

CONSTITUTIVE MODEL FOR RECYCLED AGGREGATE CONCRETE

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Abstract.

In this paper, the Performance Dependent Model previously proposed by Folino & Etse is reformulated in order to capture the mechanical behavior of Recycled Aggregate Concrete (RAC), a special type of concrete in which natural coarse aggregate content is partially or totally replaced by recycled aggregates obtained from the crushing of waste concrete. The model depending on the three stress invariants, has a non associative flow rule, a non uniform hardening law, and a softening law based on the fracture energy of modes I and II.

Regarding sustainability issues, RAC is of particular interest considering that it permits to reduce construction demolition waste and the use of natural aggregates. From the point of view of structural engineering, it is necessary to identify the main features of its mechanical behavior in order to accurately predict the structural behavior of structures using this material.

In this work, the input parameters of the model are reformulated for RAC considering the main features of its failure and mechanical behavior. Numerical predictions of peak stresses obtained with the proposed failure criterion included in the approach are validated against experimental results including uniaxial compression and triaxial compression tests, and numerical experiments showing the behavior under uniaxial compression are presented.

1 INTRODUCTION

This paper deals with a type of concrete characterized by special sustainability properties. Considering that in the construction industry concrete is the most used material and that its production consumes large amounts of energy and natural resources, any action helping to improve its sustainability conditions is of great interest for environmental protection.

The material that is particularly considered herein is *Recycled Aggregate Concrete* (RAC), in which natural coarse aggregates are partially or totally replaced by aggregates obtained from the crushing of waste concrete.

In the last years, several researchers have been devoted to the study of RACs, but there are still different open issues regarding mechanical behavior and production, and particularly regarding structural analysis. In what respect to mechanical behavior, it was demonstrated that a degradation of important concrete properties occurs when recycled aggregates are used. This fact was reported by several experimental researches (See a.o. Ryu (2002), Poon et al. (2004), Padmini et al. (2009), Buttler and Machado (2005) and Yang et al. (2008)). It was found that among other incidence factors, the percentage of replacement (R) of natural coarse aggregates by recycled ones was a fundamental parameter to be considered. A comprehensive summary of the influence of different R values is reported in Xiao et al. (2012b). From the literature, it is possible to conclude that both the Young's modulus E and the uniaxial compressive strength f'_c tend to decrease when R increases. Nevertheless, it is not possible to be so conclusive in what respects to axial and lateral strains, and neither about the resulting fracture energy.

A phenomenological continuum constitutive model is proposed in this paper to computationally predict the nonlinear behavior of the material under study. The main assumed hypothesis is that the complex behavior of concrete is influenced not only by the stresses state but also by concrete microstructure properties. For this purpose, the *Performance Dependent Model* (PDM), previously developed by Folino and Etse (2012), is considered and reformulated. It is important to point out the relevance that the development of constitutive formulations that accurately predict the mechanical behavior of structures built with new materials has for structural engineering, particularly for ensuring adequate safety factors.

2 BRIEF DESCRIPTION OF THE PERFORMANCE DEPENDENT MODEL (PDM) MAIN FEATURES

The PDM, is a constitutive theory for plain concrete based on the flow theory of plasticity. It depends on the three stress invariants, and its maximum strength surface is defined by the Performance Dependent Failure Criterion (PDFC). It considers a non uniform hardening law and an isotropic softening rule defined in terms of the concrete quality and of the first invariant of stresses in order to consider the influence of confinement on the ductility in the pre and post peak responses. Softening law is based on fracture energy concepts. It considers a volumetric non associative flow rule.

The PDFC adopted as maximum strength surface is defined in the Haigh Westergaard stress space in terms of the normalized stress coordinates (with respect to f'_c) $\bar{\xi}$, $\bar{\rho}$ and θ , depends on the three stress invariants I_1 , J_2 and J_3 .

