Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXV, pp. 1583-1592
Alberto Cardona, Norberto Nigro, Victorio Sonzogni, Mario Storti. (Eds.)
Santa Fe, Argentina, Noviembre 2006

MODEL UPDATING OF A FRAME STRUCTURE USING PENALTY FUNCTIONS BASED PROCEDURES

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Keywords: Model Updating, Indirect Methods, System Identification.

Abstract. An important requirement in design is to be able to compare experimental results from prototype structures with predicted results from a corresponding finite element model. In this context, model updating may be defined as the fit of an existing analytical model in the light of measured vibration test. After fitting, the updated model is expected to represent the dynamic behavior of the structure more accurately. An important class of model updating methods is the penalty functions based approaches (the so-called indirect methods), in which the objective is to maximize the correlation between the measured and analytical modal model. Because of its nature, the solution requires the problem to be linearized and optimized iteratively. As the advantages, these methods allow a wide choice of parameters to be updated that keeps the physical meaning and both the measured data and the initial analytical parameter estimates may be weighted, which gives the versatility to the methodology. In this way, this paper presents an indirect based updating study of a reduced scale three-story plane frame structure constructed and tested at the Laboratory of Structural Dynamics and Reliability (LDEC) of the Federal University of Rio Grande do Sul (UFRGS), Brazil. The updated results, presented in this paper, showed great accuracy. The stiffness and mass matrices were able to keep the original pattern of the finite element model of the structure.

1 INTRODUCTION

In modern dynamic design and analysis of complex structures, it is extremely necessary to deal with reliable mathematical models. This becomes especially true for structures whose operation, integrity, safety and control critically depend on the structure's dynamic characteristics. In this context, one way to verify the math model accuracy is by comparing the experimental results provided through the conduction of dynamic tests with those expected from a previous analytical analysis.

Generally smooth differences are found, and it is believed that the experimental modal data offer more confidence than the finite element model. Therefore, in order to determine the spatial properties of the structure which can reproduce the whole characteristics of the test structures (measured or unmeasured), reconciliation processes including model correlation and model updating, must be performed. This process of modifying the mathematical model in order to achieve a good agreement with the measured data is called model updating.

The model updating in structural dynamics may be divided into two main categories, that is: the direct and the iterative methods. In the first group, the model is expected to match some reference data, usually consisting of an incomplete set of eigenvalues and eigenvectors derived from measurements. These direct methods are also known as representation models because they are able to replicate the measured data, but the main drawbacks are: high quality measurements are required and accurate modal analysis, the mode shapes must be expanded to the finite element mode size and the usual loss of connectivity of the structure with updated matrices fully populated.

The second category or the iterative updating methods has as main goal to improve the correlation between the experimental and analytical models via a penalty function. Because of the general nature of penalty functions, the problem has to be linearized and thus optimized iteratively. Since the penalty function is usually non-linear, the iterations may not converge. In any case, iterative methods have two main advantages. First, a wide range of parameters can be updated simultaneously and second, both measured and analytical data can be weighted, a feature which can accommodate engineering intuition.

In this context, the main goal of the present paper is to carry out a vibration based iterative model updating approach through the penalty function based procedures using experimental data. The studied structure is a reduced scale three-story plane frame constructed and tested at the Laboratory of Structural Dynamics and Reliability (LDEC) of the Federal University of Rio Grande do Sul (UFRGS), Brazil, for different kinds of dynamic studies. The results of the finite element model are compared with those obtained experimentally and in the sequence an updating procedure is conducted. It is shown that with a little iterations the parameters and frequencies quickly converge and the model is able to keep the physical meaning.

