell frequency as random variables, with given probability distribution. It is also shown that the distribution of the frequency of passage of the unit cell is clearly not unimodal for some experimental points. It is further shown that the proposed Markov Chain model can provide a good estimate of some slug flow features, such as the intermittent factor and the unit cell frequency.

Finally, the proposed approach opens the way for further physical interpretation and insights on the complex dynamics of the slug flow.

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