IDENTIFICATION OF PARAMETERS IN THE TORSIONAL DYNAMICS OF A DRILLING PROCESS THROUGH BAYESIAN STATISTICS

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Keywords: bayesian statistical inference, parameters identification, drillstring dynamics

Abstract.

This work presents the estimation of the parameters of an experimental setup, which is modeled as a system with three degrees of freedom, composed by a shaft, two rotors, and a DC motor, that emulates a drilling process. A Bayesian technique is used in the estimation process, to take into account the uncertainties and variabilities intrinsic to the measurement taken, which are modeled as a noise of Gaussian nature. With this procedure it is expected to check the reliability of the nominal values of the physical parameters of the test rig. An estimation process assuming that nine parameters of the experimental apparatus are unknown is conducted, and the results show that for some quantities the relative deviation with respect to the nominal values is very high. This deviation evidentiates a strong deficiency in the mathematical model used to describe the dynamic behavior of the experimental apparatus.

1   INTRODUCTION

A drillstring is a device, used to drill oil wells, whose three-dimensional dynamics behavior is very complex, and subject to three mechanisms of vibration: longitudinal, transverse, and torsional (Spanos et al., 2003). The torsional mechanism is responsible for causing a type of dynamics called stick-slip. In this mode, the rotation of the column is blocked due to dry friction with the borehole, stick, and then suddenly released, slip, to be afterwards blocked again. So, due to the constant rate of rotation imposed at the top of the column, the column is severely twisted. This may cause fatigue damage, or even the rupture of the structure.

To characterize the stick-slip mode and, consequently, to develop mechanisms to avoid it, the works of Cayres et al. (2013) and Cayres (2013) present numerical studies which investigate, by means of a sensitivity analysis, the influence of the friction models and the caracterization of the structure parameters during the occurrence of the stick-slip. Experimental data obtained from a test apparatus, in laboratory scale, are used to validate the mathematical model developed.

One of the challenges associated to the use of an experimental setup to validate a mathematical model is the correct characterization of the parameters that are used to describe the behavior of the experimental setup physical system. Commonly, a physical phenomenon is not described perfectly by a mathematical model, which may be due to an inability of the model itself, or also, due to inaccuracies of the measuring instruments, ambient noise, human errors, and several others factors. In this way, the measures that one has for estimate the parameters of a physical system are subjected to variabilities (van der Heijden et al., 2004). Fortunately, the measurements erros can be circumvented with the use of stochastic techniques of estimation, in particular, the Bayesian inference, which updates the estimative of the parameters for each new evidence collected about them (Bolstad, 2007).

In order to characterize the experimental setup presented in Cayres et al. (2013) and Cayres (2013), that emulates a drilling process, this paper presents a study in which the physical parameters associated with the test rig are estimated through the Bayesian technique. The main objective that one wants to achieve with the parametric estimation is to check the reliability of the nominal values of the physical parameters of interest.

This paper is organized as follows. In section 2 the experimental apparatus is described. The mathematical modeling of the physical system appears in section 3. In section 4, the Bayesian estimation technique is briefly described. Then, in section 5, the estimation results for the experimental setup are presented and discussed. Finally, in section 6, the main conclusions are emphasized and some directions for future work outlined.

2   EXPERIMENTAL SETUP

The experimental apparatus used in the works of Cayres et al. (2013) and Cayres (2013), which is intended to emulate a drilling process, basically consists of a large flexible shaft, one DC motor and two rotors, as can be seen in the schematic representation of the Figure 1.

In this experimental setup, the flexible shaft has a very low flexural stiffness, so that it emulates the behavior of a drillstring, which is a very slender structure due to the high value of the ratio l_s/D_s, between the column length, l_s, and the diameter of the column cross section, D_s.

The DC motor has the role of imposing a constant speed on top of the column, so that it can penetrate the rock. The rotor r_2 is located between the DC motor and the shaft, and concentrates the inertia of the motor, which is denoted by j_m. On the other extreme of the shaft one can find the rotor r_1, which has an inertia j_1.