2 ITERATIVE UPDATING TECHNIQUES BASED ON PENALTY FUNCTIONS

Penalty function methods express the modal data as a function of the unknown parameters using a truncated Taylor series expansion. The series is truncated to yield the linear approximation:

$$\delta z = \boldsymbol{S}\delta\boldsymbol{\theta} \tag{1}$$

where $\delta\boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}_j$, $\boldsymbol{\theta}_j$ is the current value of the parameter vector, $\boldsymbol{\theta}$ is the estimated vector, $\delta z = \boldsymbol{z}_e - \boldsymbol{z}_j$, $\boldsymbol{z}_e$ is the measured output, $\boldsymbol{z}_j$ is the current estimate of the output, $\boldsymbol{S}$ is the sensitivity matrix containing the first derivative of the eigenvalues and mode shapes with respect to the parameters, evaluated at the current parameter estimate $\boldsymbol{\theta}_j$.

Calculate those first derivatives of the eigenvalues and mode shapes with respect to the parameters is computationally intensive and efficient methods for their computation are required. Fox and Kapoor (1968), calculated the derivative of the *ith* eigenvalue, λ_i, with respect to the *jth* parameter, θ_j, by taking the derivative of the eigenvector equation, to give:

$$\left(\frac{\delta \boldsymbol{K}}{\delta\theta_j} - \lambda_i \frac{\delta \boldsymbol{M}}{\delta\theta_j}\right)\boldsymbol{\varphi}_i - \frac{\delta\lambda_i}{\delta\theta_j}\boldsymbol{M}\boldsymbol{\varphi}_i + \left(\boldsymbol{K} - \lambda_i \boldsymbol{M}\right)\frac{\delta\boldsymbol{\varphi}_i}{\delta\theta_j} = 0 \tag{2}$$

Pre-multiplying by the transpose of the eigenvector, $\boldsymbol{\varphi}_i$, and using mass orthogonality and the original definition of the eigensystem produces:

$$\frac{\delta\lambda_i}{\delta\theta_j} = \boldsymbol{\varphi}_i^T\left(\frac{\delta \boldsymbol{K}}{\delta\theta_j} - \lambda_i \frac{\delta \boldsymbol{M}}{\delta\theta_j}\right)\boldsymbol{\varphi}_i \tag{3}$$

These authors have also suggested two methods for calculating the first derivative of the eigenvectors. Lim (1987) suggested an approximate method for calculating the first derivative of the eigenvectors which is only valid for the low frequency modes. Other methods for calculating mode shapes derivatives have been suggested by Chu and Rudisill (1975), Ojalvo (1987) and Tan and Andrew (1989).

The penalty functions methods differ in the choice of design parameters and the definition of optimization constraints. Design parameters such as individual elements of the mass and stiffness matrices, sub-matrices, geometric or material properties can be defined. Constraints are usually imposed on natural frequencies and mode shapes.

Usually, the number of design parameters and measurements is not equal and hence the matrix $\boldsymbol{S}$ in (1) is not square. The case in which there are more design parameters than measurements was considered by Chen and Garba (1980). The parameter vector closest to the original analytical parameters was sought which reproduce the required measurement change. They found the solution to the problem by seeking a set of design parameters by minimizing the norm as an additional constraint equation:

$$Q = \sum_j \Delta\boldsymbol{\theta}_j^2 \tag{4}$$

Similarly, the SVD technique was used by Hart and Yao (1977) and Ojalvo et al. (1989) for a case with less design parameters than measurements. The solution of equation 1 can be calculated by minimizing the penalty function:

$$J(\delta\boldsymbol{\theta}) = (\delta\boldsymbol{z} - \boldsymbol{S}\delta\boldsymbol{\theta})^T(\delta\boldsymbol{z} - \boldsymbol{S}\delta\boldsymbol{\theta}) \tag{5}$$

where $\varepsilon = \delta\boldsymbol{z} - \boldsymbol{S}\delta\boldsymbol{\theta}$ is the error in the predicted measurements based on the updated parameters. Differentiating J with respect to $\delta\boldsymbol{\theta}$ and setting the result equal to zero, it can be shown that the solution is given by:

$$\delta\boldsymbol{\theta} = \left[\boldsymbol{S}^T\boldsymbol{S}\right]^{-1}\boldsymbol{S}^T\delta\boldsymbol{z} \tag{6}$$

and an updated estimate of the unknown design parameter vector is obtained by:

