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MÉTODO DE HACES APLICADO A LA COORDINACIÓN HIDROTÉRMICA A CORTO PLAZO CONSIDERANDO RESTRICCIONES AMBIENTALES

Aldo J. Rubiales[a,b], **Pablo A. Lotito**[a,b], **Fernando J. Mayorano**[a,b], **Lisandro A. Parente**[b] **y Mariano A. Risso**[a,b]

[a]*PLADEMA , Universidad Nacional del Centro de la Provincia de Buenos Aires*

[b]*CONICET*

Palabras Clave: Coordinación Hidrotérmica a corto plazo, Unit Commitment, Método de Haces, Descomposición Generalizada de Benders, Restricciones Ambientales.

Resumen. La resolución del problema de coordinación hidrotérmica de corto plazo comprende tanto el pre-despacho (Unit Commitment), como el despacho económico de las unidades térmicas e hidráulicas en forma integral para un horizonte de tiempo semanal o diario con paso horario. Con el objetivo de evitar correcciones post-despacho en el presente trabajo se modelan de manera detallada las restricciones asociadas a la red de transmisión. La definición tradicional del problema de coordinación hidrotérmica de corto plazo, minimiza sólo el costo del uso de combustible y no incluye el tratamiento de la contaminación producida por la emisión de distintos tipos de gases provenientes de la operación de las centrales térmicas. Uno de los aportes novedosos del presente trabajo es la consideración de restricciones ambientales. El otro aporte es el método de resolución que combina la Descomposición Generalizada de Benders con el método de Haces. Esta combinación de ambos se asemeja a una versión estabilizada del método planos cortantes, que reduce drásticamente el conocido efecto de *tailing-off* que los métodos de Benders tienen. A través de la misma se logra descomponer el problema original en un problema maestro cuadrático entero mixto y un subproblema no lineal, de manera que el primero proponga los despachos considerando variables enteras y el segundo controle la factibilidad eléctrica del despacho propuesto considerando una linealización de las restricciones que surgen al considerar las características de la red. El esquema de resolución propuesto se aplica a un caso de prueba de 24 barras y a una red real de tamaño medio.

1. INTRODUCTION

Nowadays, the production of clean energy is an extremely important topic. Although hydraulic generation is growing nowadays, fossil fuels represent a reliable and affordable source of energy, necessary to satisfy the demand for electric energy. Coordinating thermal and hydraulic electricity generation is then a crucial problem faced generally by governmental agencies. This work aims to solve the Short Term Hydrothermal Coordination Problem (STHTC) whose objective is to decide between the two kinds of generation in a short period of time.

Economies based on fossil fuels has brought with it the potential harmful problem of the emission of gaseous and particulate products of combustion, which when reaches a pre-specified threshold, is termed pollution (Bellhouse y Whittington, 1996). Environmental concerns are becoming increasingly relevant for companies as regulations on pollutants become more stringent, therefore these concerns must be considered in scheduling models.

Conventional power generation plants causes pollution through the emission of several gases into the atmosphere. Among these gases are carbon dioxide (CO2), sulfur dioxide (SO2) and nitrogen oxides (NOx) which have a global environmental impact (greenhouse effect) and local effects such as acid rain and reduced visibility among others. In this work, environmental concerns are considered as a cost given by quadratic functions of thermal power generated by each unit. These functions are used to penalize the amount of emission of each gas.

STHTC considering a centralized dispatch has been used world-wide. Solving this problem defines the operation state and power level of each generation unit (thermal and hydraulic) of an interconnected power system achieving the lower operative cost, satisfying technical and operative constraints of generators and transmission network, among others.

The STHTC problem without considering environmental constraints has been studied considering different formulations and using different resolution techniques as dynamic programming, Lagrangean relaxation and methods based on Benders decomposition.

The use of dynamic programming to solve the STHTC problem was also mentioned in Wood y Wollenberg (1984). It provides the possibility of modeling complex objective functions and constraints, and it is both easy to understand and to implement as well as to integrate and to combine with other optimization methods. Although dynamic programming allows modeling non-linear and non-convex problems, because of its combinatorial characteristic (Hillier et al., 1990), to have reasonable calculation times only a small number of thermal units can be considered. This fact makes it impractical for large problems, as is the case of STHTC. In Rubiales et al. (2007), this approach is applied to a hydrothermal system with pumped-storage units. This article mentions the problem of dimensionality and the approach presented in Lemarechal y Sagastizabal (1997), is suggested for its resolution.

While the application of Lagrangian relaxation to Economic Dispatch (ED) problems has been done since the mid-nineties, approaches considering network issues can be seen only in the last ten years. For example, Ongsakul y Petcharaks (2005), shows the numerical solution of the ED and Unit Commitment (UC) problems addressed by a Lagrangian relaxation variation called *ILR* by *Improved Lagrangian Relaxation*. It was applied to the IEEE 24-bus test case only considering thermal units and DC network constraints.

Lagrangian relaxation and Benders method are applied in Lu y Shahidehpour (2005), to solve the problem of UC on a set of thermal units considering a detailed network. This algorithm was applied to a case of 118 buses network with a planning horizon of 24 hours. Among more recent works that consider hydroelectric units is Finardi et al. (2005), which combines the use of Lagrangean relaxation with sequential quadratic programming. Although in Finardi et al.

(2005), the authors define a detailed model of hydroelectric plants, network constraints are not considered.

Another approach which uses a combination of augmented Lagrangian relaxation and dynamic programming is presented in Wang et al. (1995b). In this work, the decomposition and coordination technique is used for generation scheduling with transmission and environmental constraints. Even though numerical results indicate that the proposed approach is fast and efficient in dealing with numerous system constraints, the network model used does not accurately represent real power networks. Therefore, post-dispatch corrections are necessary.

The approach presented in Catalão et al. (2008), allows short-term scheduling of thermal units, designed to simultaneously address the economic issue of the fuel cost incurred on the commitment of the units and the environmental consideration due to emission allowance trading. In Catalão et al. (2008), the STHTC considering emission constraints is modeled by a multi-objective optimization problem, which is solved by a combination of the weighted sum method with the ε-constraining method. However, in Catalão et al. (2008), the authors do not consider network which are necessary to avoid post-dispatch corrections.

In recent years, due to the advantages that Generalized Benders decomposition (GBD) have shown for the resolution of large scale problems, several papers that address the short-term study using GBD (Geoffrion, 1972) have been presented. A method based on Benders decomposition to solve the problem of multistage hydrothermal coordination is presented in Diniz et al. (2006). In this representation, the hydroelectric sector is modeled with a high level of detail but applies a linear DC losses model of transmission lines. T. Akbari (2009) presents a multi-stage stochastic model for short-term transmission expansion planning solved combining Benders decomposition with Montecarlo simulation method. One of the later studies that considers the application of GBD to the problem of STHTC considering environmental concerns is Rubiales et al. (2012). The principal drawback of this method is the slow convergence of the algorithm due to the *tailing-off effect* presented by this resolution scheme.