The compressive and tensile meridians are defined by two parabolic equations

$$\theta = \frac{\pi}{3} \Rightarrow A \bar{\rho}_c^{*2} + B_c \bar{\rho}_c^* + C \bar{\xi} - 1 = 0 \quad (1)$$

$$\theta = 0 \Rightarrow A \bar{\rho}_t^{*2} + B_t \bar{\rho}_t^* + C \bar{\xi} - 1 = 0 \quad (2)$$

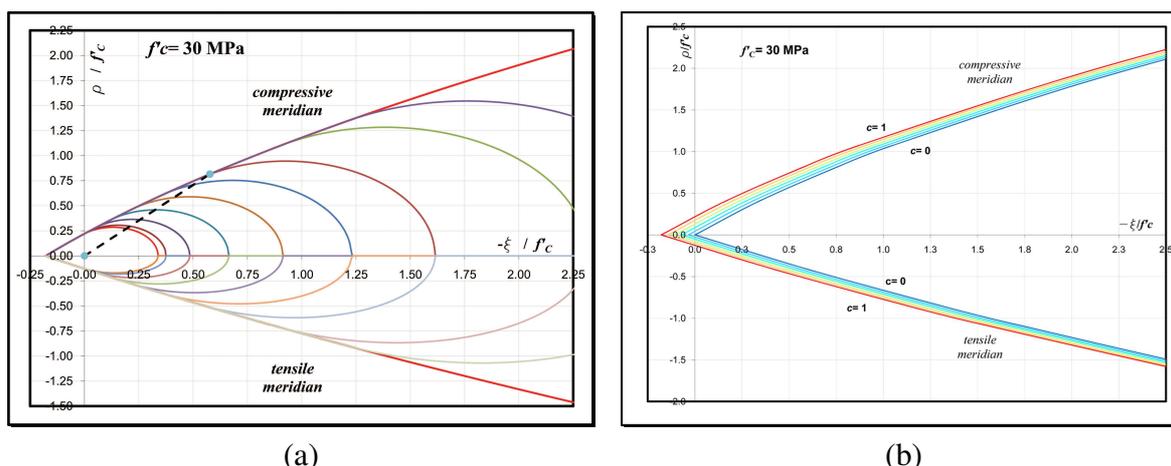


Figure 1: Compressive and tensile meridian views of the loading surfaces: (a) in hardening and (b) in softening

In the previous equations, the upper asterisk denotes failure, the subscripts "c" and "t" indicate compressive and tensile meridians respectively. In the deviatoric plane, the elliptic interpolation between the compressive and the tensile meridians by [Willam and Warnke \(1974\)](#) is followed.

$$\forall 0^0 \leq \theta \leq 60^0 \Rightarrow \bar{\rho}^* = \frac{\bar{\rho}_c^*}{r} \tag{3}$$

The ellipticity factor r is defined as

$$r = \frac{4(1 - e^2) \cos^2 \theta + (2e - 1)^2}{2(1 - e^2) \cos \theta + (2e - 1) \sqrt{4(1 - e^2) \cos^2 \theta + 5e^2 - 4e}} \tag{4}$$

where e is the eccentricity $\bar{\rho}_t^*/\bar{\rho}_c^*$.

The above equations lead to the general unified quadratic expression representing the failure surface

$$F_{max} = A r^2 \bar{\rho}^{*2} + B_c r \bar{\rho}^* + C \bar{\xi} - 1 = 0 \tag{5}$$

Coefficients A , B_c , B_t , and C defining the main meridians, are functions of material properties that are in turn expressed in terms of two fundamental parameters defining concrete quality: f'_c and the performance parameter β_P . Therefore, the PDFC adapt its shape according to the considered concrete quality. Parameter β_P , varying approximately between 0 and 1, is defined as

$$\beta_P = \frac{1}{1000} \frac{f'_c}{(w/b)} \quad f'_c \text{ in [MPa]} \tag{6}$$

where w/b is the water-binder ratio, being w and b the water and binder contents, respectively. A greater value of β_P means a more homogeneous concrete with less porosity. (See [Folino et al. \(2009\)](#) and [Folino and Etse \(2011\)](#) for further details).

