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A CHIMERA METHOD BASED ON DIRICHLET-DIRICHLET COUPLING AND PASTING PENALIZATION OPERATORS

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Abstract. A chimera method for pasting different meshes in the context of the finite element method is presented. The method is based both on transferring the solution on one mesh to the boundary of the other via Dirichlet boundary conditions and interpolation, and also with a penalization "pasting" operator. One of the advantages of the proposed method is that no changes in topology arise during the computations. The second advantage is that the solution can be obtained iteratively with a convergence rate that is similar to that one for an equivalent non-chimera mesh, and the matrix-vector operator can be computed by completely decouple operations on both meshes. For symmetric positive-definite operators the resulting system is not symmetric positive-definite, however the solution can be obtained with biconjugate gradient stabilized method, so the memory requirement is almost the same as for the conjugate gradient method, which could be used for an equivalent non-chimera mesh. Several examples are presented assessing the precision and computational cost of the method.

1 INTRODUCTION

The main purpose of the chimera method (Steger, 1991) is to avoid the difficulty that represents the generation of a general unstructured mesh for problems containing complex bodies, like an airplane wing or a windturbine blade (Andrew M. et al., 2009). The idea of the method is to use a coarse grid on the background of the entire domain while using finer meshes around the bodies. This way, the meshing work is greatly reduced while keeping the accuracy of the solution and the computational cost. It is known that the chimera fixed grid approach in combination with Arbitrary Lagrangean Eulerian (ALE) based method is a good choice for solving large deformation fluid structure interaction (FSI) problems (Peter and Wolfgang A., 2006). Also several studies (Andrew M. et al., 2009; Jean-Jaqcues Chattot, 1998; Houzeaux and Codina, 2003, 2004) have shown the advantages of this approach and its capability for engineering applications, mainly in problems that involves moving bodies. Automation of the meshing process, efficiency, accuracy and low memory overhead are the main reasons for choosing this method.

In this work we propose a chimera type domain decomposition method, presented on a two dimensional Poisson equation, for a square domain with homogeneous boundary conditions and a delta circle source term in three different positions. The chimera mesh is composed of two overlapping subdomains with different element sizes that are connected between them by sharing information across each other internal boundaries. The results are compared with a classical finite element method (FEM) solution on a uniform-structured grid and on a conformed grid, analogue to the overlapped one.

2 CHIMERA METHOD

Consider a domain Ω where we want to solve the Poisson equation, given by

$$\begin{aligned} \Delta\phi &= -f, \text{ in } \Omega, \\ \phi &= \bar{\phi}, \text{ in } \Gamma = \partial\Omega \end{aligned} \tag{1}$$

We consider the splitting in two domains $\Omega_{1,2}$ (see Figure 1). The domains have an overlapping region Ω_o. The two meshes do not coincide. Now, consider the solution on the Ω_1 region. A standard discretization with FEM gives a matricial equation of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{2}$$

The nodes in the mesh can be partitioned in four blocks:

- B: nodes in the boundary of Ω_1 and also in the boundary of Ω.
- D: nodes interior to Ω_1 and not in the overlap region.
- O: nodes interior to Ω_1 and in the overlap region.
- Z: all the interior nodes, i.e. the union of nodes in D and O.
- I: nodes in the boundary of Ω_1 and not in the boundary of Ω (they must be interior to Ω_2).

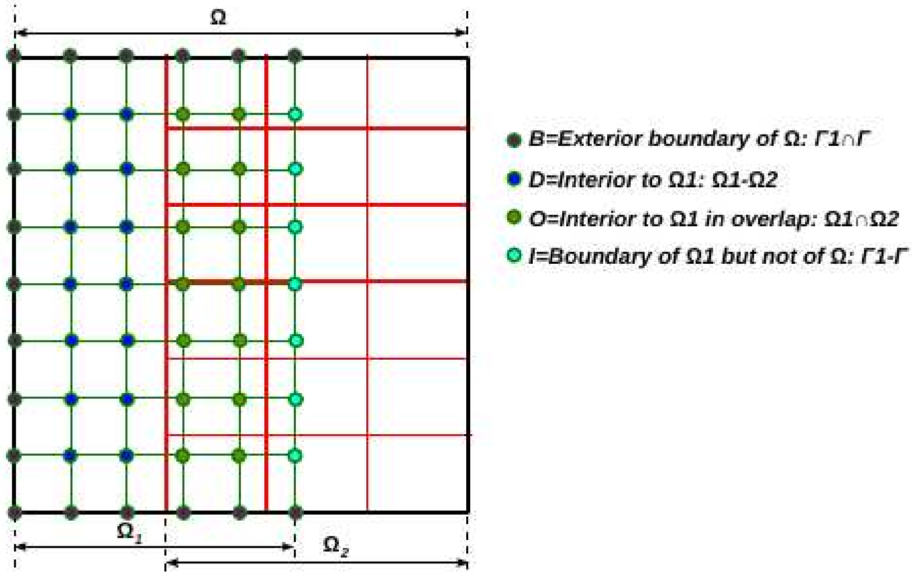


Figure 1: Description of the chimera method.