The modeling of the experimental setup assumes that it has three degrees of freedom, being the first degree of freedom (DoF) the angle of rotation of the shaft around the rotor r_1, which is denoted by θ_1, the second DoF is the angle θ_2, which measures the shaft rotation with respect to the rotor r_2, and the last DoF is the the electrical charge in the motor, denoted by q. The other physical parameters of this mechanical system, as well as their nominal values, are presented in Table 1.

For further details about this experimental apparatus and the model that is used to describe the experiments, the reader is encouraged to see the work of Cayres (2013).
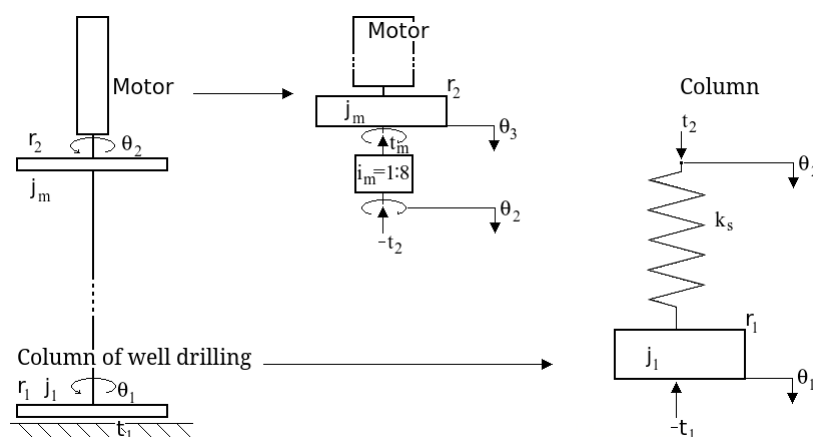

Figure 1: Schematic representation of the experimental setup used to emulates a drilling process.

3 MATHEMATICAL MODELING

The following lines reproduce the mathematical modeling, developed by Cayres (2013) and Cayres et al. (2013), of the experimental setup described in the last section.

Starting the equationing by the motor, one has that the sum of the electric potentials in the motor is equal to the external potencial applied on it, so that

$$l_m \ddot{q}(t) + r_m \dot{q}(t) + k_e \dot{\theta}_3(t) = v, \tag{1}$$

where l_m is the internal inductance of the motor, r_m is the internal resistance of the motor, k_e is a constant of voltage, and θ_3 is the angular rotation of the motor axis. Note that the upper dot denotes differentiation with respect to time.

The balance of the torques acting on the motor provides the following equation

$$j_m \ddot{\theta}_3(t) + c_m \dot{\theta}_3(t) - k_T \dot{q}(t) + t_f = -t_m, \tag{2}$$

where c_m is the internal damping of the motor, k_T is a constant of torque, t_f is the torque due to motor internal friction, and t_m is the torque on the motor due to the coupling between the motor and the gearbox.

Column

parameters	description	value	unit
j_1	moment of inertia	28.3×10^{-3}	$kg\,m^2$
l_s	length of the column	2.4	m
D_s	diameter of the column	3	mm
m_{r_1}	mass of the rotor 1	6.4	kg
k_s	torsional stiffness of the column	3.0×10^{-1}	Nm/rad
r_{r_1}	radius of the rotor 1	188	mm
t_1	friction torque in the rotor 1	0	Nm

Motor

parameters	description	value	unit
j_m	moment of inertia	4.0×10^{-4}	$kg\,m^2$
l_m	internal inductance of the motor	1.1×10^{-3}	H
r_m	internal resistance of the motor	3.3×10^{-1}	Ω
k_T	constant of torque	1.2×10^{-1}	Nm/A
k_e	constant of voltage	601.6×10^{-4}	$V/(rad/s)$
t_f	torque due to motor internal friction	1.0×10^{-1}	Nm
c_m	internal damping of the motor	18.0×10^{-5}	$Nm/(rad/s)$
i_m	gear reduction factor	1/8	8 : 1
v	voltage	8	V

Table 1: Numerical values for the physical parameters of the experimental setup.