In this paper a sophisticated version of STHTC considering environmental concerns is solved. This version covers both the unit commitment of thermal and hydropower units, and the economic dispatch of them. From a system operator point of view, solving this problem considering realistic aspects (such as those applied in this work) is an essential tool to define the daily dispatch of generating units. The main advantage of this approach to those which do not consider AC power flow is the fact that, if the latter ones are applyed, large corrections must be made for real operation use (Miguélez et al., 2004). The last fact, not only makes harder the system operator work but also many times the system is operated in a non-optimal way. All these facts lead to a very complex optimization problem, which is solved by using a novel decomposition approach based on GBD and Bundle methods. The latter drastically reduces the well-known tailing-off effect the former presents. The combination of those methods together with an adequate choice of the stopping criterion drastically reduces the total resolution time with respect to previous results presented in Rubiales et al. (2012) without any substantial differences in the final schedule.

The proposed algorithm is developed using Generalized Algebraic Modeling of System (GAMS). The applicability of this software to solve optimization problem in the power industry has been probed in several works. Among others, in A. Lashkar Ara (2010) the Optimal Power Flow problem is formulated as a nonlinear optimization problem with both equality and inequality constraints and solved using GAMS. The approach presented in S. M. Ezzati (2011) use the same software and Mixed Integer Non Linear Programming (MINLP) for solving Security Constrained Optimal Power Flow (SCOPF).

This paper is organized as follows. In section 2 the nomenclature is presented. The definition of the short-term hydrothermal coordination problem considered in this work and in Rubiales et al. (2012) is presented in section 3. Section 4 described the resolution methodology which combines GBD with Bundle methods. Then, its application to several cases is shown in section 5. And finally, concluding remarks are given in section 6.

2. NOMENCLATURE

Sets

- $t \in T$ time periods associated with the planning horizon.
- $i \in I$ thermal units.
- $j \in J$ hydroelectric units.
- $b \in B$ system buses.
- $ib \in ct(b)$ thermal units directly connected to bus b.
- $jb \in ch(b)$ hydroelectric units directly connected to bus b.
- $b' \in cb(b)$ Set of system buses directly connected to bus b.
- $r \in R$ reservoirs or dams.

Variables

- $pt_{t,i}$ active power generated by thermal unit i for period t.
- $ut_{t,i}$ state of the thermal unit i for period t.
- $st_{t,i}$ binary variable indicating that the thermal unit i has started for period t.
- $et_{t,i}$ continuous variable that is used for the purpose of checking that the minimum and maximum time of operation of each thermal unit is accomplished.
- $qt_{t,i}$ reactive power generated by thermal unit i for period t.
- $\epsilon p^-_{t,b}$ active power deficit on bus b for period t.
- $\epsilon p^+_{t,b}$ active power excess on bus b for period t.
- $\epsilon q^-_{t,b}$ reactive power deficit on bus b for period t.
- $\epsilon q^+_{t,b}$ reactive power excess on bus b for period t.
- $a_{t,r}$ water volume of the reservoir r for period t.
- $q^T_{t,r}$ across-turbine outflow of the reservoir r for period t.
- $q^I_{t,r}$ inflow of reservoir r for period t.
- $q^S_{t,r}$ spilled outflow of reservoir r for period t.

- $v_{t,b}$ voltage on bus b for period t.
- $\theta_{t,b'}$ voltage angle for period t on bus b.

$\mu_{t,i}^{k}$ lagrange multiplier associated to the active power generated by the thermal unit i at time t in the cut generated at iteration k.

$\lambda_{t,j}^{k}$ lagrange multiplier associated to the active power generated by the hydroelectric unit j at time t in the cut generated at iteration k.

$\pi_{t,i}^{k}$ lagrange multiplier associated to the thermal unit i state at time t in the cut generated at iteration k.

$\psi_{t,j}^{k}$ lagrange multiplier associated to the hydroelectric unit j state at time t in the cut generated at iteration k.

Constants

- A_i quadratic cost coefficient of thermal unit i.
- B_i linear cost coefficient of thermal unit i.
- C_i free cost coefficient of thermal unit i.
- D_i start-up cost coefficient of thermal unit i.
- Ep^{-} penalty coefficient due to active power deficit.
- Ep^{+} penalty coefficient due to active power excess.
- Eq^{-} penalty coefficient due to reactive power deficit.
- Eq^{+} penalty coefficient due to reactive power excess.
- pt_i^{LOW} y pt_i^{UP} minimum and maximum active power output of thermal unit i.
- qt_i^{LOW} y qt_i^{UP} minimum and maximum reactive power output of thermal unit i.
- ph_j^{LOW} y ph_j^{UP} minimum and maximum active power output of hydroelectric unit j.
- qh_j^{LOW} y qh_j^{UP} minimum and maximum reactive power output of hydroelectric unit j.
- on_i^{LOW} minimum on-time of thermal unit i.
- off_i^{LOW} minimum off-time of thermal unit i.
- ΔPT_i^{UP} maximum active power difference for two consecutive periods of unit i.
- ϑ_i^{UP} maximum fuel for unit i for whole planning horizon.
- ζ_t spinning reserve required for period t.
- $\Psi p_{t,b}^{-}$ active power load for period t on bus b.
- $\Psi q_{t,b}^{-}$ reactive power load for period t on bus b.

- $G_{bb'}$ and $B_{bb'}$ real and complex components of the admittance matrix at position bb'.
- a_r^{LOW} y a_r^{UP} minimum and maximum volume limits of reservoir r.
- v_b^{LOW} y v_b^{UP} minimum and maximum voltage limits of bus k. b.
- $\Omega_{bb'}^{UP}$ power flow limit between two buses b and b'.