In the PDM constitutive model, loading surfaces are constituted by a cap-cone yielding condition. The compressive cap is represented in a meridian plane by ellipses centered over the hydrostatic axis and tangents to the failure surface at the first Haigh Westergaard coordinate of a point $P1$ that continuously changes during the hardening process. The semi-axes ratio a/b remain constant. The loading surfaces (See Fig. 1(a)) are defined as

$$f_h = \begin{cases} f_h^{cone} = F_{max} = A r^2 \bar{\rho}^{*2} + B_c r \bar{\rho}^* + C \bar{\xi} - 1 = 0 & \text{if } \bar{\xi} > \bar{\xi}_{1(k)} \\ f_h^{cap} = \frac{(\bar{\xi} - \bar{\xi}_{cen(k)})^2}{a_{(k)}^2} + \frac{r^2 \bar{\rho}^2}{b_{(k)}^2} - 1 = 0 & \text{if } \bar{\xi} \leq \bar{\xi}_{1(k)} \end{cases} \quad (7)$$

Since point $P1$ lies on the maximum strength surface, its coordinates $(\bar{\xi}_1; \bar{\rho}_{c1})$ are related by Eq. 1. Each loading surface is characterized by a parameter k defined in terms of $\bar{\rho}_{c1}$ as

$$k = \frac{\bar{\rho}_{c1}}{\sqrt{2/3}} \quad (8)$$

This parameter has a minimum initial value k_o corresponding to the first loading surface where the inelastic behavior starts, and depending on concrete quality. Nevertheless, it has no upper limit, indicating that ideally the loading surfaces can evolve indefinitely. The dimensions and location of the elliptical cap are defined by the coefficients a (ellipse semi-axis on the hydrostatic axis), b (ellipse semi-axis on the deviatoric axis) and $\bar{\xi}_{cen}$ (coordinate of the center of the ellipse on the hydrostatic axis), all of them depending on k . (See Folino and Etse (2008)).

The evolution of the hardening parameter is defined as

$$k = k_o + (k_{max} - k_o) \sqrt{\kappa_h (2 - \kappa_h)} \quad (9)$$

where k_{max} is derived from the definition of k , considering the hardening parameter associated with the largest ellipse possible for a given value of confinement $\bar{\xi}$, and κ_h is a normalized work hardening measure defined as the ratio between the actual developed work hardening $\dot{\omega}_a^P$, and the total work hardening capacity W_t^P for the actual confinement level $\bar{\xi}$

$$\kappa_h = \frac{\dot{\omega}_a^P}{W_t^P} = \frac{\underline{\underline{\sigma}} : \underline{\underline{m}} \dot{\lambda}}{W_t^P} \quad (10)$$

being W_t^P evaluated in terms of concrete quality and of the actual hydrostatic normalized stress $\bar{\xi}$.

In the PDM, yielding surfaces in post peak regime are represented by the continuous contraction of the cone being each surface associated to the decohesion parameter c representing the ratio between the actual and the peak stress. The unloading surfaces (See Fig. 1(b)) are mathematically described as

$$f_s = A r^2 \bar{\rho}^2 + B_c r \bar{\rho} + C \bar{\xi} - c = 0 \quad (11)$$

Fracture energy properties are incorporated in the σ - ε relation by introducing an homogenization strategy: fracture energy G_f^I dissipated during the crack opening process along the surface of the crack A_t in a direct tensile test, is considered to be equal to the energy W dissipated during plastic softening in an equivalent continuum. Crack opening displacement \dot{u}_f is evaluated by the consideration of equivalent tensile fracture strains $\tilde{\varepsilon}_f$ uniformly distributed in the continuum in a localization width l_c which constitutes an internal characteristic length

$$\dot{u}_f = l_c \dot{\tilde{\varepsilon}}_f \quad (12)$$

In this case (mode I), the characteristic length l_c is a measure of the crack spacing in a direct tensile test h_t . This concept is extended to a general mode II of failure by considering an appropriate characteristic length which is defined in terms of fracture energy in mode II, G_f^{II} depending on confinement level and on G_f^I . An outstanding feature is that the latter in the frame of the PDM is approximated by a function depending on concrete quality and on the maximum coarse aggregate size.

The evolution of the decohesion parameter is defined by an exponential decay function as

$$c = \exp\left(\frac{-\delta \kappa_s}{u_r}\right) \quad (13)$$

where u_r represents the maximum crack opening displacement, δ is a parameter that defines the shape of the decay function, and κ_s is a fracture energy based softening measure defined as follows

$$\dot{\kappa}_s = l_c \dot{\bar{\varepsilon}}_f = l_c \|\langle \underline{\mathbf{m}} \rangle\| \dot{\lambda} \quad (14)$$

The McCauley operator extracts only the tensile components of the plastic potential surface gradient $\underline{\mathbf{m}}$. The characteristic length l_c in the PDM is determined in terms of the actual confinement level and of concrete quality.