This splitting of the nodes induces a splitting of the linear system as

$$\begin{bmatrix} \mathbf{A}_{BB} & \mathbf{A}_{BZ} & \mathbf{A}_{BI} \\ \mathbf{A}_{ZB} & \mathbf{A}_{ZZ} & \mathbf{A}_{ZI} \\ \mathbf{A}_{IB} & \mathbf{A}_{IZ} & \mathbf{A}_{II} \end{bmatrix} \begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_Z \\ \mathbf{x}_I \end{bmatrix} = \begin{bmatrix} \mathbf{b}_B \\ \mathbf{b}_Z \\ \mathbf{b}_I \end{bmatrix} \tag{3}$$

The values in $\mathbf{x}_B$ are known, due to the boundary condition. If we would know the values in $\mathbf{x}_I$ then we would have a problem with Dirichlet conditions in the whole boundary and we certainly could solve the problem

$$\mathbf{A}_{ZZ}\mathbf{x}_Z = \mathbf{b}_Z - \mathbf{A}_{ZB}\mathbf{x}_B + \mathbf{A}_{ZI}\mathbf{x}_I \tag{4}$$

Of course, we do not know $\mathbf{x}_I$ although if we consider that we can interpolate this values from the interior values in the Ω_2 domain, we get a full linear system in the interior values of both domains. We use supra-indices for the domains,

$$\begin{aligned} \mathbf{A}^1_{ZZ}\mathbf{x}^1_Z &= \mathbf{b}^1_Z - \mathbf{A}^1_{ZB}\mathbf{x}_B + \mathbf{A}^1_{ZI}\Pi_{I_{1,2}}\mathbf{x}^2, \\ \mathbf{A}^2_{ZZ}\mathbf{x}^2_Z &= \mathbf{b}^2_Z - \mathbf{A}^1_{ZB}\mathbf{x}_B + \mathbf{A}^2_{ZI}\Pi_{I_{2,1}}\mathbf{x}^1, \end{aligned} \tag{5}$$

The $\Pi_{I1,2}$ is a projection operator that interpolates the values on a x^2 vector in the nodes on I^1. It is assumed that in each domain there is at least one full element layer in the overlap region, so that the interpolation involves only nodes in the $2B$ and $2Z$ nodes, not on $2I$ nodes. Therefore the linear system (see Eqn. 5) can be solved for the interior values $\mathbf{x}^1_Z$ and $\mathbf{x}^2_Z$.

3 ITERATIVE SOLUTION

The scheme can be set as a linear operator that can be fed to a matrix-free iterative solver like Generalized Minimal RESidual (GMRES) or BiConjugate Gradient Stabilized method (BiCGStab). The system is not symmetric nor positive definite, however numerical experiments show that it is good enough so as to be solved with BiCGstab and with convergence rates similar to those obtained with Conjugate Gradient (CG) method on the whole (not using chimera) problem. This is very important because it means that the linear system has characteristics similar to the non-chimera problem. Equation (5) can be rewritten as residuals in the following form

$$\begin{aligned}\mathbf{R}_Z^1(\mathbf{x}_Z^1,\mathbf{x}_Z^2) &= \mathbf{A}_{ZZ}^1\mathbf{x}_Z^1 - \mathbf{b}_Z^1 + \mathbf{A}_{ZB}^1\mathbf{x}_B - \mathbf{A}_{ZI}^1\Pi_{I1,2}\mathbf{x}^2,\\ \mathbf{R}_Z^2(\mathbf{x}_Z^1,\mathbf{x}_Z^2) &= \mathbf{A}_{ZZ}^2\mathbf{x}_Z^2 - \mathbf{b}_Z^2 + \mathbf{A}_{ZB}^2\mathbf{x}_B - \mathbf{A}_{ZI}^2\Pi_{I2,1}\mathbf{x}^1.\end{aligned} \tag{6}$$

The algorithm that computes the residual from the interior values is as follows

1: **ALGORITHM:** compute $\mathbf{r} = f(\mathbf{x})$
2: Extract $\mathbf{x}_Z^1, \mathbf{x}_Z^2$ from $\mathbf{x} = [\mathbf{x}_Z^1; \mathbf{x}_Z^2]$
3: Interpolate $\mathbf{x}_Z^2$ on nodes $I1$
4: Compute residual $\mathbf{r}_Z^1$ on $Z1$ according to line 1.
5: Interpolate $\mathbf{x}_Z^1$ on nodes $I2$
6: Compute residual $\mathbf{r}_Z^2$ on $Z2$ according to line 2.
7: Combine $\mathbf{r} = [\mathbf{r}_Z^1; \mathbf{r}_Z^2]$

4 ESTIMATION OF THE CHIMERA ERROR

Due to the decomposition of the domain Ω in two subdomains, Ω_1 and Ω_2, each one has its own error according to its element size. We define the error in Ω_1 as follows:

$$||u_h - u_{ref}||_{(L2,\Omega_1)} = \sqrt{\sum_{i=1}^{(N_1+1)^2} (u_{h_1} - u_{ref})^2 w_{h1}} \tag{7}$$

Where:

- u_{h_1}: Chimera solution on Ω_1.
- u_{ref}: solution for a uniform-structured grid of 2048^2 elements.
- w_{h_1}: area of the element $(h_{1x}h_{1y}/2)$. Where h_{1x} and h_{1y} are the length of the triangular element in x and y directions respectively.