Due to the reduction factor in the gearbox, there is a relationship between the angles of rotation θ_2 and θ_3, which is given by

$$i_m\theta_3 = \theta_2, \tag{3}$$

being i_m the gear reduction factor. This reduction factor also provides a coupling between t_m and the torque on the second DoF, t_2, which is expressed by

$$t_m = i_m t_2. \tag{4}$$

Moreover, the balance of the torques which are acting on the rotor r_2 provides

$$-k_s\left(\theta_1(t) - \theta_2(t)\right) = t_2, \tag{5}$$

being k_s is the torsional stiffness of the shaft.

Likewise, the balance of the torques acting on the rotor r_1 provides the following differential equation

$$j_1\ddot{\theta}_1(t) + k_s\left(\theta_1(t) - \theta_2(t)\right) = -t_1. \tag{6}$$

Combining the Eqs.(2) to (5), one can write

$$j_m\ddot{\theta}_2(t) - i_m^2 k_s\left(\theta_1(t) - \theta_2(t)\right) - i_m k_T \dot{q}(t) + c_m\dot{\theta}_2 = -i_m t_f. \tag{7}$$

Thus, through a rearrangement of the Eqs.(1), (6), and (7), it is possible represent the dynamics of the experimental setup by the following state space equation

$$\dot{\mathbf{y}}(t) = [A]\,\mathbf{y}(t) + \mathbf{F}(t), \tag{8}$$

where the state vector $\mathbf{y}(t)$, the control vector $\mathbf{F}(t)$, and the system matrix $[A]$ are, respectively, defined as

$$\mathbf{y}(t) = \begin{pmatrix} \theta_1(t) \\ \theta_2(t) \\ q(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \dot{q}(t) \end{pmatrix}, \quad \mathbf{F}(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -t_1/j_1 \\ -i_m t_f/j_m \\ v/l_m \end{pmatrix}, \tag{9}$$

and

$$[A] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -k_s/j_1 & k_s/j_1 & 0 & 0 & 0 & 0 \\ i_m^2 k_s/j_1 & -i_m^2 k_s/j_m & 0 & 0 & -c_m/j_m & i_m k_T/j_m \\ 0 & 0 & 0 & 0 & -i_n k_e/l_m & -r_m/l_m \end{pmatrix}. \tag{10}$$

Later in this work will be necessary to integrate the initial value problem defined by the ODE of the Eq.(8), and an appropriate initial condition, assumed as the zero vector for all the simulations reported in this work. This integration is carried out numerically, using the routine ODE23T of MATLAB, that uses a trapezoidal rule with "free" interpolant to construct the time response of the dynamical system under analysis. The choice of this ODE solver was motivated by the presence of a moderately stiffness in the system of equations to which one wish to solve.

4 PARAMENTERS IDENTIFICATION

The Bayesian statistical inference theory provides the link between the results of the experimental observations with the theoretical predictions. This can be tapped for parametric estimation techniques. Thus, given the known distribution of the observations conditioned on the parameters $p(\mathbf{y}|\mathbf{x})$ (forward PDF), it is desired to find a probability distribution of the parameters influenced by the observations $p(\mathbf{x}|\mathbf{y})$ (posterior PDF), (Bolstad, 2007).

Thus,

$$p(\mathbf{x}|\mathbf{y}) = \frac{1}{p(\mathbf{y})}\, p(\mathbf{y}|\mathbf{x})\, p(\mathbf{x})\ , \tag{11}$$

where $\mathbf{x}$ is the parameters vector and $\mathbf{y}$ is the sample vector.

The Eq.(11) is the Bayes theorem, and states that the posterior PDF can be obtained from the forward PDF, determined by the experiment (or simulation). The a priori PDF, $p(\mathbf{x})$, is calculated based on what is known (or believed to be known) about the parameters before observations. The ratio $1/p(\mathbf{y})$ is a normalization constant.

To obtain the desired parametric estimation through Bayesian inference, it is necessary to seek the set of parameter that maximize the Eq.(11). For this purpose, the natural logarithm is applied to the posterior PDF, considering Bayes law and under the assumption that the values of the parameters before the observation do not change i.e., $p(\mathbf{x})$ is constant. In this way one obtains that

$$\ln\big(p(\mathbf{x}|\mathbf{y})\big) \propto \ln\big(p(\mathbf{y}|\mathbf{x})\big), \tag{12}$$

that is, the posterior PDF is proportional to the forward PDF. Therefore, to maximize the left hand side of (12) is similar to maximize its right hand side.