A non associative flow rule is adopted by defining a plastic potential introducing a modification on the dependence of the loading/unloading surfaces on the hydrostatic coordinate $\bar{\xi}$. A non associative parameter η_o is considered and evaluated in terms of concrete quality and of the hydrostatic normalized stress $\bar{\xi}$.

3 APPROACH FOR THE MODELING OF RACS WITH THE PDM

Recycled coarse aggregates obtained from the crushing of waste concrete are characterized by a greater porosity and a lower strength than natural coarse aggregates. When are used to partially or totally replace natural aggregates, mix proportioning needs to include a greater water content due to the greater absorption of recycled aggregates.

Regarding that the PDM constitutive model involves concrete quality in the definition of all its functions, it was considered to be suitable for the modeling of RACs by conveniently adapting the definition of the two main input parameters: the performance parameter β_P and the uniaxial compression strength f_c' .

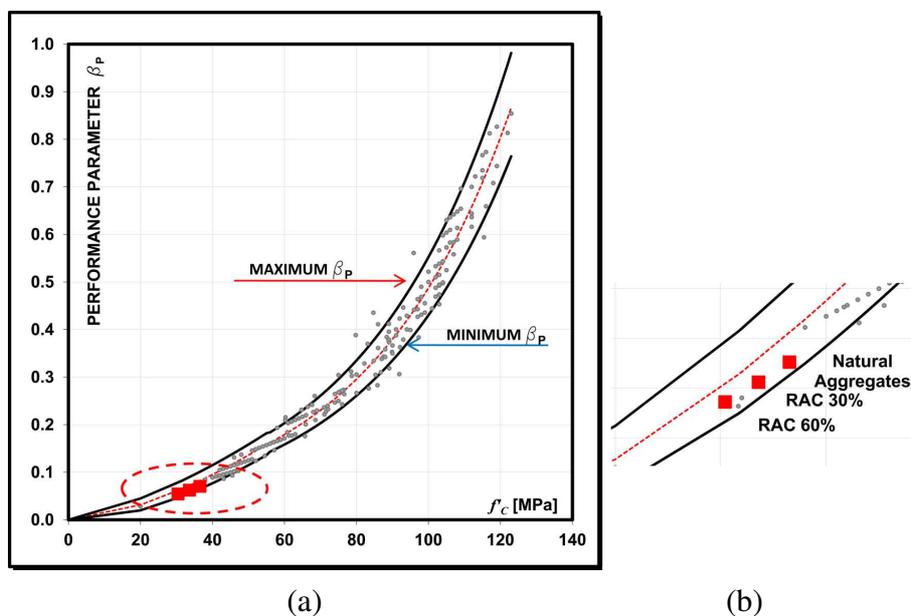


Figure 2: Performance parameter vs. f_c' : (a) global view for ordinary and recycled concretes with $f_c' \leq 120$ MPa, and (b) zoom view for RAC case

Index β_P in Eq. 6 was defined after analyzing a large concrete mixtures database. Those original data only considered ordinary or natural coarse aggregate concrete (NAC). It was found that a very good

correlation exists between β_P and f_c' as can be observed in Fig. 2(a), where the grey points correspond to the obtained β_P for different real concrete mixtures.

In order to include the RAC case, in the frame of this work β_P was evaluated for different RAC concrete mixtures. The red points in Fig. 2(a) represent the obtained values for RAC concrete mixtures elaborated at FIUBA laboratory, for different replacement percentages R of natural by recycled coarse aggregates. A zoom view can be seen in Fig. 2(b). It can be noted that an increasing degradation in both parameters is observed for increasing values of R . The same tendency was also observed for other data extracted from the literature.

The approach presented herein for the RAC constitutive modeling, involve the assumption of the following hypotheses

- It is assumed that a RAC is obtained, for a desired value of R , by adjusting the water content in the concrete mix proportioning of a target NAC, while all the other contents remain the same.
- With respect to the target NAC, the obtained RAC presents lower values of β_P and f_c' , and also a lower Young's Modulus E .
- In the numerical approach in this work it is considered that f_c' and E degradations only depend on R and on the NAC concrete quality. This is a simplifying assumption considering that degradation in mechanical properties produced by the use of recycled aggregates depends also on the recycled aggregates quality, which is strongly influenced by the parent concrete quality from which the recycled aggregates are obtained, and by the crushing procedure. Moreover, that quality can be considerably improved by a cleaning mechanical process of the recycled aggregates.
- The failure surface of concrete is represented by the PDFC, even in the case of RACs. In other words, the partial or total replacement of natural coarse aggregates by recycled ones produces degradation in mechanical properties like the uniaxial compressive strength, and on the performance parameter, but once these are determined, peak stresses for different load scenarios are the same than that of a natural aggregate concrete with the same f_c' and β_P .