3. PROBLEM DEFINITION

In this section the STHTC problem applied to centralized electricity markets based on audited costs is defined. As it was stated in Rubiales et al. (2012) this definition consider Environmental concerns. The minimization problem objective function is given by (1). If environmental constraints are not considered, it corresponds to the cost related to produce the electricity needed to meet a fixed demand, which is estimated for each period. Emission control may be included as an extra cost of generation (Ramanathan, 1994) or as an extra constraint which limits the total emission generated by each thermal unit during the planning horizon. In this approach the former methodology is chosen and different types of emissions (CO2, SO2, NOx, etc.) are considered. Like fuel cost curves, the CO2, SO2 and NOx emission functions can be expressed as quadratic costs for each emission type. The total cost function fo summarize costs associated with fuel consumption and startup of thermal units and penalties related with different types of emissions. This function is defined as follows:

$$fo = \sum_t \sum_i P_{t,i}(pt_{t,i}, ut_{t,i}, st_{t,i}) + \sum_q \sum_t \sum_i w_q E_{q,t,i}(pt_{t,i}, ut_{t,i}) \tag{1}$$

$$P_{t,i} = A_i pt_{t,i}^2 + B_i pt_{t,i} + C_i ut_{t,i} + D_i st_{t,i} \tag{2}$$

$$E_{q,t,i} = X_{q,i} pt_{t,i}^2 + Y_{q,i} pt_{t,i} + Z_{q,i} ut_{t,i} \tag{3}$$

Hence, power generation cost $P_{t,i}$ and pollution generated $E_{q,t,i}$ for each unit are defined as a quadratic curve. The coefficients $X_{q,i}$, $Y_{q,i}$ and $Z_{q,i}$ are generally obtained by curve fitting. The number of terms and segments in the emission curve depends upon the characteristic of the unit (Ramanathan, 1994). It should be mentioned that hydrothermal units does not have costs associated to environmental concerns and to power generation. Environmental costs of hydroelectric units are not present in fo because only emission costs are considered. The electricity generated by these units is derived from the force or energy of falling water which is accumulated in unit reservoir and they do not consume any kind of fuel. However, the use of water to generate power hydroelectrically in a given time comprise the use of water for future generation and vice-versa. The main issue is to know the total volume of water to be spent in the planning horizon. In the literature, there are two methods for dealing with this issue Wood y Wollenberg (1984). The first one considers that the total amount of water in the reservoir is available in the short term, but a value to the amount of water that is not spent is assigned to motivate hydroelectric plants to keep water beyond the horizon of analysis. The second approach considers that a known fixed volume of water is available to be used in the planning horizon (obviously, less than the total volume of water in the reservoir) as a result of long-term programming that takes into account other modeling aspects (uncertainty in weather, demand, etc.). In this work, the second approach is adopted, avoiding the need to assign the value of water. This fixed volume of water available during the horizon of analysis is considered in the definition of the initial and final volume for each reservoir.

The constraints were divided into five groups, which are detailed below.

3.1. Constraints associated with thermal units only

$$ut_{t,i}pt_i^{LOW} \leq pt_{t,i} \leq ut_{t,i}pt_i^{UP} \tag{4}$$

Equation (4) represents box constraints associated with the active power of each thermal unit. Thus, given the discontinuity the power of a thermal unit has, it is necessary to introduce binaries variables to properly address possible states of operation.

$$ut_{t,i}qt_i^{LOW} \leq qt_{t,i} \leq ut_{t,i}qt_i^{UP} \tag{5}$$

Equation (5) represents box constraints associated with the reactive power of each thermal unit. As for active power, binary variables to represent the possible states of operation should be introduced.

$$\begin{gathered} ut_{t,i} - ut_{t-1,i} = st_{t,i} - et_{t,i} \\ st_{t,i} + et_{t,i} \leq 1 \end{gathered} \tag{6}$$

In order to determine when a unit is powered-on or powered-off, in (6) a binary $st_{t,i}$ and a continuous $et_{t,i}$ variable (between 0 and 1) are defined. Only at this time, they take the value 1 if it corresponds, for any other condition these value is 0. More precisely, $st_{t,i}$ takes the value 1 if the unit i is turned on on period t (0 for other cases). On the other hand, $et_{t,i}$ takes the value 1 if the unit i is turned off on period t. These variables were introduced not only to consider the starting cost of a thermal unit but also to model minimum on and off time of each unit.

$$ut_{t,i} + ut_{t-1,i} + ... + ut_{t+on_i^{LOW}-1,i} \geq st_{t,i}on_i^{LOW} \tag{7}$$

$$(1 - ut_{t,i}) + (1 - ut_{t-1,i}) + ... + (1 - ut_{t+off_i^{LOW}-1,i}) \geq et_{t,i}off_i^{LOW} \tag{8}$$

Constraints modeling minimum on and off time of each unit are shown in (7) and (8).

$$-\Delta PT_i^{UP} \leq (pt_{t-1,i} - pt_{t,i}) \geq \Delta PT_i^{UP} \tag{9}$$

Equation (9) defines ramping constraints for thermal units.

$$\Delta T \sum_t f(pt_{t,i}) \leq \vartheta^{UP} \tag{10}$$

The maximum amount of fuel available for a thermal unit during planning horizon is considered in (10). In some papers this constraint groups a set of units within a plant.

3.2. Constraints associated with hydro power units only

$$uh_{t,i}ph_j^{LOW} \leq ph_{t,j} \leq uh_{t,i}ph_j^{UP} \tag{11}$$

$$uh_{t,i}qh_j^{LOW} \leq qh_{t,j} \leq uh_{t,i}qh_j^{UP} \tag{12}$$

Equations (11) and (12) represent minimum and maximum active and reactive power output of hydraulic unit generation. In order to avoid losing generality, discontinuities in hydraulic units are also considered.

$$ph_{t,j} = q_{t,j}^T \beta_j \tag{13}$$

Equation (13) represents the linear relationship between water flow across turbine and the power generated by each hydraulic unit. There are several approaches to model this relationship. In those applied to systems mainly served by hydropower, such as Brazil, great importance is given to the accuracy of this relationship (Diniz y Maceira, 2008). The expression considered in this work is the one used in most of the referenced works where the power generation units are mostly thermal. The chosen relationship allows to solve the optimization problem using decomposition techniques and to have enough known parameters. In other works, because of the linear nature of the production function for the case of plants with a great fall, the flow variable is eliminated leaving everything in terms of power generated. However, it is preferred to explicitly maintain the variable representing the flow; sometimes these variables are eliminated to make the problem more compact.

3.3. Constraints associated with both types of generation

$$\sum_i (ut_{t,i}pt_{t,i}^{UP} - pt_{t,i}) + \sum_j (uh_{t,j}ph_{t,j}^{UP} - ph_{t,j}) \geq \zeta_t \tag{14}$$

Constraint (14) represents the spinning reserve of the whole system for each period.