In the same way, we define the error in Ω_2 as follows:

$$||u_h - u_{ref}||_{(L2,\Omega_2)} = \sqrt{\sum_{i=1}^{(N_2+1)^2} (u_{h_2} - u_{ref})^2 w_{h2}} \tag{8}$$

Where:

- u_{h_2}: Chimera solution on Ω_2.
- u_{ref}: solution for a uniform-structured grid of 2048^2 elements.
- w_{h_2}: area of the element $(h_{2x}h_{2y}/2)$. Where h_{2x} and h_{2y} are the length of the triangular element in x and y directions respectively.

Aiming to compare the results obtained by the chimera method in contrast with a uniform-structured grid, we define the total error in Ω for the chimera solution as it is shown below:

$$||u_h - u_{ref}||_{(L2,\Omega)} = \sqrt{||u_h - u_{ref}||^2_{(L2,\Omega_2)} + ||u_h - u_{ref}||^2_{(L2,\Omega_1)}} \tag{9}$$

For the overlapping zone, it would be unfair for the chimera scheme if we take into account the error of both subdomains, because we would be adding the error of the overlap region twice. To overcome this problem, we take into account only the error of the coarse mesh in the overlap zone, which is the most adverse case.

5 NUMERICAL EXAMPLE

We consider a singular source term as follows:

$$\begin{aligned} f &= \frac{\delta}{\cosh((r-0.2)/\delta)^2}, \\ r &= \sqrt{(x-x_c)^2+(y-y_c)^2}, \\ \delta &= 0.025; \end{aligned} \tag{10}$$

This represents a concentrated source term on the circle $r = 0.2$, with its center placed in (x_c, y_c). This is a monopolar layer, and the value of ϕ is almost constant in the interior of the circle. The interest in this case is in the large curvature of the solution, due to the highly concentrated source term.
The problem is solved with two domains spanning 0.525 of the x side, i.e.

$$\begin{aligned} \Omega_1 &= [0, 0.525] \times [0, 1], \\ \Omega_2 &= [0.475, 1] \times [0, 1], \end{aligned} \tag{11}$$

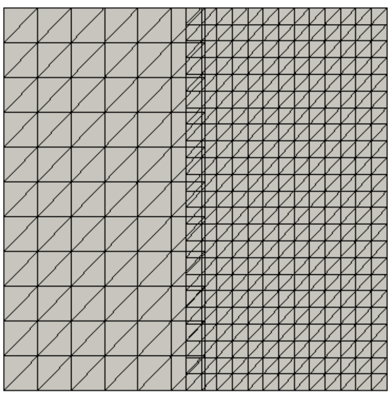
The overlap region is then

$$\Omega_1 \cap \Omega_2 = [0.475, 0.525] \times [0.1]$$

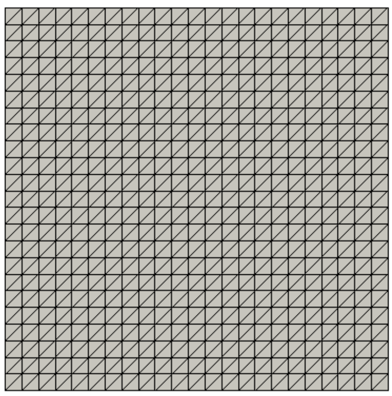
The meshes are Cartesian homogeneously refined in both domains with $N_{1y} = 11$ elements in y direction for Ω_1 and $N_{2y} = 23$ for Ω_2. Therefore, the meshes are not coincident. The mesh sizes in the x direction are chosen so as to keep the aspect ratio (AR) of the elements. The mesh is then refined in steps of $\sqrt{2}$ i.e. the mesh in refinement step $n+1$ is $1/\sqrt{2}$ the one in step n.

5.1 Chimera vs Structured-uniform grid

We consider three different cases. In case 1 we have the center of the source in Ω_2, over the fine mesh. In case 2 we have it on the overlap region, in the center of Ω. And in case 3 it is in Ω_1, over the coarse mesh. For each test, we analyze the convergence of the proposed method for an overlapped grid in contrast with the convergence of the conventional FEM method solving a uniform structured grid (see Fig. 2). Structured triangular elements are used in both meshes.



(a) Chimera grid.



(b) Uniform-structured grid.