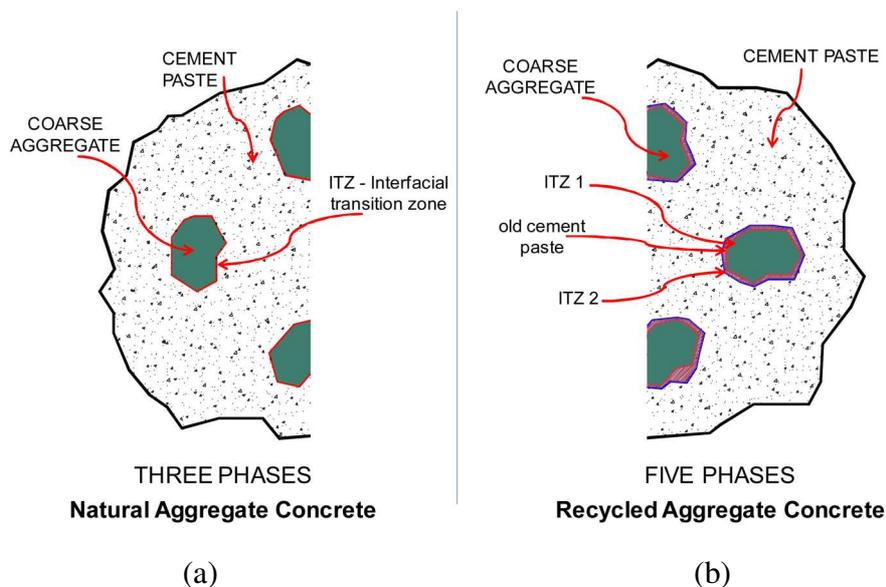


Figure 3: Interface Transition Zone for concrete with: (a) natural coarse aggregates, and (b) recycled coarse aggregates

- From a mesoscopic point of view, it is understood that a standard concrete is a composite material constituted by three main phases: the cement paste, the natural coarse aggregates, and the interface transition zone (ITZ) between paste and coarse aggregates. Following Xiao et al. (2012a), it is assumed that in a recycled coarse aggregate concrete, the cement paste that remains attached to the surface of the natural coarse aggregate, introduce two more constituents: an ITZ corresponding to the interface between the natural coarse aggregate and this old cement paste, and another one, constituted by the interface between the old and the new cement pastes. Consequently, five main phases are involved in the mesoscopic composition of RACs. This is illustrated in Fig. 3.
- Considering that fracture energy in mode I, G_f^I , is strongly influenced by the maximum coarse aggregate size (van Mier (1997)), it can be concluded that depending on the relative quality of the two ITZ present in RACs, the resulting fracture energy is expected to be less or at least equal of that of the original concrete, for the same maximum global aggregate size. This conclusion may not be valid in the case of high and very high strength concretes, where cracks can cross along the coarse aggregates. As in the constitutive model applied in this paper G_f^I is evaluated by a function depending on concrete quality and on the maximum coarse aggregate size Φ_{max} , an *effective* maximum size Φ_{max}^{RAC} will be considered for the RAC case in order to estimate the reduction above mentioned.
- It is assumed that the internal functions of the PDM are also suitable for the case of RACs. Therefore, it is accepted that RACs mechanical behavior is equivalent to that of a natural aggregate concrete with the same f'_c , β_P , E , and Φ_{max} values.