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \delta\boldsymbol{\theta}_j \tag{7}$$

In practical situations, all measured data do not have the same accuracy. Usually, mode shape data are less accurate than natural frequency data. Also the higher natural frequencies are not measured as accurately as the lower ones. The relative accuracy of measured data can be incorporated into the updating process by including a diagonal positive definite weighting matrix $\boldsymbol{W}_{\varepsilon\varepsilon}$, whose elements are given by the reciprocals of the variance of the corresponding measurements. Equation 5 becomes:

$$J(\delta\boldsymbol{\theta}) = (\delta\boldsymbol{z} - \boldsymbol{S}\delta\boldsymbol{\theta})^T \boldsymbol{W}_{\varepsilon\varepsilon}(\delta\boldsymbol{z} - \boldsymbol{S}\delta\boldsymbol{\theta}) \tag{8}$$

The minimization of equation 8 yields:

$$\delta\boldsymbol{\theta} = \left[\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\boldsymbol{S}\right]^{-1} \boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\delta\boldsymbol{z} \tag{9}$$

or in full,

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \left[\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\boldsymbol{S}\right]^{-1} \boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}(\boldsymbol{z}_m - \boldsymbol{z}_j) \tag{10}$$

In either solutions equation 6 or equation 10, the number of measurements was assumed to be larger than the number of parameters. Under this assumption the matrix is square being full rank, so the equations may be solved. However, in almost all practical cases this situation will not occur, *i.e.*, the number of unknown parameters will exceed the number of measured data points. Due this problem $\boldsymbol{S}^T\boldsymbol{S}$ will be rank deficient because the number of equations in equation 1 is less than the number of unknowns. An alternative approach (Natke, 1988) is to add an extra term to minimize the change of the design parameters. The extended weighted penalty function can be expressed as:

$$J(\delta\boldsymbol{\theta}) = \varepsilon^T \boldsymbol{W}_{\varepsilon\varepsilon}\varepsilon + \delta\boldsymbol{\theta}^T \boldsymbol{W}_{\theta\theta}\delta\boldsymbol{\theta} \tag{11}$$

where again $\varepsilon = \delta\boldsymbol{z} - \boldsymbol{S}\delta\boldsymbol{\theta}$ is the error in the predicted measurements based on the updated parameters. Next is added a positive definite weighting matrix $\boldsymbol{W}_{\theta\theta}$ chosen to be a diagonal matrix with the reciprocals of the estimated variances of the corresponding parameters as the elements. These variances are not an easy task and some engineering insight is required. The solution of $\delta\boldsymbol{\theta}_j$ is given by:

$$\delta\boldsymbol{\theta} = \left[\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\boldsymbol{S} + \boldsymbol{W}_{\theta\theta}\right]^{-1} \boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\delta\boldsymbol{z} \tag{12}$$

or in full,

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \left[\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\boldsymbol{S} + \boldsymbol{W}_{\theta\theta}\right]^{-1} \boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}(\boldsymbol{z}_m - \boldsymbol{z}_j) \tag{13}$$

A similar approach to obtaining a well conditioned set of equations is to weight the initial estimates of the unknown parameters. This more accurately reflects the engineer's desire to weight the change in parameter from the initial estimated values, rather than the parameter change at every iteration. Thus, the new penalty function is given by:

$$J(\delta\boldsymbol{\theta}) = \varepsilon^T \boldsymbol{W}_{\varepsilon\varepsilon}\varepsilon + (\boldsymbol{\theta} - \boldsymbol{\theta}_0)^T \boldsymbol{W}_{\theta\theta}(\boldsymbol{\theta} - \boldsymbol{\theta}_0) \tag{14}$$

where the solution is:

