

A HAUSDORFF STABLE METHOD FOR FINDING SINGULARITIES WITH APPLICATION TO THE INTERSECTIONS OF SAMPLED MANIFOLDS

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Abstract. The processing and analysis of digital images represent fundamental tasks for anyone who rely on images to make decisions. Such decisions depend mostly on the detection and measurement of features and structures that the images might reveal and that are specific to the application area. A common assumption for the analysis and detection of a given feature consists in identifying it with a singularity of the image. One can reveal a feature to the extent that such feature contrasts with its neighbourhood background. Current techniques of feature detection, in fact, can be generally seen as development of methods that aim at the enhancement and/or selection of the singularity representative of the feature. In the class of methods adapted from the analysis of digital signals, these are obtained by comparing pixel values in a predetermined mask using some ad hoc problem-designed convolution function. In those methods that use partial differential equations or variational principles, on the other hand, one usually assumes that a starting curve, surface or image is deformed so to obtain the desired result. It is therefore clear that such methods appear very specific to the problem setting used for their development. This make them difficult to apply to real problems or to adapt them to problems different from the idealized setting. We will present a family of novel methods for feature detection and image restoration which have a very clear geometrical interpretation, though the application areas, are not only limited to these ones. Our methods rely on the idea of realizing a close smooth approximation of the digital image or of a modified image which creates the singularity at the feature of interest. Given the input function, by close smooth approximation we mean that our transformation outputs a smooth function which coincides with the input function in the neighbourhood where the function is smooth. As a result, by difference one gets a neighbourhood of the singularity. With this respect, we could term them as geometric based methods for singularity detection. By such transformation, we are able to develop multi-scale, parametrised methods for identifying singularities in functions. These tools can then be used, via a numerical implementation, to detect features in images or data (e.g. edges, corner points, blobs, etc.), remove noise from images, identify intersections between surfaces, etc, and thus produce new geometric techniques for image processing, feature extraction and geometric interrogation.