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ASSESSMENT OF DIFFERENT FRACTURE REGIMES IN ANDESITE ROCK VIA ACOUSTIC EMISSION, WAVELET TRANSFORM AND ENERGY B-VALUE

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Abstract. The damage process occurring in Andesite rock specimens subjected to uniaxial compression tests up to failure is investigated. During the test, the propagating elastic waves (Acoustic Emission, AE) due to micro and macro-crack growth are detected. Damage assessment is performed by determining and processing individual AE hits with a large number of overlapping transients with variable intensities. It is important to detect the sudden transition from diffused micro-cracking to localized macro-cracks which characterizes the catastrophic failure in brittle materials; these sudden transitions can be associated with critical values of AE parameters. It is known that different fracture regimes in brittle materials are connected with different characteristic frequencies of AE signals. The AE hits were analysed with the Continuous Wavelet Transform, and Wavelet Entropy was calculated in order to detect relevant scale (frequency) bands. A modification of the original b-value (Gutenberg-Richter parameter), was afterwards used to assess damage in different frequency bands. This modification, called energy b-value, is obtained (for each relevant frequency band) as the local slope of the log-log cumulative distribution of hits as a function of their AE energy. Local b-value minima indicate critical AE energy values. Thus, AE hits related to critical damage can be located along the test. Results were corroborated by comparison with the Cumulative AE (CAE) energy in each band. The hits with near-critical energy values preceded CAE jumps, which indicate coalescence of micro-cracks into macro-cracks.

1 INTRODUCTION

Elastic waves are produced inside the materials submitted to stress due to the sudden release of mechanical energy. The processes frequently involved are deformation, dislocations movement, inclusions rupture and cracks nucleation, growth, friction and closure. The Acoustic Emission technique gathers information about these processes by detecting and measuring elastic waves with piezoelectric sensors. These sensors are placed on the surface of the material and transform mechanical signals into electrical ones, i.e. AE signals. They are in the form of bursts called hits and are processed for further analysis. AE signals are very low in amplitude ($\approx 10\mu V$) and high in frequency ($\approx 1kHz - 1MHz$) (Grosse and Ohtsu, 2008).

Rocks are brittle compound materials and contain cracks: when loaded, these flaws concentrate stresses. In this paper, we continue previous work on Andesite rock from Cerro Blanco, Cordillera de los Andes, an earthquake-prone zone (Filipussi et al., 2015; Muszkats et al., 2019). The rocky material was subjected to uniaxial compression tests and the resulting damage processes were monitored with AE sensors. Analytical seismic models (Aki and Richards, 1980) and the Gutenberg-Richter law used in seismology were adapted to AE signals by different authors (Rao and Prasanna Lakshmi, 2005).

Stress concentrations lead cracks to link up and form advancing features, such as geological faults. This is how earthquakes, landslides, and volcanic eruptions start: processes occurring at a small scale have large scale consequences. Experiments have shown that there is a rapid transition in the behaviour of porous materials under stress: when reaching a certain point, the cracks interact and coalesce in a narrow zone, rather than being distributed throughout the material. Therefore, it is important to apply techniques that quantify the transition between distributed (stable) deformation and localized (unstable) deformation (Rizzo et al., 2017). Thus, micro-cracking activity occurs in a phased manner while the rock is deforming under stress. It has been successfully monitored in the laboratory, using AE as a tool (Rao et al., 2011) and including AE parameters such as amplitude distribution (b-value), cumulative energy and ring down count.

Fracture in brittle materials, and the related AE, involve different spatial and temporal scales ranging from microscopic events to seismic faults. In previous work, a model of a source crack propagating in a homogeneous and isotropic material was studied with the Fourier Transform (Filipussi, 2012), obtaining a characteristic frequency for Andesite rock. But different mechanisms related to fracture, such as nucleation, advance, widening, closure and interface friction, occur during different stages of the whole process. And these mechanisms can be related to different scales or frequency bands. AE is then essentially a non-stationary process and CWT has proven to be well suited to detect the frequency band corresponding to dangerous cracks (Sagasta et al., 2018). The Hilbert Huang Transform, an alternative time-frequency analysis (Huang et al., 1998), can also be applied to AE signals. It provides an empirical basis, obtained from data. On the other hand, CWT with the Morlet wavelet offers very good frequency discrimination. And as it consists of an analytical basis, it can eventually be used for a fracture model improving the one described by Filipussi (2012).