$$\delta\boldsymbol{\theta} = \left[\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\boldsymbol{S} + \boldsymbol{W}_{\theta\theta}\right]^{-1} \left(\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon}\delta\boldsymbol{z} - \boldsymbol{W}_{\theta\theta}(\boldsymbol{\theta}_j - \boldsymbol{\theta}_0)\right) \tag{15}$$

or in full,

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \left[\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon} \boldsymbol{S} + \boldsymbol{W}_{\theta\theta}\right]^{-1} \left(\boldsymbol{S}^T \boldsymbol{W}_{\varepsilon\varepsilon} \left(\boldsymbol{z}_m - \boldsymbol{z}_j\right) - \boldsymbol{W}_{\theta\theta} \left(\boldsymbol{\theta}_j - \boldsymbol{\theta}_0\right)\right) \tag{16}$$

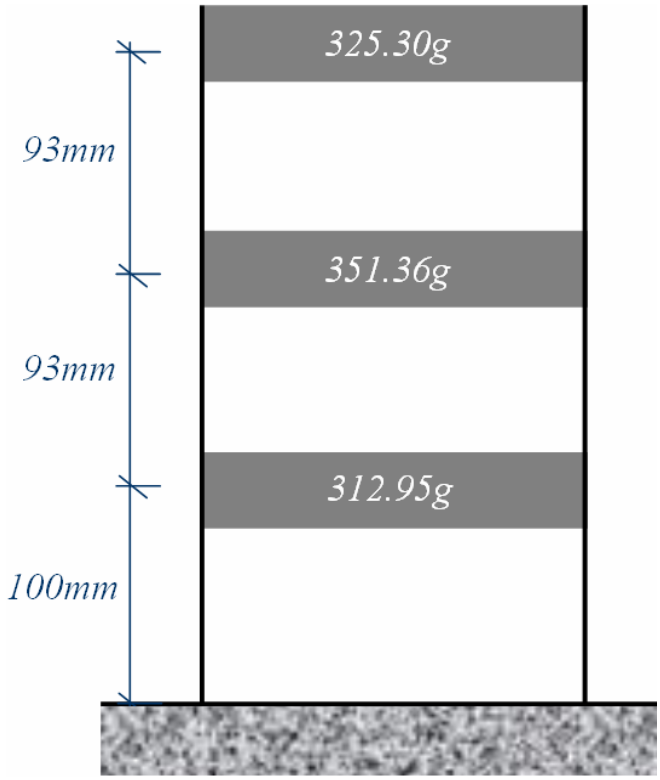
3 APPLICATION: SHEAR BUILDING PLANE FRAME

The iterative updating approach is verified using a typical reduced scale three-story plane frame shown in Figure 1(a). This model was constructed and tested at the Laboratory of Structural Dynamics and Reliability (LDEC) of the Federal University of Rio Grande do Sul (UFRGS), Brazil, for different kinds of dynamic studies.

The model has three stories, which can be considered as rigid plates, and two elastic columns. This assumption is valid because the stiffness of the girders is much higher than the stiffness of the columns, which allows neglecting the flexibility of the former.

Each one of the two steel columns has cross section dimensions of $b = 19mm \times t = 0.62mm$ and Young's modulus equal to 2×10^{11} N/m^2. The two highest stories have a floor-to-ceiling height, h, of 93mm and the lowest story has a 100mm floor-to-ceiling height. The structural columns are tightly clamped at each floor. The mass of each degree of freedom takes account besides the floors' masses, the columns' masses, the accelerometers masses and accelerometers' supports masses. Geometrical and physical details may be seen in Figure 1(b).