Nodal balance of active power for each period is defined in equation (15), where $P_{t,bb'}$ (16) represents the real part of power flow (active) presented in the line between bus b and b'. The set $cb(b)$ on which the sum is applied, corresponds to the buses directly connected to the bus b.

$$\sum_{ib \in ct(b)} pt_{t,ib} + \sum_{jb \in ch(b)} ph_{t,jb} + \Psi p_{t,b} = \sum_{b' \in cb(b)} P_{t,bb'} \tag{15}$$

$$P_{t,bb'} = v_{t,b}v_{t,b'}(G_{bb'}\cos(\theta_{t,b} - \theta_{t,b'}) + B_{bb'}\sin(\theta_{t,b} - \theta_{t,b'})) \tag{16}$$

Equation (17) defines the nodal balance of reactive power for each period. $Q_{t,bb'}$ (18) represents the complex part of power flow (reactive) presented in the line between bus b and b'

$$\sum_{ib \in ct(b)} qt_{t,ib} + \sum_{jb \in ch(b)} qh_{t,jb} + \Psi q_{t,b} = \sum_{b' \in cb(b)} Q_{t,bb'} \tag{17}$$

$$Q_{t,bb'} = \\ v_{t,b}v_{t,b'}(G_{bb'}\sin(\theta_{t,b}-\theta_{t,b'}) - B_{bb'}\cos(\theta_{t,b}-\theta_{t,b'})) \tag{18}$$

A deeper explanation about how equations (15-18) are obtained goes beyond the scope of this work and is presented in classical books such as (Wood y Wollenberg, 1984) and (Grainger y Stevenson, 1994).

3.4. Hydraulic related constraints

$$a_{t+1,r} = a_{t,r} + \Delta T(q^I_{t,r} - q^T_{t,r} - q^S_{t,r}) \tag{19}$$

Reservoir water balance is represented in (19). Although only one unit per reservoir is considered, it should be easily extended to several units for the same reservoir. Equation (20) represents box constraints to reservoir water volume.

$$a^{LOW}_r \leq a_{t,r} \leq a^{UP}_r \tag{20}$$

3.5. Network related constraints

$$-\Omega^{UP}_{bb'} \leq v_{t,b}v_{t,b'}[G_{bb'}\cos(\theta_{t,b}-\theta_{t,b'}) - B_{bb'}\sin(\theta_{t,b}-\theta_{t,b'})] - G_{bb'}v^2_{t,b} \leq \Omega^{UP}_{bb'} \tag{21}$$

$$v^{LOW}_b \leq v_{t,b} \leq v^{UP}_b \tag{22}$$

Constraints associated with transmission lines and transformers capacity are defined in (21), while allowed voltage levels for each bus are considered in (22).

3.6. Maintenance of system components

In order to address constraints associated with system elements which are temporarily out of service, or conversely, whose operation is forced for some other reason, the above constraints should be modified. For instance, the availability of thermal or hydraulic units for a given period can be previously defined forcing binaries variables $ut_{t,i}$ or $uh_{t,j}$.

4. RESOLUTION METHOD

To simplify the resolution of the problem avoiding falling into infeasible solutions, penalties for being unable to provide active or reactive power to the system are include into the problem formulation. They are represented by variables $\epsilon p^-_{t,b}$, $\epsilon p^+_{t,b}$, $\epsilon q^-_{t,b}$ and $\epsilon q^+_{t,b}$. They allows closing the nodal balance (active and/or reactive) for any condition, preventing the occurrence of infeasibility in the optimization problem. If these variables are different from zero at the final solution then the proposed generating schedule cannot satisfy the active and/or reactive power demand in any bus. Following these considerations, equations (15) and (17) are redefined as (23) and (24) respectively.

$$\sum_{ib\in ct(b)} pt_{t,ib} + \sum_{jb\in ch(b)} ph_{t,jb} + \epsilon p^-_{t,b} - \epsilon p^+_{t,b} - \Psi p_{t,b} = P_{t,bb'} \tag{23}$$

$$\sum_{ib\in ct(b)} qt_{t,ib} + \sum_{jb\in ch(b)} qh_{t,jb} + \epsilon q^{-}_{t,b} - \epsilon q^{+}_{t,b} - \Psi q_{t,b} = Q_{t,bb'} \tag{24}$$

And the objetive function (1) including deficits and excesses penalizations is redefined as follows:

$$fo = \sum_t \sum_i A_i pt^2_{t,i} + B_i pt_{t,i} + C_i ut_{t,i} + D_i st_{t,i} \tag{25}$$
$$\sum_t \sum_b Ep^{-} \epsilon p^{-}_{t,b} \quad + Ep^{+} \epsilon p^{+}_{t,b} + Eq^{-} \epsilon q^{-}_{t,b} + Eq^{+} \epsilon q^{+}_{t,b}$$

When applied to real cases the scale of the resulting problem formulation is usually large. Therefore, many authors have considered decomposition methods (Baptistella y Geromel, 1980; Pereira y Pinto, 1983; Habibollahzadeh y Bubenko, 1986; Carneiro et al., 1990; Conejo y Medina, 1994; Bai y Shahidehpour, 1996; Demartini et al., 1997; Enamorado et al., 2000; Alguacil y Conejo, 2000; Finardi y da Silva, 2006; Sifuentes y Vargas, 2007; Norbiato dos Santos y Diniz, 2009; Takigawa et al., 2010; Rubiales et al., 2012).

4.1. Benders method

The Benders method presented in (Benders, 1962) showed its utility in many applications, specially for large scale problems with variables that when are fixed the problem became a linear programming problem. In 1972 A. M. Geoffrion (Geoffrion, 1972) generalized the approach allowing for subproblems not necessarily linear.

As any variable partitioning method, it applies when the problem can be formulated in the form

$$\begin{array}{l} \text{mín } f_1(x) + f_2(y) \\ x \in X, \; y \in Y, \\ g(x,y) \leq 0. \end{array} \tag{26}$$

and fixing the value of x the resulting problem is an easier solved problem.

Calling $\varphi(x)$ the optimal value of the subproblem

$$\varphi(x) = \begin{array}{l} \text{mín } f_2(y) \\ y \in Y, \\ g(x,y) \leq 0, \end{array} \tag{27}$$

the original problem can be written in the following form

$$\begin{array}{l} \text{mín } f_1(x) + \varphi(x) \\ x \in X, \end{array} \tag{28}$$

where we have considered that $\varphi(x) = +\infty$ in the case that there is no $y \in Y$ such that $g(x,y) \leq 0$.