In the present work, AE signals were filtered for different frequency bands with the Continuous Wavelet Transform (CWT) (Torrence and Compo, 1998). The premise is that different frequency bands are associated with different mechanisms of the overall fracture process. The detection of characteristic frequencies in our previous work (Muszkats et al., 2019) has been reinforced in the present work with a criterion based on the Wavelet Entropy, as defined by Piotrkowski et al. (2009). Moreover, a similar entropy criterion has already been successfully applied by these authors in a previous presentation concerning another brittle material (Sassano et al., 2017). Thus, in the selected band we have studied parameters associated with the coalescence of micro-cracks into dangerous macro-cracks. A parameter presented by Sagasta et al. (2018), the energy b-value (b_E), was calculated for the chosen band.

In our contribution, the detailed calculation of the energy b-value allowed us to highlight different alternating fracture regimes as a function of energy. Results might, in future work, be compared with those obtained by other researchers (Stormo et al., 2015; Wang et al., 2018) and interpreted as an expression of impulsive systems (Mancilla-Aguilar and Haimovich, 2019).

2 EXPERIMENT

Following the work reported by Muszkats et al. (2019), four cylindrical Andesite rock samples from Cerro Blanco, San Juan, Argentina, were tested as shown in Fig. 1. These specimens were $75mm$ in diameter, $150mm$ in length and were subjected to single uniaxial compression tests up to rupture. The equipment consisted of a CGTS machine with a $100tons$ capacity of servo-hydraulic type and a closed-loop. The actuator displacement speed was $0.12mm/min$. Three piezoelectric sensors were used to monitor AE. Only the result of one of the tests is presented, considering the broadband sensor $(100 - 1000kHz)$. The AE system was a PCI-2 two-channel PAC plate and the commercial software AEWIN was used for initial determination of classical AE parameters. The present work focuses on the results obtained in one of the samples, as recorded by the broadband sensor with a $1MHz$ sampling frequency.

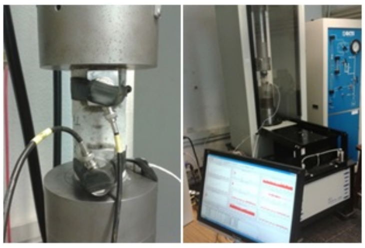

Figure 1: The experimental setup and the AE system

3 MATHEMATICAL RESOURCES

3.1 Continuous Wavelet Transform

Following Boggess and Narcowich (2009), the Continuous Wavelet Transform (CWT) is defined by means of a continuous wavelet function $\psi(t)$, with exponential decay that also verifies $\int_{\mathbb{R}} \psi = 0$. Under these assumptions, the CWT of a function $f \in L^2(\mathbb{R})$ is defined as $c : \mathbb{R}^2 \to \mathbb{C}$, where

$$c(j,k) = \frac{1}{\sqrt{|j|}} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-k}{j}\right)} dt \tag{1}$$

if $j \neq 0$, while $c(0,k) = 0$. In each CWT coefficient c_{jk} the value of j indicates a scale (and therefore a frequency), while k denotes time displacement. In the present work we used the Morlet wavelet defined by

$$\psi(t) = \pi^{-\frac{1}{4}} \cdot e^{6it} \cdot e^{-\frac{t^2}{2}} \tag{2}$$

Implementation details can be found in Torrence and Compo (1998). As an example, the plot of a hit is illustrated in Fig. 2, with its corresponding scalogram (that is, the distribution in time and frequency of the wavelet energy density).