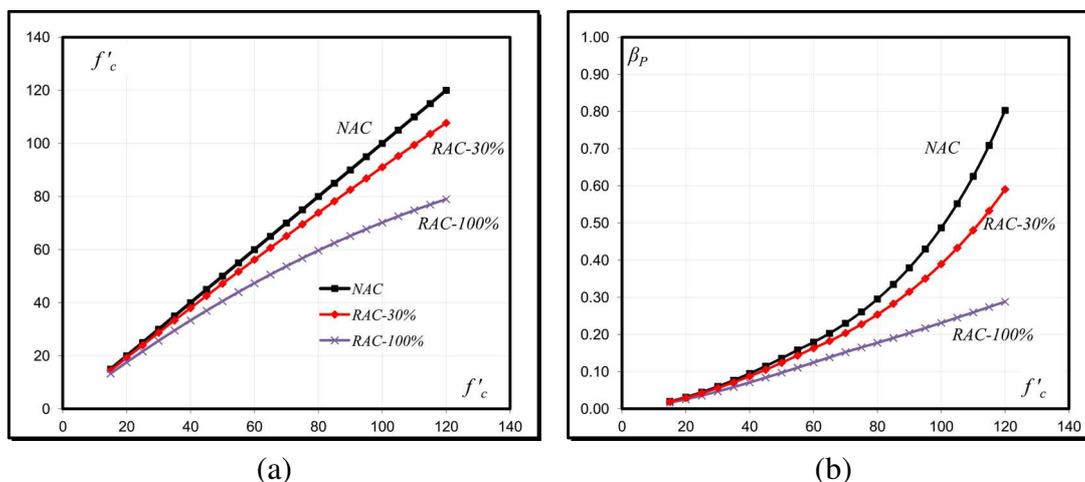


Figure 4: Proposed variation of main parameters for different percentages of replacement R and different f'_c NAC: (a) Uniaxial compressive strength and (b) Performance parameter

For the description of RAC quality degradation, different approximation functions were considered in the frame of this work. Firstly, it was proposed to linearly modify β_P in terms of R , but this approach led to some problems for the f'_c determination regarding that β_P has a different variation law for concretes below and over 55MPa, point were concrete mixtures usually starts incorporating mineral and chemical admixtures. For this reason it was finally proposed to linearly modify f'_c in terms of R , with a coefficient depending on the NAC f'_c strength, and obtain then β_P from the approximation functions originally proposed for NAC (Folino et al. (2009)). The proposed variation is defined by the following expression

$$f'_c{}^{RAC} = f'_c{}^{NAC} (1 - \alpha_{f_c} R) \quad (15)$$

where f_c^{RAC} and f_c^{NAC} are the uniaxial compressive strengths for the recycled and natural concretes, respectively, expressed in [MPa], R is expressed in [%], and α_{fc} is a coefficient defined as

$$\alpha_{fc} = 0.08 \left(1 + 1.50 \frac{f_c^{NAC}}{55} \right) \quad (16)$$

Then, index β_P is determined in terms of f_c^{RAC} by

$$\beta_P = \begin{cases} 0.00026 \cdot (f_c^{RAC})^{1.60} & \text{if } f_c^{RAC} \leq 55 \text{MPa} \\ 0.04 \cdot e^{(0.025 f_c^{RAC})} & \text{if } f_c^{RAC} > 55 \text{MPa} \end{cases} \quad (17)$$

The plots in Fig. 4 show the resulting variation law of the main input PDM parameters, in terms of R and f_c^{NAC} .

4 NUMERICAL RESULTS

Coefficients A , B_c , B_t , and C in Eq. 5 representing the mathematical expression of the failure surface in the PDFC, are completely defined once that β_P and f_c' are determined.