In most of the practical cases, the optimal value function φ is convex and it is easy to compute one subgradient using the Lagrange multipliers associated to the constraints in (27). In those cases it is possible to approximate the function φ by a cutting plane model

$$\varphi_k(x) = \sup\{f_i + \xi_i^T (x - x_i), \; i = 1, \ldots, k\} \tag{29}$$

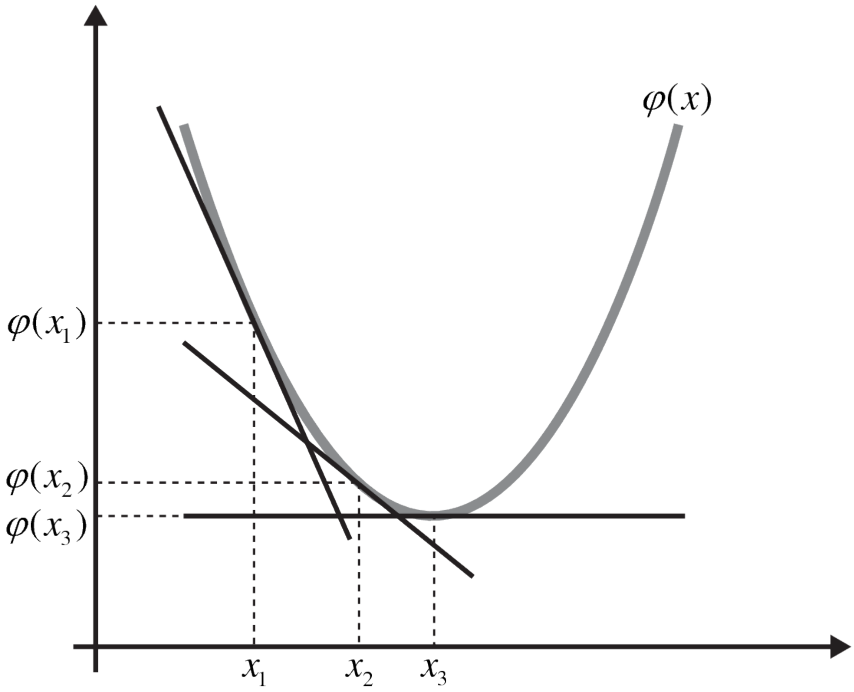

Figura 1: Evolution of the cutting planes algorithm

where f_i and ξ_i are the function value and a subgradients of φ for some points x_i in X.

In fact, it can be shown that under some hypothesis the function φ is equivalent to

$$\varphi(x) = \sup\{\varphi(y) + \xi^T(x - y), y \in X,\ \xi \in \partial\varphi(y)\}.$$

Now, the definition in (29) can be seen as a finite evaluation approximation of this formula, made upon cutting planes that in the convex case are support planes, see figure 1 for a graphical example. The interest of this formulation is that subgradients of φ are automatically obtained at each evaluation of φ through formula (27).

It may happen that the subproblem for some value of x is infeasible and there are no multipliers to build a cutting plane. In those cases it is possible to include feasibility cuts, as it is shown in (Bonnans et al., 2006). However, in this work feasibility cuts will not be necessary because subproblems are always feasible.

The Benders method starts with an initial solution $x_0 \in X$ and obtains the actual value $\varphi(x_0)$ solving the subproblem (27). After computing the first $k-1$ iterations there is a point x_{k-1} and a cutting plane approximation φ_{k-1}. Then it solves the master problem

$$\begin{aligned} &\text{mín}\, f_1(x) + \varphi_{k-1}(x) \\ &x \in X, \end{aligned} \tag{30}$$

that, considering the formula (29), can be reformulated as

$$\begin{aligned} &\text{mín}\, f_1(x) + z \\ &x \in X, \\ &z \geq f_i + \xi_i^T(x - x_i),\ i = 1, \ldots, k. \end{aligned} \tag{31}$$

Calling x_k the solution to the master problem, the algorithm keeps iterating until the gap between upper and lower bounds is small enough. The lower bound is given by the solution of the master problem and the upper bound is given by the solution to the subproblem.

4.2. Bundle Method

Benders method can be seen as a cutting planes method and, as it is shown in the literature (Bonnans y Lemaréchal, 2006), algorithms based on cutting planes can present instabilities and bad numerical behavior which is observed as a very poor convergence rate. To cope with those problems Lemaréchal in (Lemarechal, 1978; Lemaréchal et al., 1995) introduced the *Bundle method* which can see as a stabilized cutting planes method. The stabilizing property of that method is explained in the next section and inspires the algorithm presented in this work.

Bundle methods were designed to accelerate the convergence of the cutting planes method. In order to avoid oscillations it is crucial that the algorithm at each step keeps the best obtained solution. With this extra information, two sequences of points are generated:

- the sequence of points y^k, called *candidates,* over which the cutting plane model $\hat{f}_k$ is made up.
- the sequence of points x^k, called *stability centers,* where the objective function sufficiently decreased.

In the case presented here (taken from (Bonnans y Lemaréchal, 2006)), the stabilization is obtained thanks to a quadratic penalization introduced in the objective function of the subproblem. This penalization can be seen as a mechanism to not letting the next candidate point going far away from the last stability center.

The general bundle algorithm can be resumed as:

Step 0: $k = 0$, $\delta_0 = \infty$, $tol = \epsilon$, $m \in (0, 1)$, for a given x_0 compute $f(x_0)$ and $\xi_0 \in \partial f(x_0)$.

Step 1: If $\delta_k \leq tol$ END.

Step 2: Solve the stabilized optimization problem

$$\min_{y} \varphi_k(y) + \frac{1}{2}\tau^k \|y - x^k\|^2 \tag{32}$$

obtaining y^{k+1} and the subgradient ξ^{k+1}.

Step 3: Compute

$$\delta_{k+1} = f(x^k) - \varphi_k(y^{k+1}) - \frac{1}{2}\tau^k \|y - x^k\|^2.$$

Step 4: Test

$$f(x^k) - f(y^{k+1}) \geq m\delta_{k+1}. \tag{33}$$

True: Accept candidate as new stability center $x^{k+1} = y^{k+1}$.

False: $x^{k+1} = x^k$.

Step 5: Improve cutting plane model with y^{k+1} and ξ^{k+1}. Set k to k+1 and go to Step 1.

The quadratic term in the objective function of problem (32) together with the penalization parameter τ^k focus the search around the last good obtained solution (the current stability center). The parameter can be updated at each iteration to constraint the search around the stability center. There is no an optimal accepted rule to do it. In (Bonnans et al., 2006) the authors study the so called reversal form for the update given by

$$\frac{1}{\tau^{k+1}} = \frac{1}{\tau^k} + \frac{\left\langle x^{k+1} - x^k, \xi^{k+1} - \xi^k \right\rangle}{\|\xi^{k+1} - \xi^k\|^2}, \tag{34}$$

and also mention on other works which study different rules, some of them based on variable metrics. The use of variable metrics allows to obtain in some difficult cases, superlinear convergence (see for example Parente et al. (2011)). For the numerical tests, and after many experiments in this work we propose the rule

$$\tau^k = \frac{\alpha \sum_k f(y^k)^{\beta+1}}{\left(f(y^k) - \hat{f}(y^k)\right)^\beta \sum_k \|x^k\|^2}, \tag{35}$$

where α and β are parameters calibrated to improve the convergence rate. For the parameter m in the decrease test, many tests were made to adjust it to a good value.