(a)



(b)

Figure 1: Shear Building Model

After known the model geometrical and physical characteristics, it is conducted a theoretical analysis through finite element method to obtain the initial stiffness and mass matrices. During the dynamic tests, piezoelectric accelerometers (Bruel & Kjaer), signal amplifiers (Bruel & Kjaer), an acquisition board (ComputerBoards) and the software HP VEE

5.0 (Hewlett Packard) are used to measure the response of the structure. The experimental frequencies are selected as the peaks of the response spectrum when the structure is subjected to an impulsive loading. Table 1 presents a comparing analysis of the finite element model and the experimental results.

Modes	Theoretical Analysis (Hz)	Experimental Results (Hz)
1^{st}	5.47	5.2
2^{nd}	16.04	16.3
3^{rd}	23.27	23.0

Table 1: Compared frequency results.

As may bee seen, the obtained results showed very close but not identical. In this way, a model updating study must be carried out. In this situation, six parameters will be considered in the process (the stiffness and the mass of each degree of freedom) and just three measured data (the three frequencies) are available. Thus the number of unknown parameters is higher than the number of measurements, which lead the solution of updating problem via equation 16.

The sensibility matrix is a non-square $\boldsymbol{S}_{3x6}$ matrix and it was determined through Fox and Kapoor (1968) procedure (given in equation 2). The weighting matrices $\boldsymbol{W}_{\varepsilon\varepsilon}$ and $\boldsymbol{W}_{\theta\theta}$ were formed by the reciprocals of the variance of the corresponding measurements and by the reciprocals of the variance of the corresponding parameters, as was pointed out in section 2.

Observing Table 2 it is clear that the initial frequencies have moved closer to the measured values, reproducing them almost exactly. It is very interesting to note that the convergence is very fast, just after four steps, and the results are very accurate.

<table>
<tr><th colspan="2">Modes</th><th>1st</th><th>2nd</th><th>3rd</th></tr>
<tr><td colspan="2">Theoretical Analysis (Hz)</td><td>5.47</td><td>16.04</td><td>23.27</td></tr>
<tr><td rowspan="6">Iterations</td><td>2</td><td>5.1999</td><td>16.3234</td><td>23.0452</td></tr>
<tr><td>4</td><td>5.2015</td><td>16.2955</td><td>23.0011</td></tr>
<tr><td>8</td><td>5.2015</td><td>16.2955</td><td>23.0011</td></tr>
<tr><td>12</td><td>5.2015</td><td>16.2955</td><td>23.0011</td></tr>
<tr><td>20</td><td>5.2015</td><td>16.2955</td><td>23.0011</td></tr>
<tr><td>100</td><td>5.2015</td><td>16.2955</td><td>23.0011</td></tr>
<tr><td colspan="2">Experimental Results (Hz)</td><td>5.2</td><td>16.3</td><td>23.0</td></tr>
</table>

Table 2: Convergence of the natural frequencies.

Figures 2 to 4 illustrate the graphical iteration procedure convergence behavior to the three analyzed frequencies.