Figure 2: Meshes

5.1.1 Case 1. Source term contained in fine mesh region

In this case we solve the Poisson equation (see Eqn. 1) for a singular source term (see Eqn. 10) with its center located in $x_c = 0.75$ and $y_c = 0.50$. The total overlapped length is 0.05, so for the first refinement level, only one column of the elements belonging to Ω_2 are overlapped with the elements of the other subdomain (see Fig. 2a). Mesh data and the computed errors are presented in Table 1 and Table 2, respectively.

	Chimera overlapped grids					Uniform grid		
ref	N_{1_y}	N_{2_y}	h_{1_y}	h_{2_y}	Elements	$N_{x,y}$	h	Elements
1.00	11	23	9.0909e-02	4.3478e-02	730	23	4.3478e-02	1058
1.41	16	33	6.2500e-02	3.0303e-02	1476	33	3.0303e-02	2178
2.00	23	47	4.3478e-02	2.1277e-02	2948	47	2.1277e-02	4418
2.82	32	66	3.1250e-02	1.5152e-02	5708	66	1.5152e-02	8712
4.00	45	93	2.2222e-02	1.0753e-02	11274	93	1.0753e-02	17298
5.65	63	131	1.5873e-02	7.6340e-03	22362	131	7.6340e-03	34322
8.00	89	185	1.1236e-02	5.4050e-03	44626	185	5.4050e-03	68450
11.31	125	261	8.0000e-03	3.8310e-03	88536	261	3.8310e-03	136242

Table 1: Mesh information for case 1.

	Chimera error			Uniform grid error
ref	Ω_1	Ω_2	Ω	Ω
1.00	1.518e-02	1.215e-02	2.445e-02	7.5035e-03
1.41	1.417e-03	3.276e-03	3.717e-03	1.3559e-03
2.00	6.295e-05	3.593e-04	3.689e-04	5.9680e-04
2.82	5.409e-05	1.423e-04	1.537e-04	1.5154e-04
4.00	1.363e-05	7.261e-05	7.418e-05	7.4389e-05
5.65	7.482e-06	3.655e-05	3.742e-05	3.7537e-05
8.00	3.872e-06	1.815e-05	1.862e-05	1.8758e-05
11.31	2.011e-06	9.080e-06	9.331e-06	9.3992e-06

Table 2: Errors.

It can be seen (see Fig. 3) that when the major gradients are placed in the fine mesh the error of the chimera method (Eqn. 9) is very similar to the error for a full uniform-structured mesh which its element sizes are equal to the element sizes of the fine mesh in the chimera domain. As we have two different element sizes for both subdomains, the smallest is used for error comparison. Also we can see that the convergence is $O(h^2)$ as expected.

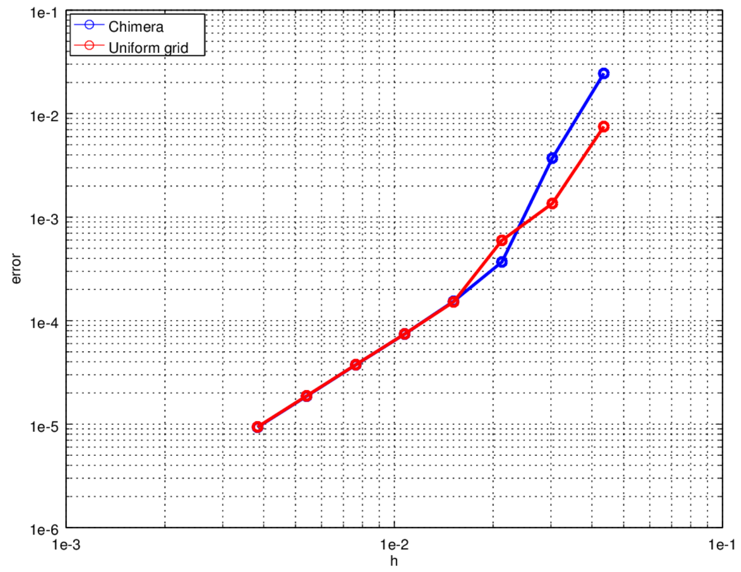
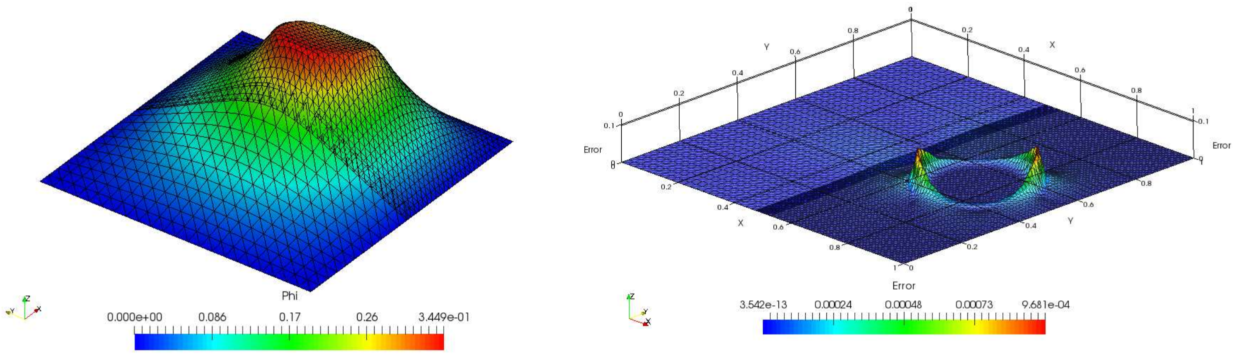


Figure 3: Convergence of both, chimera method and uniform and structured grid.