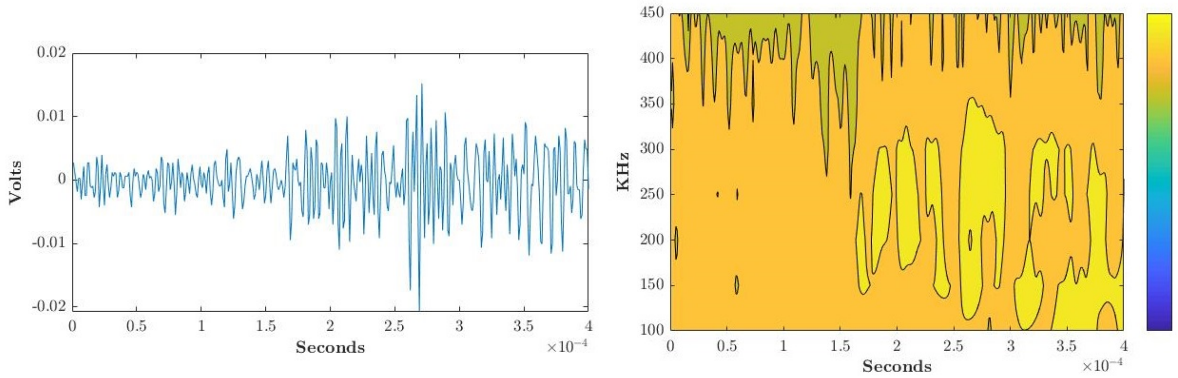

Figure 2: Plot of a hit and its scalogram

A simple bandpass filtering can be performed by reconstructing the signal only with the CWT coefficients of the desired frequencies. Fig. 3 illustrates the same hit filtered at $250kHz$. As we shall see later, this particular frequency will prove to be very relevant in the macro-fracture process.

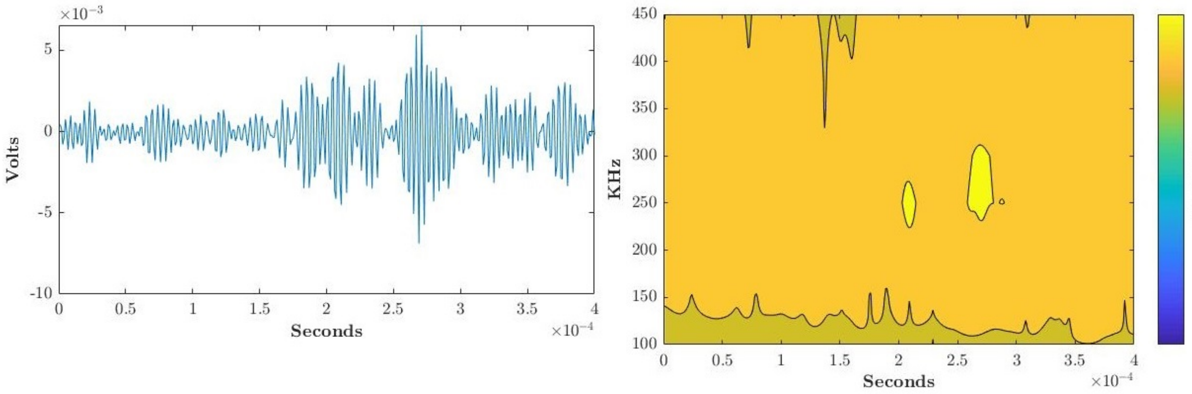

Figure 3: Plot of the filtered hit (same as in Fig. 2) and scalogram

3.2 Wavelet Entropy

What follows is an adapted version (Piotrkowski et al., 2009) of the Shannon Entropy for the wavelet coefficients obtained with formula Eq. (1). Given that in practice only a finite quantity of c_{jk} coefficients is obtained, the Wavelet Power (WP) corresponding to the j-th scale is defined by

$$WP_j = \sum_{k=1}^{N} |c_{jk}|^2 \tag{3}$$

The coefficients p_{jk} express the fraction of Wavelet Power corresponding to time k:

$$p_{jk} = \frac{|c_{jk}|^2}{WP_j} \tag{4}$$

The definition of the j-th band Wavelet Entropy is formally equal to the Shannon Entropy:

$$S_j = -\sum_{k=1}^{N} p_{jk} \log p_{jk} \tag{5}$$

For the present purposes, entropy is a measure of the intrinsic order in a signal. Fig. 4 shows the values of entropy calculated for the different scales (frequencies) of the previously studied hit. Relative minima suggest the presence of more organized phenomena corresponding to a certain frequency.