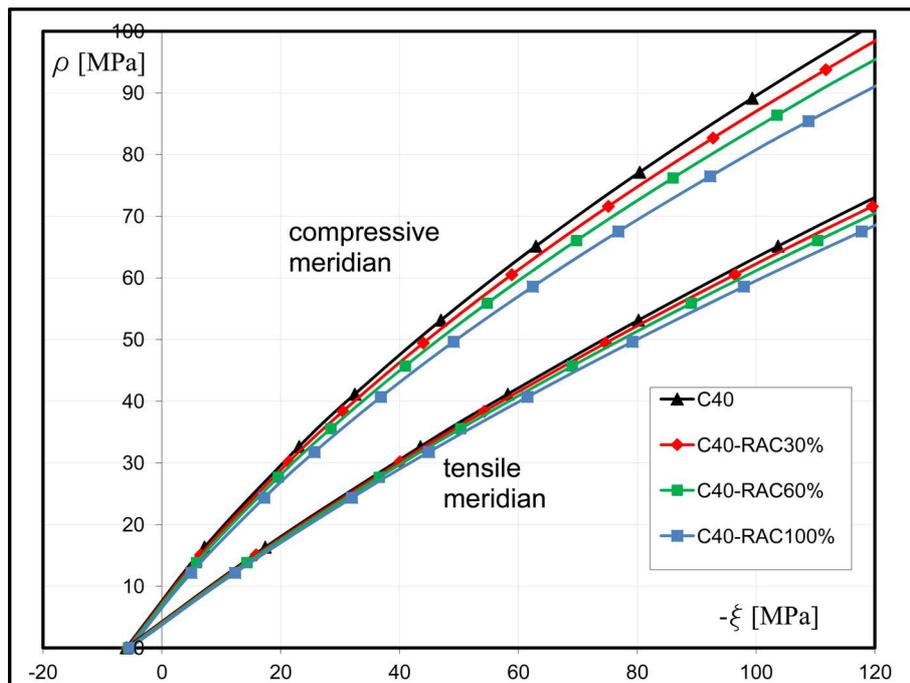


Figure 5: Compressive and tensile failure meridians numerical predictions applying the PDFC for different percentages of replacement R and $f'_c = 40 \text{MPa}$

In Fig. 5 the PDFC compressive and tensile meridians numerical predictions are presented for a NAC with $f'_c=40 \text{MPa}$ in which the natural coarse aggregate content was replaced by recycled aggregates in different percentages R from 0% to 100%. It can be observed that the failure surface experiments a contraction when R increases, which is more evident in the case of the compressive meridian.

Some triaxial experimental tests on RACs have been performed at FIUBA Materials and Structures Laboratory. A NAC with $f'_c=40 \text{MPa}$ was elaborated and two different values of R were considered corresponding to 30% to 60%, respectively. Cylindrical samples 200mm height and a diameter of 100mm were used and subjected to uniaxial compression and to three different confinement levels of 4.5MPa,

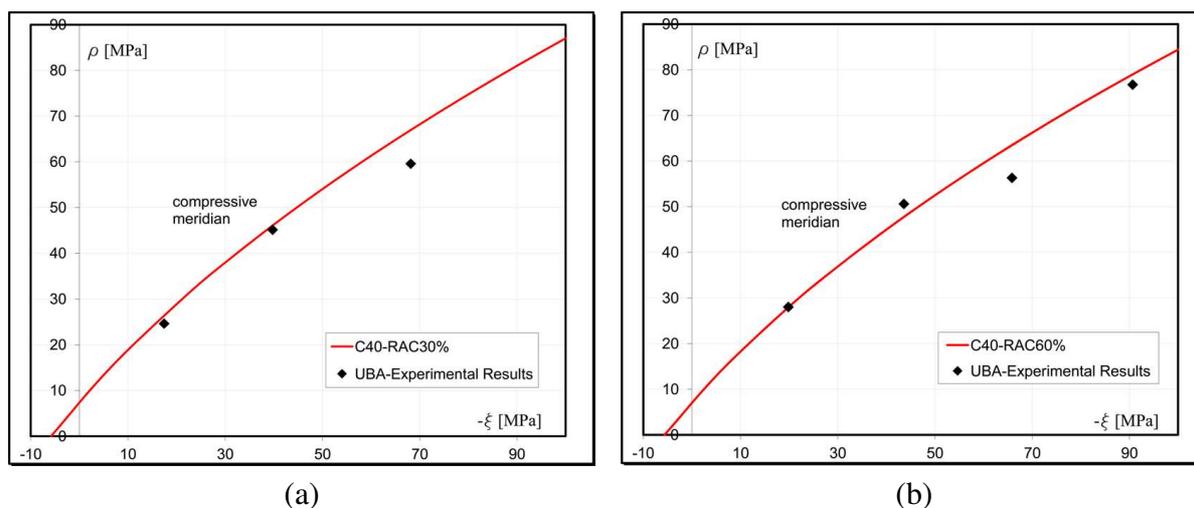


Figure 6: Comparison between triaxial experimental and numerical results applying the PDFC for $f'_c = 40$ MPa and: (a) $R = 30\%$; (b) $R = 60\%$

15MPa and 21MPa, respectively. Recycled aggregates were obtained from the same concrete by the crushing of samples previously tested.