(a) Chimera solution

(b) Absolute error

Figure 4: Solution(a) and error(b) to Poisson equation with the source placed in the fine mesh.

The solution as well as the absolute error in both meshes for case 1 are shown in Figure 4.

5.1.2 Case 2. Source term placed in overlap region.

For this test, we solve the Equation 1 for a singular source term (see Eqn. 10) with its center placed in $x_c = 0.5$ and $y_c = 0.50$, over the overlapped region. The total overlap length is 0.05. The main purpose of this test is to ensure the convergence of the method even when the largest curvature of the solution is placed in the overlapping area. The mesh data and the computed errors of the overlapped and uniform grids are presented in Tables 3 and 4 respectively, for each refinement level.

	Chimera overlapped grids				Uniform grid		
ref	N_{1_y}	N_{2_y}	h_Ω	Elements	$N_{x,y}$	h	Elements
1.00	11	23	5.3149e-02	730	23	4.3478e-02	1058
1.41	16	33	3.7216e-02	1476	33	3.0303e-02	2178
2.00	23	47	2.6463e-02	2948	47	2.1277e-02	4418
2.82	32	66	1.8932e-02	5708	66	1.5152e-02	8712
4.00	45	93	1.3482e-02	11274	93	1.0753e-02	17298
5.65	63	131	9.5655e-03	22362	131	7.6340e-03	34322
8.00	89	185	6.7623e-03	44626	185	5.4050e-03	68450
11.31	125	261	4.8005e-03	88536	261	3.8310e-03	136242

Table 3: Mesh information for case 2.

	Chimera error			Uniform grid error
ref	Ω_1	Ω_2	Ω	Ω
1.00	1.433e-02	1.648e-02	2.979e-02	9.4406e-03
1.41	6.627e-03	2.220e-03	7.057e-03	3.5459e-03
2.00	2.270e-03	1.562e-03	2.893e-03	1.2711e-03
2.82	2.034e-03	1.151e-03	2.394e-03	1.6490e-04
4.00	5.953e-04	3.377e-04	7.115e-04	7.4229e-05
5.65	1.265e-04	5.965e-05	1.432e-04	3.7984e-05
8.00	5.686e-05	2.234e-05	6.248e-05	1.8966e-05
11.31	2.732e-05	7.076e-06	2.853e-05	9.5043e-06

Table 4: Errors.

For this particular case, we define a resulting element size for the chimera mesh according to Jinsheng Cai (2005) as follows:

$$h_\Omega = \left[\frac{2}{N_{12}} \left(\sum_{i=1}^{N_{1_D}} \Delta A_{1_i} + \sum_{i=1}^{N_2} \Delta A_{2_i} \right) \right]^{1/2} \tag{12}$$

where ΔA_{1_i} is the area of the $i-th$ element in Ω_1 and N_{1_D} is the number of elements in $\Omega_{1_D} = \Omega/\Omega_2$, i.e. the elements of Ω_1 that are not inside the overlap region. ΔA_{2_i} is the area of the $i-th$ element in Ω_2 and N_2 is the number of elements in Ω_2, i.e. the elements inside and outside of the overlap region that belongs to Ω_2. Then we define $N_{12} = N_{1_D} + N_2$ for compute the chimera element size.
We can see from Figure 7 that the second order convergence is maintained and the error is similar for the uniform grid.