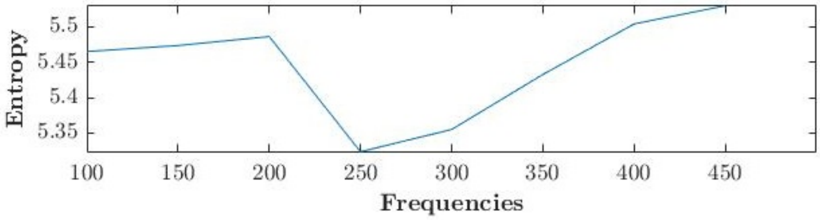

Figure 4: Wavelet Entropy vs frequency for the hit studied in Fig. 2

3.3 Energy b-value

In the present context, AE may be interpreted as small scale seismology. Following previous work (Sagasta et al., 2018), the Gutenberg-Richter law used in seismology was adapted in order to detect dangerous cracks in the present experiment. The Acoustic Emission Energy (AEE) of a hit is defined by

$$AEE = \sum x_i^2 \cdot \Delta t \tag{6}$$

and was calculated after band filtering and reconstructing each hit. $N(AEE)$ counts the number of hits with energy greater than the given value of energy. Fig. 5 relates AEE and $N(AEE)$ for all the hits filtered at $250kHz$. Additionally, there is a colour scale indicating the time of occurrence of each hit. It can be appreciated that the most energetic hits tend to occur by the end of the mentioned experiment.

This data can be linearly fit according to the following theoretical distribution

$$\log_{10} N(AEE) = a - b_E \log_{10}(AEE) \tag{7}$$

The b_E obtained is the Energy b-value. In the present work, it was locally calculated by taking sequences of 3000 consecutive hits.

Large b_E values denote a rapid change in the value of N with respect to energy (that is, slow change of energy with respect to N). This suggests a relatively stable micro fracturing regime. On the other hand, a small b_E value implies a rapid change of energy for a few hits, which might indicate the formation of macro fractures or rapid growth of fractures.