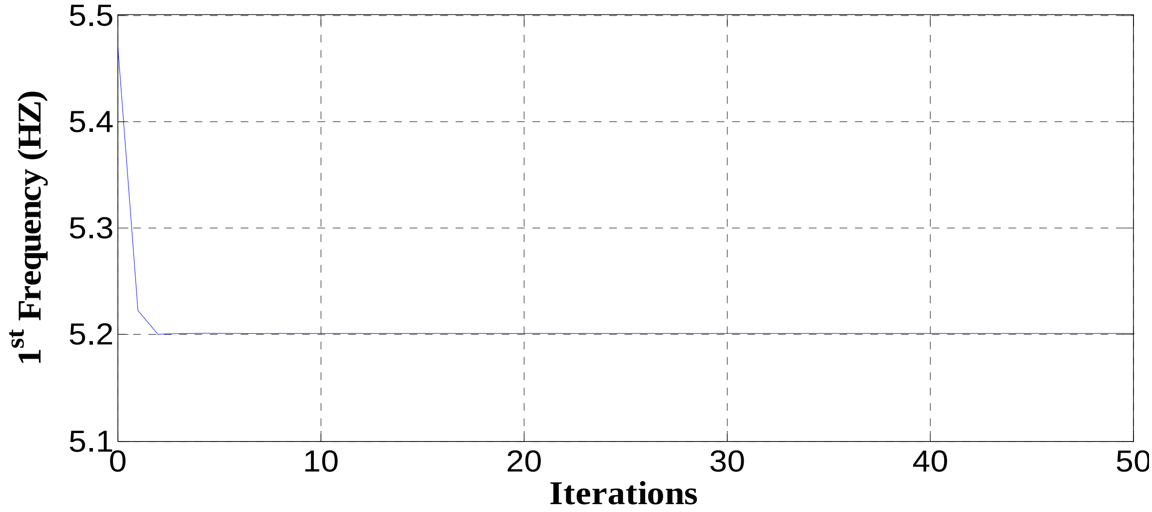


Figure 2: Convergence of the 1st natural frequency.

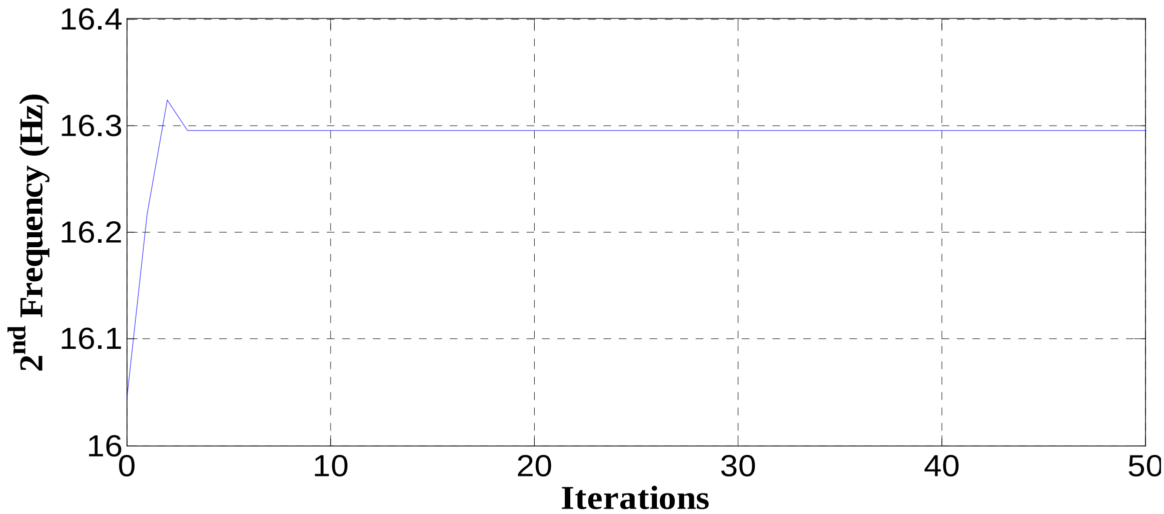


Figure 3: Convergence of the 2nd natural frequency.

