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FROM EXACT DIFFERENCE SCHEMES TO ADVANCED NODAL SCHEMES: FROM NEUTRONICS TO NAVIER-STOKES

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Abstract. Roots of coarse mesh, or advanced, nodal methods [1] can be traced to "exact finite difference schemes." After a brief overview of exact finite difference schemes, a nodal scheme will be developed for the scalar convection-diffusion PDE [2].

To address some of the limitations on classical nodal schemes, our efforts have focused on the development of: 1) a modified nodal method for the time-dependent Navier-Stokes (N-S) equations and its parallel implementation [3]; 2) hybrid methods for domains with curved boundaries [4]; and 3) adaptive mesh refinement (AMR) capability for nodal schemes [5]. The modified nodal method for the time-dependent, incompressible N-S equations incorporates two major modifications over nodal schemes developed earlier. First, rather than using the conventional continuity equation or the vorticity-stream function formulation, we replace the conventional continuity equation by a Poisson-type continuity equation written in terms of pressure, and retain the momentum equations in primitive variables. The second modification is introduced in the development of the numerical scheme. Here, rather than using only the diffusion term to obtain the homogeneous part of the solution of the momentum equations, a "linearized" convection term—based on previous time step velocity—is also retained on the left hand side of the transverse-integrated equations, leading to a local homogeneous solution for the transverse-integrated velocities in each spatial direction that is a combination of a constant, a linear and an exponential term.

Restriction on domain geometry has been removed by developing hybrid schemes in which nodal methods are restricted to the interior of the domains and along boundaries that are parallel to the coordinate axes, while a second scheme—such as finite element, more suitable for complex boundaries—is used along curved boundaries. For problems that require fine mesh in some regions and coarse in other, AMR capability has been developed for nodal methods to retain the coarse mesh efficiency by allowing high degree of resolution in specific localized areas only where it is needed, and using a lower resolution everywhere else.

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