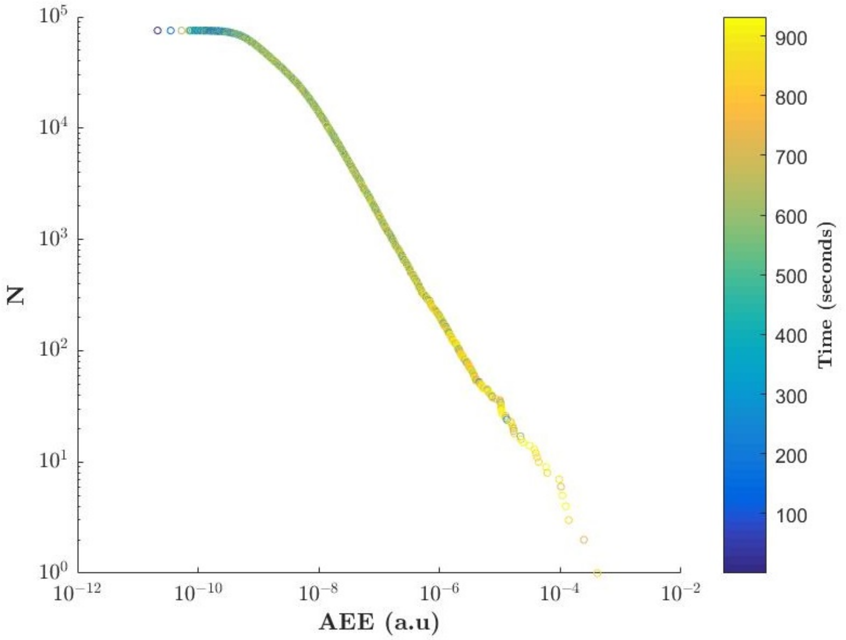

Figure 5: N vs AEE for the $250kHz$ band

4 RESULTS AND DISCUSSION

4.1 Identification of relevant frequencies

As previously said, damage in brittle materials occurs via the alternation of stable and unstable fracture regimes. This pattern respectively implies micro-cracking and coalescence into rapid advancing macro-cracks. Therefore, our first goal was to identify, among the impulsive signals, those for which the energy showed to be more concentrated in the time-frequency plane.

In order to choose a trustworthy frequency resolution, several features were taken into account: the range of the broadband sensor $(100 - 1000kHz)$, the sampling frequency $(1MHz)$, the duration of shortest hits $(\approx 10\mu s)$ and the Heisenberg uncertainty principle ruling in harmonic analysis. Under these premises, the width of frequency bands was chosen to be $50kHz$.

The CWT defined in Eq. (1) allows a choice of scales (and frequencies) that grow linearly, instead of exponentially. Fig. 2 and Fig. 3 illustrate the $50kHz$ step.

As entropy is a measure of the intrinsic order in a signal, finding the frequency band for which relative minima mostly occur suggests the presence of more organized phenomena. Every single hit was analysed as in Fig. 4. That is, frequencies with relative minimum entropy were detected according to a threshold criterion. It is relevant to make clear that each hit might present a single relative minimum, several or none. The results are charted in Fig. 6: each single dot represents a hit which attains (relative) minimum entropy for a certain frequency at a given time. Fig. 7 is a histogram illustrating the density of hits for a certain frequency at a given time. Table 1 displays basic statistical information about hits and their entropies. A significant number of minima occur in only three bands: 150, 200 and $250kHz$. A slight descending tendency for the mean entropy values can be observed for these bands, followed by an increase after $250kHz$.

Several interconnected phenomena occur during the fracture of brittle materials. Among them are: nucleation, advance, bifurcation, closure, and friction of interfaces. Some of them, which are irrelevant for the present study, manifest themselves from the beginning and keep going on along the test. This is the case of the closure of initial previous fractures and the

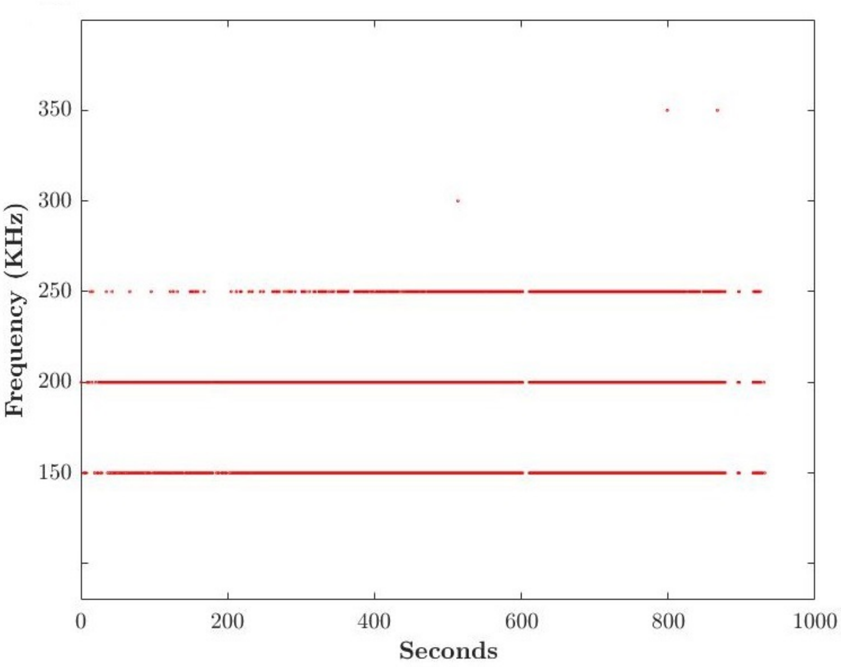

Figure 6: Frequencies with minimum entropy and their time location

Frequency (kHz)	Number of minima	μ (entropy mean value)	σ (entropy standard deviation)
100	0	5.81	0.44
150	4978	5.65	0.51
200	3675	5.60	0.54
250	1234	5.60	0.53
300	1	5.64	0.51
350	2	5.63	0.51
400	0	5.59	0.52
450	0	5.54	0.54