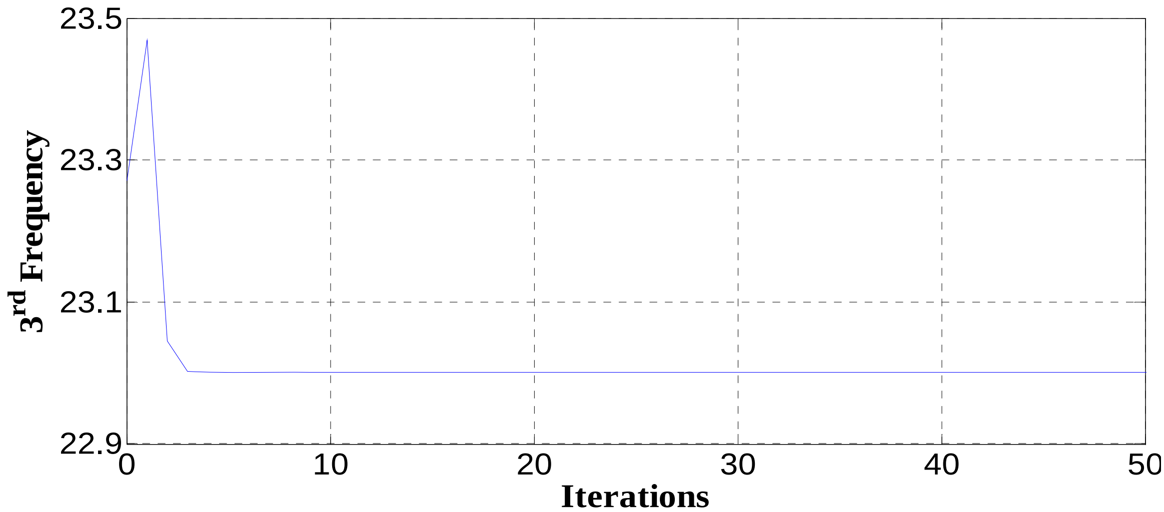


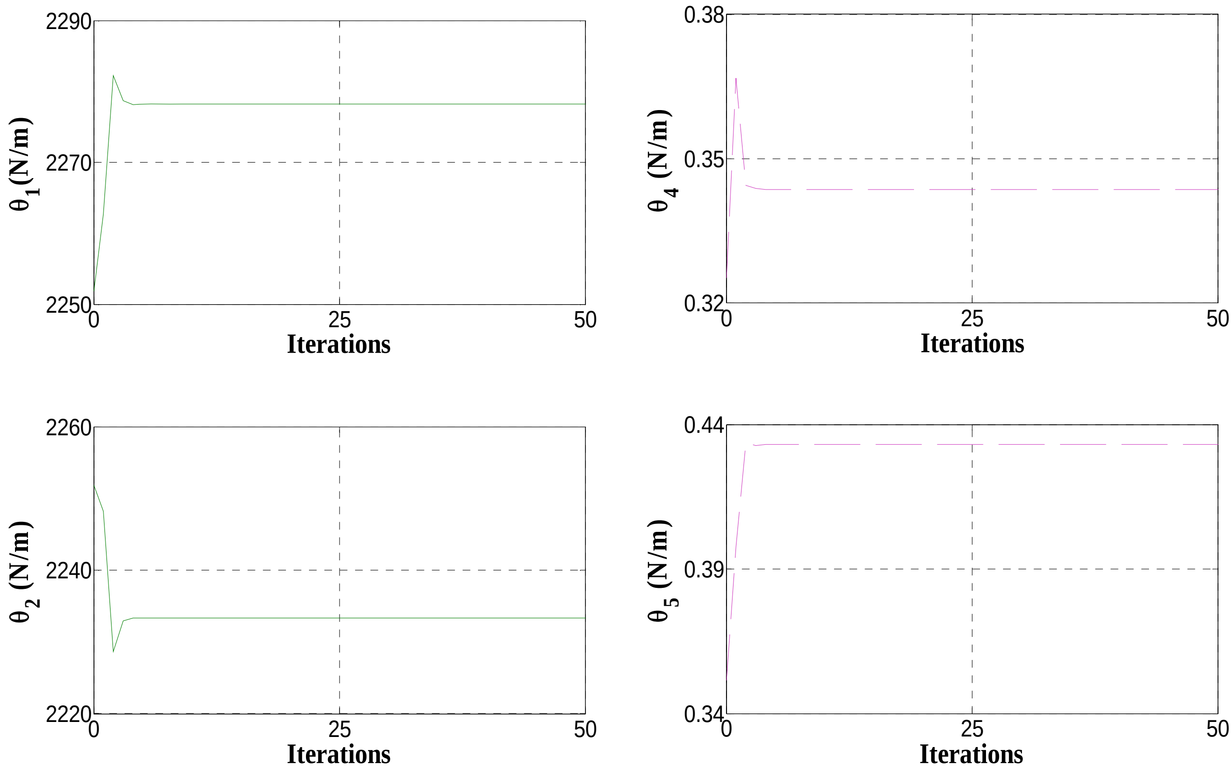
Figure 4: Convergence of the 3rd natural frequency.