In general, the measurements of an experiment have some type of dispersion, associated with each observation, and this dispersion can be modeled as an error of random nature, which will be called $\mathbf{n}$. This error is added to the value given by the physical theory $\hat{\mathbf{y}}$, which is considered free of error. Thus, one has

$$\mathbf{y} = \hat{\mathbf{y}} + \mathbf{n}. \tag{13}$$

It is assumed that the first two moments of $\mathbf{n}$ are known (the average value μ_n and the covariance matrix $[\Gamma_n]$) and that its distribution is a Gaussian. Also, it is supposed that the measurements are independent and identically distributed (i.i.d.) events, so that the covariance matrix is $[\Gamma_n] = \sigma_n^2\,[I]$, where $[I]$ is the identity matrix and σ_n is the Gaussian standard deviation.

Therefore, the misfit function is defined as

$$\varepsilon(\mathbf{x}\,;\ \mathbf{y}) = \frac{1}{\sigma_n^2}(\mathbf{y}-\hat{\mathbf{y}})^T(\mathbf{y}-\hat{\mathbf{y}}) = \frac{1}{\sigma_n^2}\|\mathbf{y}-\hat{\mathbf{y}}\|^2, \tag{14}$$

and must be minimized in order to estimate the parameters. This procedure is called least-squares method.

The procedure followed in order to estimate the parameters of the experimental apparatus consists of collecting the experimental data, construct the misfit function, Eq.(14), and then minimize it. The minimum of the function given by the Eq.(14) is searched with the aid of the function FMINSEARCH of MATLAB, which used M-simplex method (Nelder and Mead, 1965).

For further information, the reader can see van der Heijden et al. (2004).

5 RESULTS AND DISCUSSION

5.1 Verification of the technique effectiveness

First, to verify the effectiveness of the method, only two parameters are estimated, namely the constant voltage k_e and the theoretical internal damping of the motor c_m. Thus, the parameter vector is written as

$$\mathbf{x} = \begin{pmatrix} c_m \\ k_e \end{pmatrix}. \tag{15}$$

The other parameters values are are assumed to known, and are obtained from the Table 1.

Given that, in the laboratory, it is only possible to measure the values of the rotations, θ_1 and θ_2, and their respective time derivatives, $\dot{\theta}_1$ and $\dot{\theta}_2$, a vector of samples is

$$\mathbf{y} = \begin{pmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{pmatrix}^T = \hat{\mathbf{y}} + \mathbf{n}, \tag{16}$$

where $\hat{\mathbf{y}}$ is given by the numerical solution of the Eq.(8), and the noise $\mathbf{n}$, as previously stated, is assumed to be i.i.d. and Gaussian.

Thus, the misfit function, according to Eq.(14), is given by

$$\varepsilon(\mathbf{x}; \mathbf{y}) = \left(\frac{1}{\sigma_n^2} \right) \sum_{i=1}^{2} \left[\left(\boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i(\mathbf{x}) \right)^2 + \left(\dot{\boldsymbol{\theta}}_i - \hat{\dot{\boldsymbol{\theta}}}_i(\mathbf{x}) \right)^2 \right] \tag{17}$$

with $i = 1, 2$. In this effectiveness verification test, the $\hat{\boldsymbol{\theta}}_i(\mathbf{x})$ and $\hat{\dot{\boldsymbol{\theta}}}_i(\mathbf{x})$ are given by the mathematical model, while $\boldsymbol{\theta}_i$ and $\dot{\boldsymbol{\theta}}_i$ are obtained from $\hat{\boldsymbol{\theta}}_i(\mathbf{x})$ and $\hat{\dot{\boldsymbol{\theta}}}_i(\mathbf{x})$ by adding a noise.

Estimates of the two unknown parameters, for different values of the standard deviation σ_n, using as initial guess $c_m = 0.001$ and $k_e = 0.01$, and the corresponding deviation, computed with respect to the nominal values of the Table 1 can be seen in Table 2.