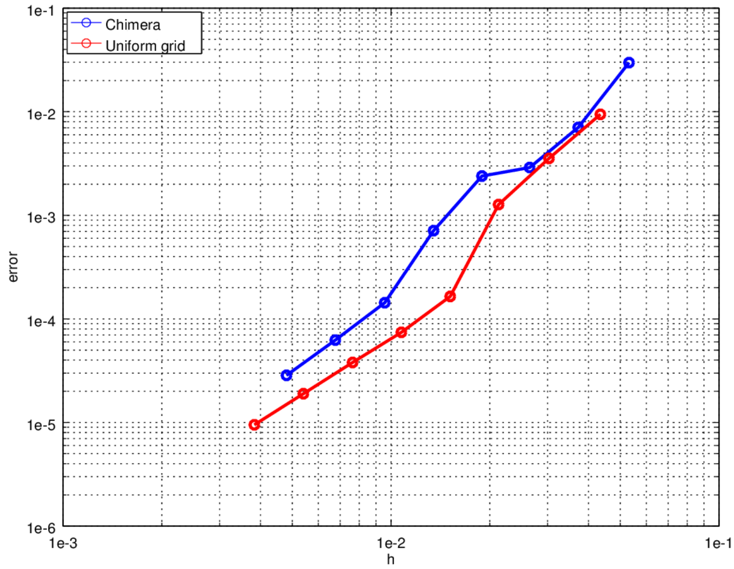
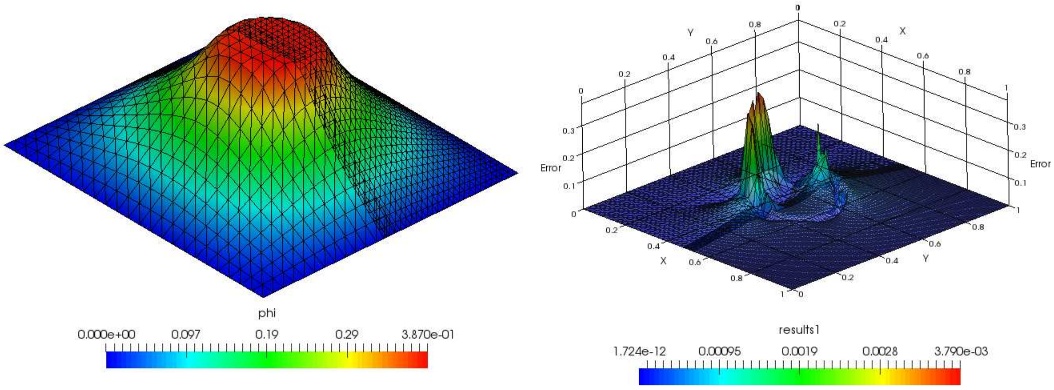


Figure 5: Convergence of chimera method and a uniform-structured grid, with the source placed in the center of the domain.



(a) Chimera solution

(b) Absolute error

Figure 6: Solution(a) and error(b) to Poisson equation with the source placed in the mesh center

The solution and the absolute error in both meshes for case 2 are shown in Figure 6.

5.1.3 Case 3. Source term contained in coarse mesh region.

Here we solve Eqn. 1 with the singular source term (see Eqn.10) with its center positioned in $x_c = 0.25$ and $y_c = 0.5$. We keep the overlapped region length of 0.05. The analyzed meshes and the computed error are shown in Table 5 and Table 6, respectively.

	Chimera overlapped grids					Uniform grid		
ref	N_{1_y}	N_{2_y}	h_{1_y}	h_{2_y}	Elements	$N_{x,y}$	h	Elements
1.00	11	23	9.0909e-02	4.3478e-02	730	11	9.0909e-02	242
1.41	16	33	6.2500e-02	3.0303e-02	1476	16	6.2500e-02	512
2.00	23	47	4.3478e-02	2.1277e-02	2948	23	4.3478e-02	1058
2.82	32	66	3.1250e-02	1.5152e-02	5708	32	3.1250e-02	2048
4.00	45	93	2.2222e-02	1.0753e-02	11274	45	2.2222e-02	4050
5.65	63	131	1.5873e-02	7.6340e-03	22362	63	1.5873e-02	7938
8.00	89	185	1.1236e-02	5.4050e-03	44626	89	1.1236e-02	15842
11.31	125	261	8.0000e-03	3.8310e-03	88536	125	8.0000e-03	31250

Table 5: Meshes for case 3.

	Chimera error			Uniform grid error
ref	Ω_1	Ω_2	Ω	Ω
1.00	2.084e-02	1.805e-02	3.373e-02	3.6717e-02
1.41	5.104e-03	1.099e-03	5.263e-03	1.0113e-02
2.00	4.938e-03	1.939e-03	5.406e-03	7.5035e-03
2.82	1.294e-03	1.574e-04	1.306e-03	2.2885e-03
4.00	8.868e-04	3.104e-04	9.560e-04	9.2744e-04
5.65	1.772e-04	3.293e-05	1.809e-04	2.0521e-04
8.00	8.100e-05	9.039e-06	8.167e-05	8.0990e-05
11.31	4.101e-05	4.752e-06	4.137e-05	4.1253e-05

Table 6: Errors for case 3.