Table 1: Number of relative minima, entropy mean and standard deviation considering all the hits in each frequency band

friction of fracture faces. For this reason, we are only interested in those frequency bands which show entropy minima starting at advanced states of the test. This kind of minima can be attributed to macro-cracks generation and advancement. Both Fig. 6 and Fig. 7 as well as Table 1 point to the $250kHz$ band, which begins to neatly manifest after 200 seconds of the test have elapsed.

4.2 Energy b-value and its minima

After selecting the $250kHz$ frequency as the most relevant, every hit was analysed and reconstructed with the coefficients corresponding to this band. Once sorted, the resulting signals (shown in Fig. 5) allowed a local estimation of energy b-value. Nevertheless, as energy level increases and plot points become sparser, the least-squares approximation results coarser. The result, shown in Fig. 8, is a fairly smooth function in the low energy region, were local minima can be spotted.

The energy values for which local minima occur are of special interest. Fig. 9 illustrates the

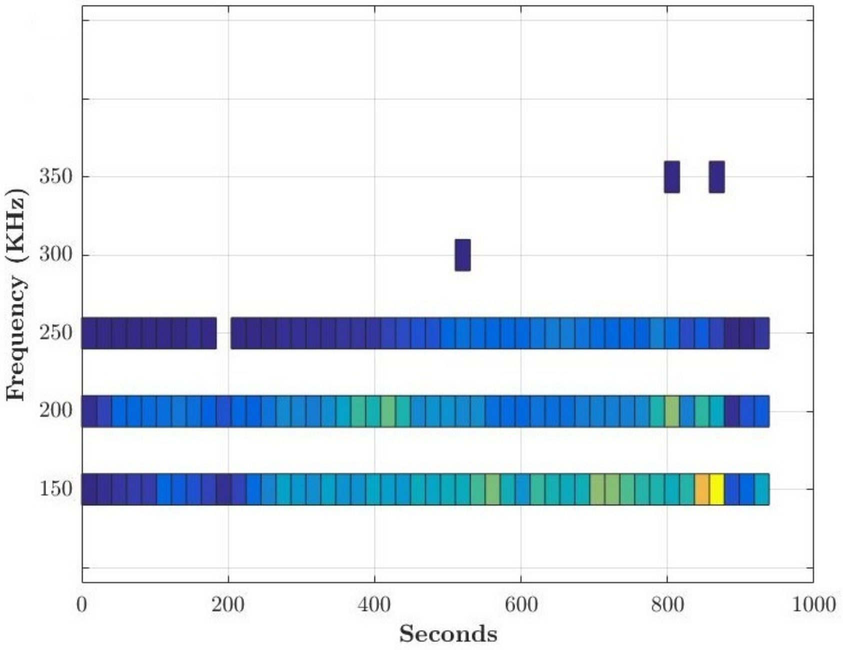

Figure 7: Density of frequencies with minimum entropy

cumulative energy as a function of time. It also shows those hits with energy values closest to the b-value local minima: on the left side, those closest to $1.5257 \cdot 10^{-9}$ and on the right side those closest to $2.0585 \cdot 10^{-9}$. Remarkably, for every sudden change in the energy level, at least one of these hits can be found.

5 CONCLUSIONS

The entropy criterion showed to be useful and reliable for the detection of relevant frequency bands. It also proved to be coherent with previous work and other criteria (Muszkats et al., 2019).