The results reported in Table 2 show a good agreement between the values estimated by the Bayesian technique and the reference values shown in Table 2. Relative deviations less than 1% can be seen in the case of c_m, and negligible values are observed for k_e. These results provide an indication that the available nominal values for c_m and k_e are realistic.

	c_m		k_e	
σ_n	estimation	relative deviation	estimation	relative deviation
0.001	18.9×10^{-5}	$0.77\,\%$	601.6×10^{-4}	$0.0064\,\%$
0.01	19.1×10^{-5}	$0.77\,\%$	601.6×10^{-4}	$0.0068\,\%$
0.1	19.0×10^{-5}	$0.60\,\%$	601.6×10^{-4}	$0.0051\,\%$
1.0	18.9×10^{-5}	$0.82\,\%$	601.9×10^{-4}	$0.0061\,\%$

Table 2: Parameters of the experimental setup estimated for different values of σ_n, using as initial guess $c_m = 0.001$ and $k_e = 0.01$.

5.2   Estimation of the experimental apparatus parameters

In this section are presented the results of the estimation of all the nine physical parameters that are associated to the experimental setup. In this way, the parameter vector is given as follows

$$\mathbf{x} = \begin{pmatrix} j_m & c_m & k_e & r_m & k_t & l_m & k_s & j_1 & t_f \end{pmatrix}^T. \tag{18}$$

For this case the expression of the misfit function is similar to the one presented in Eq.(17), but now the sample vectors are no longer calculated based on the mathematical model, but are obtained experimentally.

The first thing to notice is that when one tries to estimate all the nine parameters, simultaneously, using the Bayesian approach used in the estimation procedure reported in the last section, convergence problems in the minimization procedure are observed.

To circumvent these problems of convergence, a heuristic procesudre to minimize the misfit function is proposed. In this heuristic algorithm, first it is assumed that the parameters $\mathbf{x}_1 = (t_f \ j_m)^T$ are unknown, and that the other parameters are given by the values of the Table 1. Then, to obtain estimates for t_f and j_m, the least-squares method is executed for 10 iterations, using an arbitrary initial guess.

In the next step of the heuristic algorithm, the unknown vector is assumed to be $\mathbf{x}_2 = (j_m \ c_m)^T$. The value of the parameter t_f and the initial guess for j_m are given by the estimation of the last step. Again the least-squares method is executed for 10 iterations, and an estimation for j_m and c_m is obtained.

In what follows, the procedure above continues using the following order to the vector of unknowns: $\mathbf{x}_3 = (c_m \ k_e)^T$, $\mathbf{x}_4 = (k_e \ k_t)^T$, $\mathbf{x}_5 = (k_t \ r_m)^T$, $\mathbf{x}_6 = (r_m \ l_m)^T$, $\mathbf{x}_7 = (l_m \ k_s)^T$, $\mathbf{x}_8 = (k_s \ j_1)^T$, $\mathbf{x}_9 = (j_1 \ t_f)^T$.

The overall procedure is repeated until a steady-state is reached for the misfit function. A schematic representation of this heuristic can be seen in Figure 2. See Sandoval (2013) for further details.
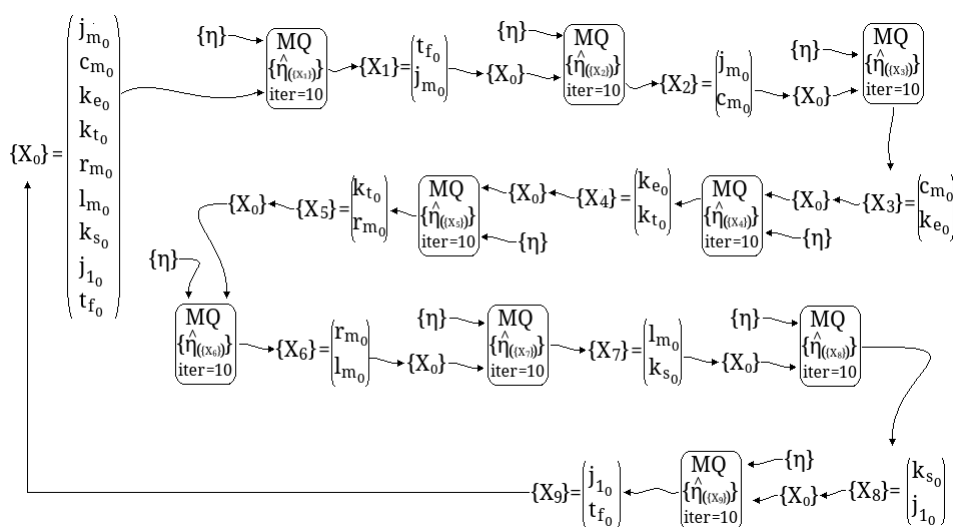

Figure 2: Schematic representation of the proposed heuristic.