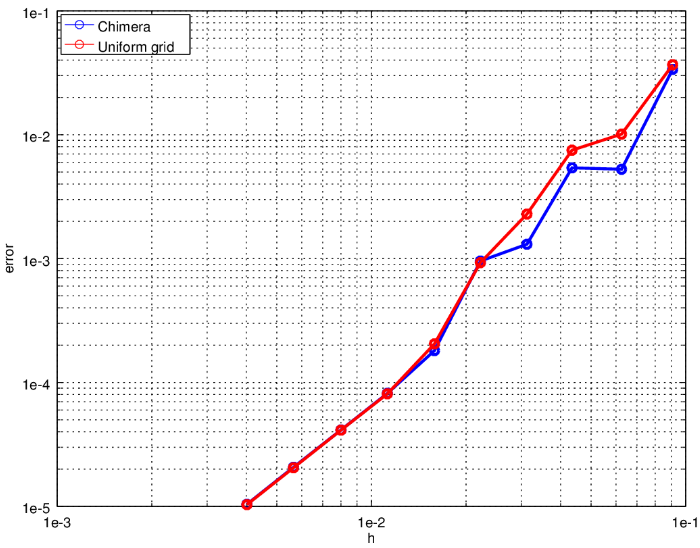
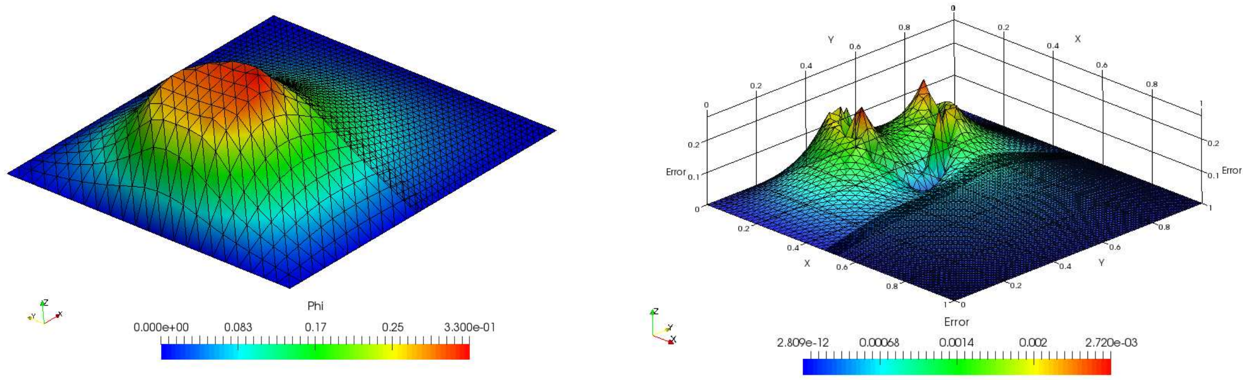


Figure 7: Convergence of chimera method and a uniform-structured grid, with the source placed in the coarse mesh.



(a) Chimera solution

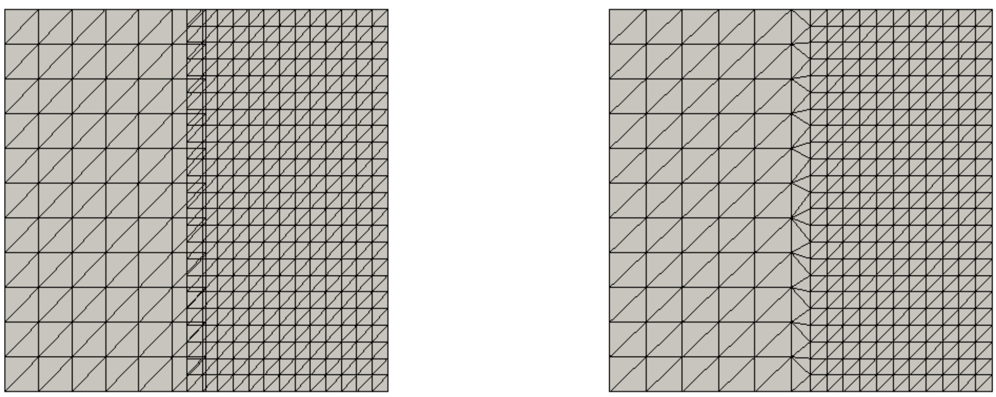
(b) Absolute error

Figure 8: Solution(a) and error(b) to Poisson equation with the source placed in the coarse mesh

The solution and the absolute error in both meshes for case 3 are shown in Figure 8.

5.2 Chimera vs conformed mesh

A complementary convergence study was carried out in contrast with a conformed mesh, as similar as possible to the overlapped meshes (see Fig. 9), that has a very well define coarse zone as well as the fine zone, connected by an adaptive scheme where the overlapped region is supposed to be. The conformed mesh was made with the Open-source software **Salome** (Ribes and Caremoli, 2007.). We test eight refinement levels and we compare the results for both schemes. In this study we analyze the Poisson equation for the same source term as the previous case (see Eq. 10) although only with the circle center placed in the fine zone ($x_c = 0.75$, $y_c = 0.5$). Results are shown in Table 7.



(a) Chimera grid

(b) Conformed grid

Figure 9: Meshes of the two cases

	Chimera overlapped grids		Conformed grid	
ref	Elements	Error	Elements	Error
1.00	730	2.445e-02	650	4.8696e-03
1.41	1476	3.717e-03	1361	2.1124e-03
2.00	2948	3.689e-04	2736	5.7245e-04
2.82	5708	1.537e-04	5214	2.8146e-04
4.00	11274	7.418e-05	11604	1.2195e-04
5.65	22362	3.742e-05	20596	6.9399e-05
8.00	44626	1.862e-05	46202	3.1310e-05
11.31	88536	9.331e-06	81114	1.7930e-05

Table 7: Error of Overlapped and Conformed meshes.