As it is noted in (Lemarechal y Sagastizabal, 1997), the principal advantages of this method are:

- converges toward an optimal point,
- higher robustness,
- better stability properties,
- possibility of reducing the used memory without compromising the convergence rate.

Bundle ideas can be applied to Benders method to improve its stability. To do so, the Bundle methodology is applied to the problem (31) for the minimization of the function (nondiferentiable in general) $f_1(x) + \varphi(x)$.

4.3. Decomposed problem

The optimization problem of minimizing the function (1) constrained to (4-22) is written under the following form

$$\min_{y_m} f_m(y_m) + \varphi(y_m), \tag{36}$$

where y_m and f_m represents the variables and the objective function of the master problem respectively. The function φ represents the subproblem, the variable of the subproblem is called y_{sp} and $f_{sp}(y_m, y_{sp})$ is the objective function of the subproblem. Now, the constraints must be included in each one of the problems. Different choices will correspond to a different behavior of the Benders algorithm. In this case all the binary variables and the active power variables are considered for the master problem letting all the others variables belong to the subproblem. We have then

$$y_m = (ut_{t,i}, pt_{t,i}, uh_{t,j}, ph_{t,j}, st_{t,i}, et_{t,i}). \tag{37}$$

For the master-problem objective function, we considered the start-up costs of the thermal units together with the quadratic terms of the thermal generation costs and those related to the environmental concerns. Thus, we obtain

$$f_m(y_m) = \sum_t \sum_i P_{t,i}(pt_{t,i}, ut_{t,i}, st_{t,i}) + \sum_q \sum_t \sum_i w_q E_{q,t,i}(pt_{t,i}, ut_{t,i}) \tag{38}$$

The constraints considered for the master problems are all those that not contain the variables of the subproblem: (4), (6-11), (13), (14), (19), (20).

Now the subproblem objective function becomes

$$f_{sp} = \sum_t \sum_b Ep^- \epsilon p^-_{t,b} + Ep^+ \epsilon p^+_{t,b} + + Eq^- \epsilon q^-_{t,b} + Eq^+ \epsilon q^+_{t,b}. \tag{39}$$

Together with the variables that appear in this function, the subproblem has also the reactive power variables $qt_{t,i}$, $qh_{t,j}$ the corresponding slack variables ($\epsilon p^-_{t,b}$, $\epsilon p^+_{t,b}$, $\epsilon q^-_{t,b}$, $\epsilon q^+_{t,b}$), the angles $\theta_{t,b}$ and the voltages $v_{t,b}$. The remaining constraints (5), (12), (15), (17), (21), (22) are considered in the subproblem with the values of master variables fixed by the master problem.

With the proposed decomposition the resulting master problem is numerically more complex than the subproblem. Indeed, it is a quadratic mixed integer problem. The subproblem has a linear objective function and non linear constrains, but the non linearity of the constraints is non harmful in practice and the solvers used can deal well with them.

It is worth mentioning also that the subproblem becomes temporally uncoupled obtaining several optimal flow problems where the values of the start-up variables $ut_{t,i}$, $uh_{t,j}$ and the active power generation $pt_{t,i}$, $ph_{t,j}$ are given by the master problem solution at each iteration.

In order to simplify the introduction of cutting plane equations some dummy equations were added:

$$\begin{array}{rcl} pt_{t,i} = pt^k_{t,i} & : & \mu^k_{t,i}, \\ ph_{t,j} = ph^k_{t,j} & : & \lambda^k_{t,j}, \\ ut_{t,i} = ut^k_{t,i} & : & \pi^k_{t,i}, \\ uh_{t,j} = uh^k_{t,j} & : & \psi^k_{t,j}, \end{array} \tag{40}$$

where $uh_{t,j}$, $ph_{t,j}$, $ut_{t,i}$ y $pt_{t,i}$ are now subproblem variables, $uh^k_{t,j}$, $ph^k_{t,j}$, $ut^k_{t,i}$ y $pt^k_{t,i}$ are given by the master problem at the k iteration and $\mu^k_{t,i}$, $\lambda^k_{t,j}$, $\psi^k_{t,j}$, $\pi^k_{t,i}$ are the corresponding multipliers.

The addition of cutting planes to the master problem is made in the same way that in (30) and (31). The master objective function has the term $\sum_t z_t$ which corresponds with $\varphi(y^k_m)$, and the cutting planes are:

$$\begin{aligned} z_t \geq {z_t}^k &+ \sum_j \lambda^k_{t,j}(uh_{t,j} - uh^k_{t,j}) + \\ &+ \sum_j \psi^k_{t,j}(ph_{t,j} - ph^k_{t,j}) + \\ &+ \sum_i \mu^k_{t,i}(pt_{t,i} - pt^k_{t,i}) + \\ &+ \sum_i \pi^k_{t,i}(ut_{t,i} - ut^k_{t,i}). \end{aligned} \tag{41}$$

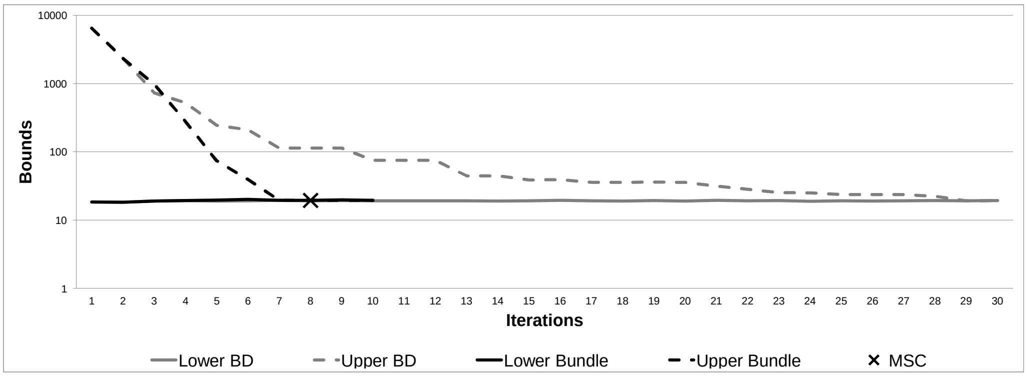

Figura 2: Convergence comparison - 24 buses test case

5. RESULTS

In Rubiales et al. (2012) the STHTC problem stated previously was solved using a Generalized Benders Decomposition approach. The results presented in this work make emphasis in analyzing the strategy proposed by the algorithm and in evaluating the correctness of the approach comparing it with Powerworld Simulator results. The present work improves the performance of the GDB approach including a stabilization mechanism based on Bundle method. This section compare the performance obtained by both techniques. The previously detailed algorithm (section 4) is applied to the IEEE 24-buses test system and to a larger real power system.