Despite its heuristic nature, the algorithm shows good convergence properties, as can be seen in the Figures 3 and 4, which respectively show the value of the misfit function for each iteration, and the comparison between the rotations and their time derivatives, obtained from the experimental measurements and from the estimated curves. Note that the good agreement between the estimated and measured curves increases the degree of confidence in the heuristic estimation.
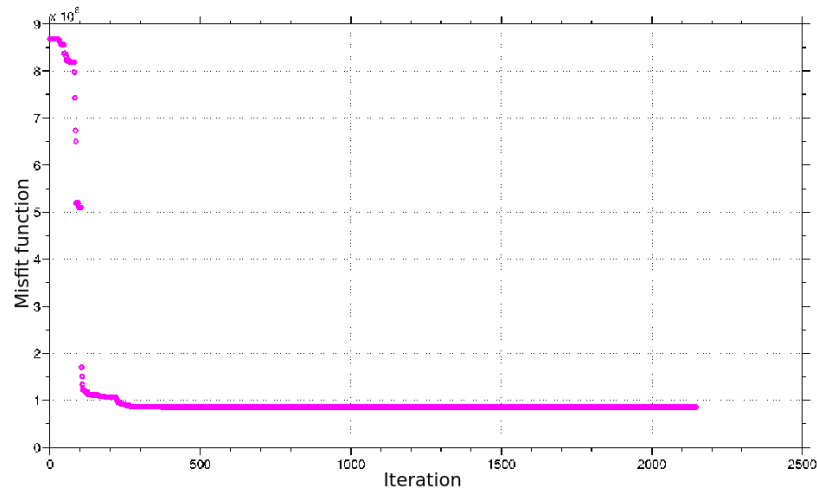

Figure 3: Value of the misfit function for each iteration index of the heuristic algorithm.
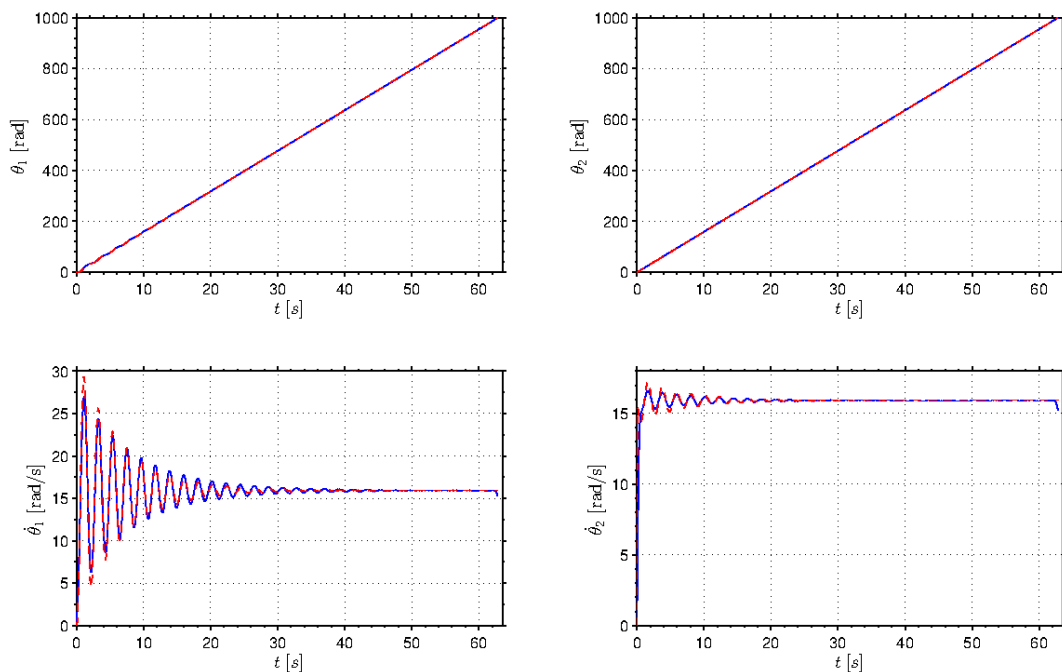

Figure 4: Values measured (blue line) in the experimental setup and curves (red marks) constructed by the Bayesian estimation method.