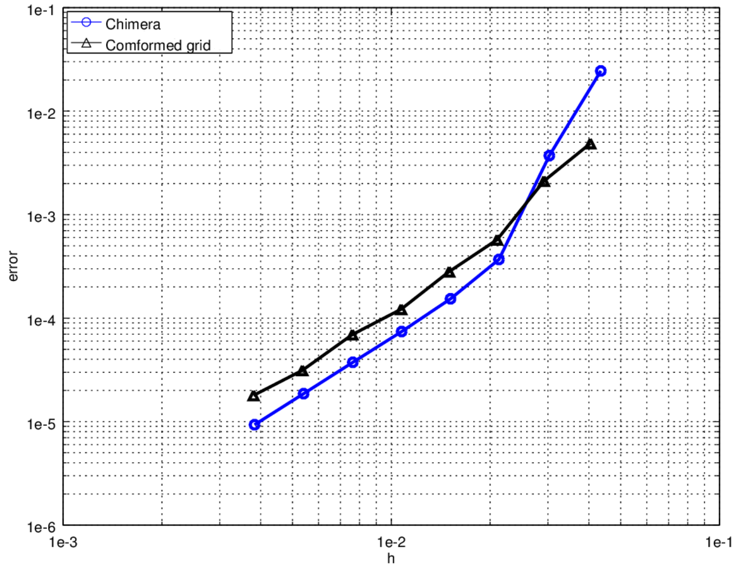


Figure 10: Convergence of chimera method and a conformed grid, with the source placed in the fine grid.

Once we had the solution on the nodes for the conformed mesh, we interpolate these values on the reference mesh used as the "exact solution" and the error is computed. As it can be seen from Figure 10 the chimera error is slightly lower than the error for a Conformed grid. With this results we proved that we are having a resulting error of the overlapped grid scheme which is very similar to the error solving a conformed grid with a classical FEM method, saving time on meshing the geometry.

6 RESIDUAL ANALYSIS

The convergence of BiCGstab and Conjugate Gradient iterative methods were studied for case 1. A special analysis was made for this case because we will always want for the discontinuities and the greatest gradients to be in the finest mesh of the chimera grid. The study was carried out solving a chimera grid with 125 x 64 elements and then split in triangles for the coarse mesh, and 261 x 144 elements (also split) for the fine mesh, with BiCG iteration method. Meanwhile, a conformed adaptive mesh of triangular elements was solved with Conjugate Gradient without any preconditioning. In contrast, the same problem was solved for a uniform-structured coarse grid of 125x125 split in triangular elements, and for uniform structured fine grid of 261x261, split in triangular elements too, with Conjugate Gradient without any preconditioning. All cases were solved for a relative residual tolerance of 1e-08.

Figure 11: Convergence of BiCGstab (for chimera grid) and CG (for comforming and uniform grids).

	Total elements	Iterative solver	Iterations	Error
Chimera	88536	BiCGstab	503	9.3310e-06
Comformed	81114	CG	699	1.7930e-05
Coarse	31250	CG	327	8.0000e-03
Fine	136242	CG	687	9.3992e-06

Table 8: Convergence analysis.

As can be seen from the Figure 11 the chimera method solved with BiCGstab shows a very good convergence, reaching the preset tolerance in 503 iterations, proving a better convergence than the adaptive mesh, that takes it 699 iterations to converge.Also, the total number of iterations that takes chimera method to reach the convergence, keeps lower than for the fine uniform-structured grid.

7 CONCLUSIONS

A chimera scheme for overlapped grids was presented as well as the mathematical formulation and the algorythm for solving it iteratively.

It was defined an estimation of the resulting error for the chimera method that takes into account the error of both subdomains, which allowed us to compare it against other methods errors.

The scheme was tested solving a poisson equation in a two dimensional domain for three different cases. It was shown that the chimera solution maintain its accuracy and good convergence even if the largest curvature of the solution is placed over the coarse zone of the overlapped grids. We did special emphasys on the first case in which the major gradients are located in the fine mesh, because that is the main purpose of the scheme presented. A comparison with a equivalent conformed mesh was made. It was shown that the error for the overlapped grids is slightly lower than it is for the conformed mesh from the third refinement level onwards.

Also a convergence analysis was carried out solving the overlapping grid scheme with BiCGstab in contrast with a classical FEM scheme, solving for a fine, a coarse and a conformed mesh with Conjugated Gradient without preconditioning. The results showed that in the case of the overlapped subdomains, the problem was well defined, proving a good convergence for the iterative method. Also it was proved that the robustness of the chimera method in conjunction with the BiCGstab solver lead us to a lower number of iterations to achieve convergence.

As future work we are aiming to improve the algorithm for solving Stokes equation on unstructured meshes. Also we are in the way to solve transient problems with moving bodies.

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