The characteristics of the electrical systems used in tests are described below:

5.1. IEEE 24-bus system

In this case, the system consists of 26 units of which, 18 are supposed to be thermal and 8 hydraulic. Units characteristics, power line characteristics and Demand profiles are obtained from (Wang et al., 1995a).

In figure 2 a comparison between algorithms convergence is presented. Gray lines show upper and lower bounds values of Generalized Benders Decomposition (BD) algorithm presented in (Sifuentes y Vargas, 2007) and (Rubiales et al., 2012). These are ones of the first approaches which consider AC power flows network constraints and serve as a starting point for the present work. Black lines show upper and lower bounds values of the novel decomposition approach presented in this work, which combines Generalized Benders Decomposition with Bundle methods. As it was shown in figure 2, the later methodology considerably decreases the *tailing-off effect* which is the main drawback of the former approach. Furthermore, deficits or excesses of active and reactive power are even lower than 10^{-3} MW, which are negligible in electric power systems. Remarkably, over several runs, these values make the method converge slowly. Because they are heavily penalized (in order to avoid network misconfigurations) small changes in these values greatly influence the objective function. These changes do not impact in the power generated in both, hydro and thermal power units, and even in the configuration of the network. Therefore, one possible implementation of the stopping criterion may consider a greater tolerance for the difference between the bounds (10^{-1}) not allowing that excess and deficit values in each bus be more than a given tolerance.

Reducing those values, or their impact in the total cost, may require much more iterations at the ending of the algorithm execution. During these iterations, neither the operation cost nor the power generated by each thermal unit, present significant changes. These facts require the

Tabla 1: IEEE 24-buses Problem Resolution Characteristics

	Time(s)	Iterations	Number of Equations of the Problem		
			Master (Initial)	Master (Final)	Subproblem
Benders	303	34	2651	3443	24*273
Bundle	125	13	2651	2939	24*273
MSC	69	8	2651	2819	24*273

use of a new convergence criterion based not only on the difference between bound levels, but also on the deficit or excess of active and reactive power in each bus. This new criterion makes a significant reduction of the number of iterations. However, the power generated in each unit does not present significant changes. The previously mentioned fact is valid as long as the non-zero values of the slack variables are not due to the inability of the system to meet the required demand. Remarkably, when such cases happen the magnitudes of these values are larger and do not fluctuate along iterations, moreover, they are stabilized in the values of power deficit that effectively present the power dispatch. This technique is called Modified Stopping Criterion (MSC) and results using it are also represented in figure 2 with a black cross.

Numerical experiments were performed on a virtual machine with 1 GB RAM running on a PC AMD Athlon 2.96 GHz X3 435. The GAMS version used is 23.6 and the solver used for solving the problem are CPLEX and CONOPT. The resolution time, number of equations and iterations of different approaches are presented in 1. Both were executed with a relative tolerance of 10^{-7}.

The size of the solved master problem is 985 variables (and 336 discrete variables). For the best case (MSC), the number of equations starts at 2651 growing to 2819 at the eighth iteration. This size growth corresponds to the cuts added at each iteration. One cut is added at each time. The subproblem is decomposed in 24 problems (each one corresponds to a given time interval), with 174 variables and 273 equations each one.

5.2. Mid-size real power system

The proposed algorithm was also applied to a section of the Argentinean National Interconnected System which is operated by Transcomahue. This network is located in the Upper Valley zone and includes the provinces of Neuquen and Rio Negro. The extension of this power network is medium size and has thermal and hydraulic generation. The network modeled considers the 132 kV voltage level areas and lower voltage buses and lines that reach the generators. The system demands are considered as bus connections which consume power at 132 kV.

This system has 87 buses, 23 thermal and 6 hydraulic units. The one-line diagram is presented in Figure 3.

Table 2 shows the maximum and minimum active and reactive power characteristics of each thermal unit. Table 3 represents thermal units quadratic operational costs (A, B and C), startup costs and environmental coefficients (X, Y and Z). This table also shows the minimum on and off time of each unit. Note that the cost data are fictitious because they were not provided by the system operator.

Hydraulic units characteristics are shown in table 4. As cost coefficients, data from the reservoirs were not provided; consequently, fictitious values are considered and shown in Table 4. For the sake of simplicity, line characteristics, and demand profile in each bus are not shown.

As in previous case, the problem is solved using a virtual machine of 1GB RAM running on