Estimated values for the physical parameters of interest are presented in Table 3, as well as the respective initial guess used in the estimation, and the relative deviations, which are measured with respect to the nominal values shown of the Table 1. Note that some parameters exhibit a small relative deviation, with a magnitude smaller than 7%, while others estimates are very distant from the nominal value, with a deviation of up to 60%.

Note that the nominal values in the Table 1 are in fact not the correct value of the parameters of the experimental setup, but, in fact, what one believes to know about them. Besides that, as the heuristic used showed itself reliable, one can argue that the nominal values of the parameters j_m, r_m, and l_m are not close to the real values of the test rig parameters.

However, it is the authors belief that this discrepancy in deviations has another explanation. First one can note that all the parameters have large deviations are associated with the motor. Note also that the modeling of the motor assumes a linear behavior, which does not correspond to reality, since torsional dynamics of the shaft is inherently nonlinear due to its coupling with the motor.

Another factor worthy of discussion is the representativeness of a model with only two degrees of freedom of rotation to described the dynamics of a flexible shaft, which is actually a deformable body with infinite degrees of freedom.

Therefore, it is the main conclusion of this study that the deficiencies of the model used to describe the dynamics of the experimental setup are responsible for causing the large deviations, with respect to the reference values, observed in the estimated parameters. The authors believe that better estimates can be obtained if a more realistic model is employed. For example, using a system with distributed parameters for modeling the shaft and to include a transistor in the description of the motor.

parameters	initial guess	estimation	relative deviation
j_m	4.0×10^{-4}	5.0×10^{-4}	$28.0\,\%$
c_m	19.0×10^{-5}	18.0×10^{-5}	$6.6\,\%$
k_e	601.6×10^{-4}	587.7×10^{-4}	$2.3\,\%$
k_T	1.2×10^{-1}	1.3×10^{-1}	$5.0\,\%$
r_m	3.3×10^{-1}	5.3×10^{-1}	$60.0\,\%$
l_m	1.1×10^{-3}	0.8×10^{-3}	$23.3\,\%$
k_s	2.6×10^{-1}	2.7×10^{-1}	$3.0\,\%$
j_1	28.3×10^{-3}	29.8×10^{-3}	$5.4\,\%$
t_f	1.0×10^{-1}	1.0×10^{-1}	$0.0\,\%$

Table 3: Parameters of the column-motor physical system estimated by the Bayesian estimation method.

Further results and discussion about the estimation of parameters of the experimental setup of interest can be seen in Sandoval (2013).

6   CONCLUDING REMARKS

In this work was presented a procedure for estimating the parameters of an experimental apparatus, composed by a shaft, two rotors, and a DC motor, that emulates a drilling process. A Bayesian approach was used to address the variabilities that are present in the experimental measurements due to factors such as measurement error, noise, etc.

For a verification test, to check the effectiveness of the Bayesian estimation technique, first only two parameters were considered as unknown. The results obtained with the estimation procedure for this two parameters problem present a very low deviation from the nominal values of the experimental apparatus for both parameters.

On the other hand, in a estimation process which considers nine parameters of the experimental apparatus as unknown, convergence problems of the Bayesian method were observed. A heuristic procedure was proposed to overcome this problem. The estimation results obtained with the proposed algorithm show some parameters with low relative deviation with respect to the nominal values, and others with large deviation. The presence of these large deviations evidentiates a strong deficiency in the model used to describe the dynamic behavior of the experimental apparatus, especially by not considering the continuous nature of the shaft and nonlinearities intrinsic to the shaft/motor coupling.

ACKNOWLEDGMENTS

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