$250kHz$ was considered to be a characteristic frequency of macro fracture mechanisms. Therefore, all the subsequent study was performed for hits previously band-filtered around this value. The energy b-value reached two local minima, and their respective energy levels are of great importance. Those hits with energy closest to the mentioned levels are precursors of dangerous fracture: the cumulative energy plot shows that they precede every major change in energy level.

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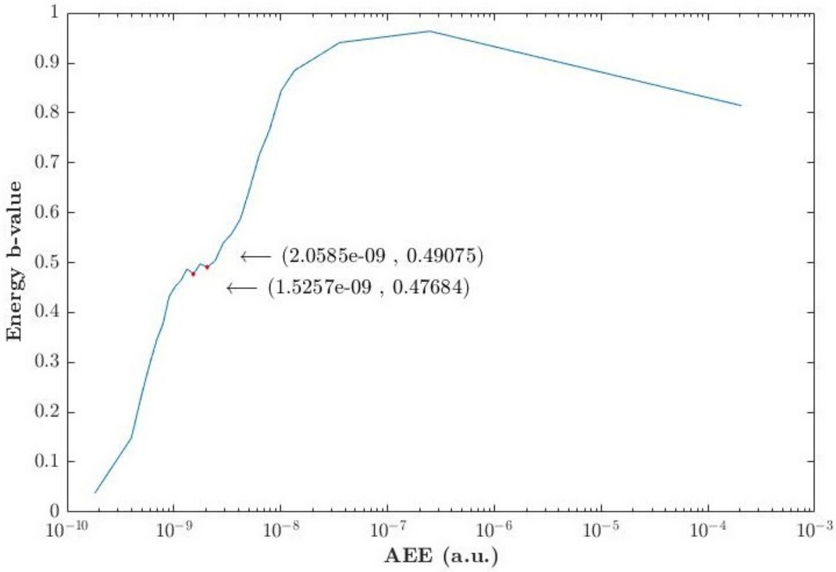

Figure 8: Energy b-value as a function of energy

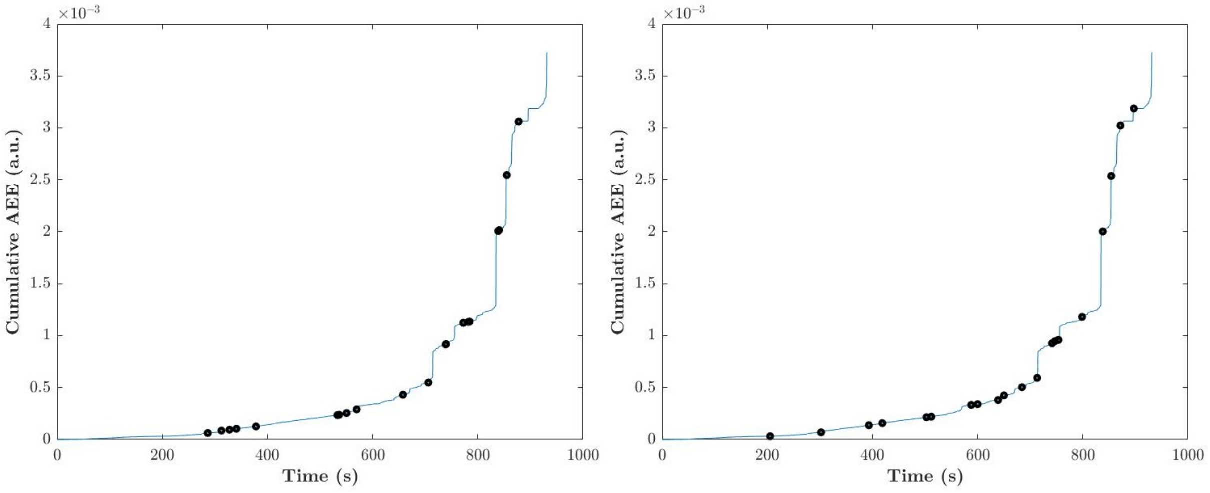

Figure 9: Cumulative energy as a function of time and hits closest to b-value minima

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