The updated parameters may be seen in Table 3 and in Figure 5, and as for the natural frequencies, they converge very fast. It can be believed that these updated values have physical meaning since the experimental natural frequencies are almost exactly reproduced and the mass and stiffness matrices preserve the original pattern.

In the seventh iteration the updated parameters achieve, for this numerical precision, the final values showed in Table 3 keeping constant until the end of the procedure. Graphically this behavior may be seen in Figure 5.

Modes		θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
Initial Values		2251.8509	2251.8509	1811.292	0.3253	0.3514	0.3129
Iterations	2	2282.2665	2228.6539	1789.9862	0.3444753	0.4338752	0.2587534
	4	2278.163	2233.342	1787.5456	0.3436723	0.4332875	0.261158
	8	2278.2043	2233.3502	1787.4844	0.3436736	0.4332632	0.2611685
	12	2278.2043	2233.3502	1787.4844	0.3436736	0.4332632	0.2611685
	20	2278.2043	2233.3502	1787.4844	0.3436736	0.4332632	0.2611685
	100	2278.2043	2233.3502	1787.4844	0.3436736	0.4332632	0.2611685
Units		N/m	N/m	N/m	Kg	Kg	Kg

Table 3: Convergence of the updated parameters.



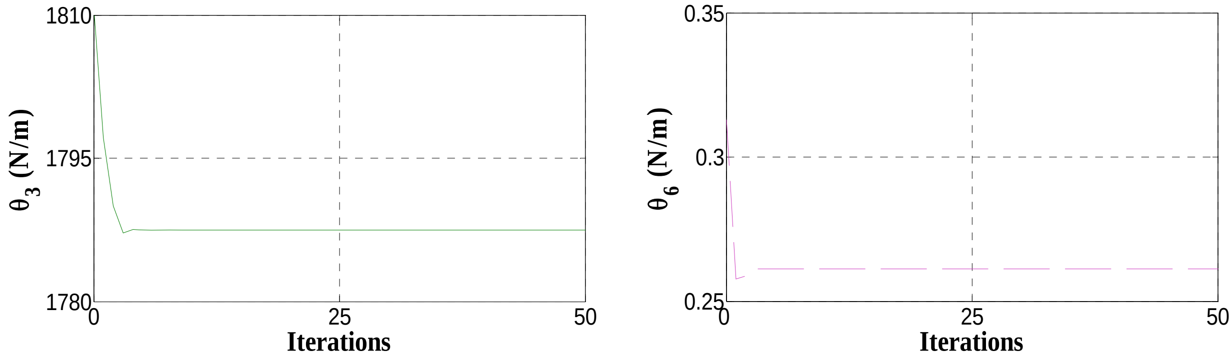


Figure 5: Convergence of the updated parameters.

4 CONCLUSIONS

In this paper was carried out an iterative model updating study, which uses modal data in order to improve the correlation between the experimental and analytical models. Iterative methods have one main advantage: they maintain the original matrices pattern, so the updated model is able to keep the physical meaning.

It is shown that one important step is the sensibility matrix computation. This may be done evaluating the first derivative of the eigenvalues with respect to the updating parameters via, for example, the Fox and Kapoor (1968) procedure. Another powerful characteristic is the ability to weigh individually both measured and analytical data, which allow the properly consideration of the uncertainty contained in this values.

An example with experimental data of a typical reduced scale three-story plane frame was carried out showing that the methodology was able to correct update both mass and stiffness matrices and reproduce correctly the tested data.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the research by CAPES, CNPQ and the Federal University of Rio Grande do Sul, Brazil.

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