The plots in Fig. 6 present the comparison between the experimentally obtained peak stresses and numerical predictions obtained by applying the PDFC. It can be observed a good accuracy in the numerical predictions.

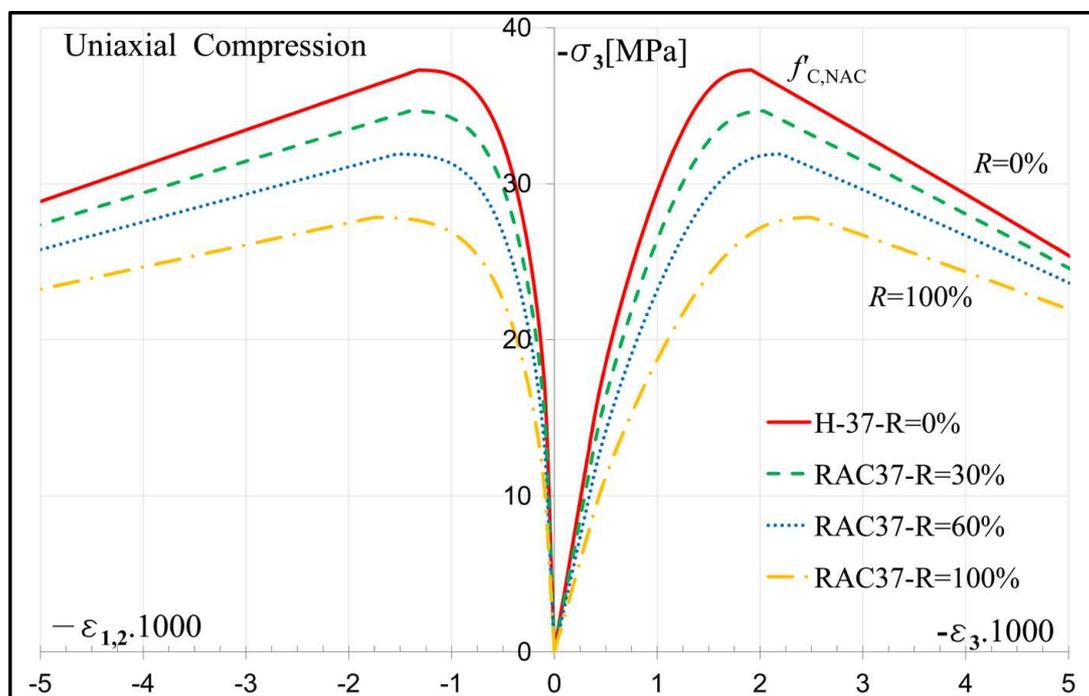


Figure 7: Uniaxial compression numerical results applying the PDM for different percentages of replacement R and $f'_c = 40$ MPa

Finally, numerical tests applying the PDM constitutive model are presented in Fig. 7 representing RACs behavior under uniaxial compression, including different R percentages. A good agreement with general behavior reported in the literature and observed in experimental tests at FIUBA is achieved,

showing the increasing degradation in strength and in elastic properties for increasing R .

5 CONCLUSIONS

In this work an approach for the modeling of the failure and mechanical behavior of Recycled Aggregate Concrete, obtained by the partial or total replacement of natural coarse aggregates by recycled ones, was presented.

Recycled aggregates, obtained from the crushing of waste concrete, are characterized by a greater porosity and a lower strength than those of natural aggregates. Consequently, concrete obtained using these aggregates generally presents lower strength and a degradation in elastic properties.

With the aim of formulating a constitutive model valid for RAC, the main input parameters of the Performance Dependent Model (PDM) by Folino and Etse (2012) were reformulated for the RAC case based on its main features.

Numerical approximations of the degradation on mechanical and microstructural properties were proposed and incorporated to the model. The failure criterion included in the approach was validated by the comparison of numerical and experimental results. A good agreement with the general tendencies that were observed in experimental tests performed at FIUBA were observed in what respect to peak stresses under different confinement levels. Numerical experiments showing the behavior under uniaxial compression were presented. Although the results so far demonstrated that the PDM is suitable for the RAC modeling, further research is needed in order to verify the model capabilities under different loading paths.

6 ACKNOWLEDGMENTS

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