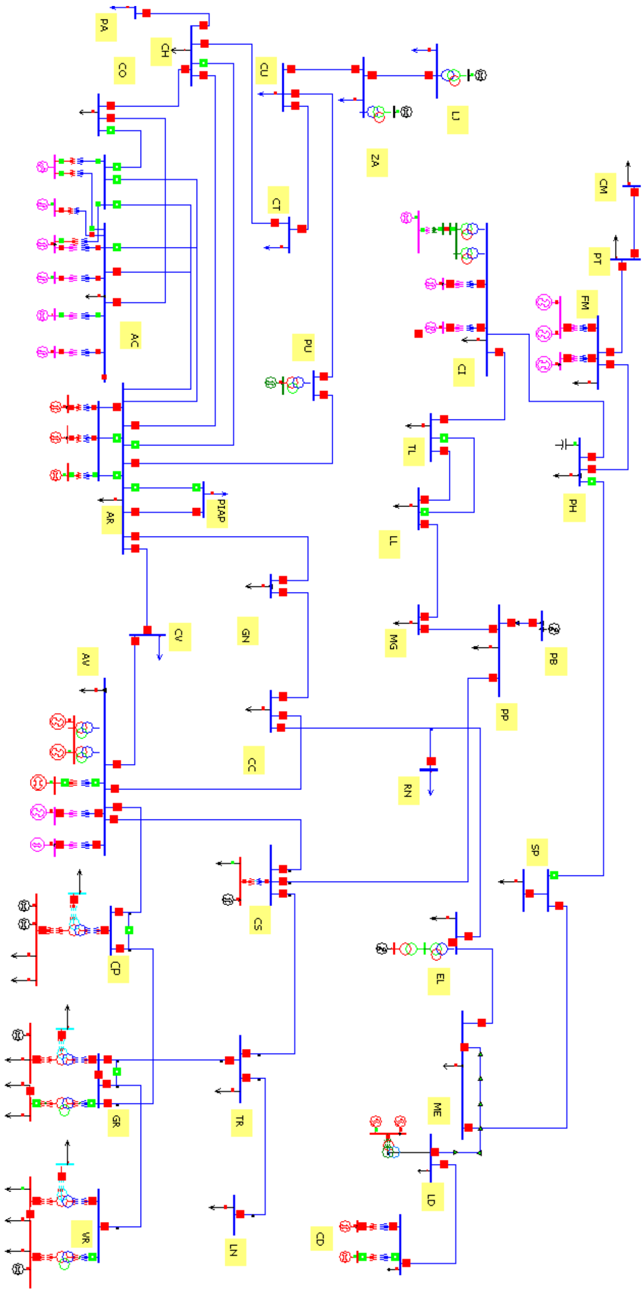

Figura 3: Mid-size real power network

Tabla 2: Thermal Units Power Characteristics

Name	Active Power		Reactive Power	
	Min [MW]	Max [MW]	Min [MW]	Max [MW]
P.BAND.	0	70	-50	50
ACAJTG06	40	130	-67.5	82.5
ACAJTG01	15	51	-13.38	19.88
ACAJTG02	15	51	-13.38	19.88
ACAJTG03	15	51	-13.38	19.88
ACAJTG04	15	51	-13.38	19.88
ACAJTG05	15	51	-13.38	19.88
AVALTV	3	30	-30	37.6
AVALTG21	0	17	-100	100
AVALTG22	5	26	0	14
AVALTG23	5	26	0	14
FILOTG	7	23.6	-4	23
CHIUTG02	5	19.4	-10.3	10.81
CHIUTG01	5	19.4	-10.3	10.81
HUINTG01	0	42.73	-8.6	30
CP_13	0	10	-5	10
GR_13A	0	5	-2.5	5
VR_13B	0	5	-2.5	5
CS_13_1	0	5	-2.5	0
RI_33	0	25	0	25
ELOM2 TG	0	18	-5	8
FILOTG3	7	23.6	-7.05	17.14
PHFICT	0	70	-50	50

Tabla 3: Thermal Units Cost

Name	Costs			Env. Coef.			Operation Min. Time		
	A	B	C	X	Y	Z	Startup	On	Off
P.BAND.	0.11	5	150	0.54	13	320	500	6	6
ACAJTG06	0.13	5.5	160	0.52	12.5	310	500	6	6
ACAJTG01	0.15	6	170	0.5	12	300	500	6	6
ACAJTG02	0.17	6.5	180	0.48	11.5	290	500	6	6
ACAJTG03	0.19	7	190	0.46	11	280	500	6	6
ACAJTG04	0.21	7.5	200	0.44	10.5	270	500	6	6
ACAJTG05	0.23	8	210	0.42	10	260	500	6	6
AVALTV	0.25	8.5	220	0.4	9.5	250	500	6	6
AVALTG21	0.27	9	230	0.38	9	240	500	6	6
AVALTG22	0.29	9.5	240	0.36	8.5	230	500	6	6
AVALTG23	0.31	10	250	0.34	8	220	500	6	6
FILOTG	0.35	11	270	0.3	7	200	500	6	6
CHIUTG02	0.37	11.5	280	0.28	6.5	190	500	6	6
CHIUTG01	0.39	12	290	0.26	6	180	500	6	6
HUINTG01	0.41	12.5	300	0.24	5.5	170	500	6	6
CP_13	0.45	13.5	320	0.2	4.5	150	500	6	6
GR_13A	0.47	14	330	0.18	4	140	500	6	6
VR_13B	0.49	14.5	340	0.16	3.5	130	500	6	6
CS_13_1	0.51	15	350	0.14	3	120	500	6	6
RI_33	0.53	15.5	360	0.12	2.5	110	500	6	6
ELOM2 TG	0.43	13	310	0.22	5	160	500	6	6
FILOTG3	0.33	10.5	260	0.32	7.5	210	500	6	6
PHFICT	0.11	5	150	0.1	2	100	500	6	6

Tabla 4: Hydraulic Units Power Characteristics

Name	Active Power		Reactive Power	
	Min [MW]	Max [MW]	Min [MW]	Max [MW]
DIVIHI	0	5	-1.34	4.5
ARROHI01	0	42.5	-29.03	26.51
ARROHI02	0	42.5	-29.03	26.51
ARROHI03	0	42.5	-29.03	26.51
CDPIHI01	0	30	-25.4	20
CDPIHI02	0	30	-35	24.23

Tabla 5: Mid-size Problem Resolution Characteristics

	Time(s)	Iterations	Number of Equations of the Problem		
			Master (Initial)	Master (Final)	Subproblem
Benders	2812	49	7789	8941	24*815
Bundle	468	11	7789	8029	24*815
MSC	401	10	7789	8005	24*815

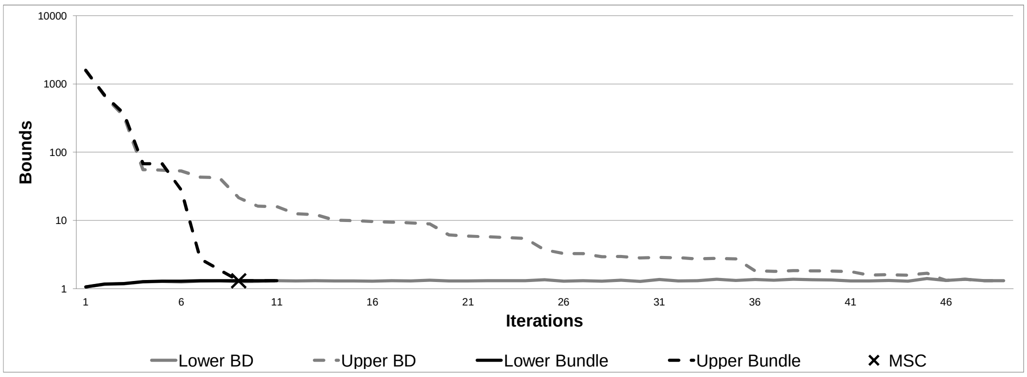

Figura 4: Convergence comparison - Medium size real case

a PC AMD Athlon 2.96 GHz X3 435. The GAMS version used is 23.6 and the solver used for solving the problem are CPLEX and CONOPT. The resolution time, number of equations and iterations of different approaches are presented in Table 5. Both were executed with a relative tolerance of 10^{-3}.

The size of the solved master problem is 2665 variables (and 1104 discrete variables). For the worst case (MSC), the number of equations starts at 7789 growing to 8005 on iteration number 10. This size growth corresponds to the cuts added in each iteration. One cut is added for each time. The subproblem is decomposed in 24 problems (each one corresponds to a given time interval), with 601 variables and 815 equations each one. A significant reduction in time resolution was obtained. Comparing the latter methodology with the one presented in (Sifuentes y Vargas, 2007) the reduction achieved was from 2812 to 401 seconds.

Figure 4 shows comparison between algorithms convergence. As depicted in section 5.1 gray lines show upper and lower bounds values of Generalized Benders Decomposition (BD) algorithm and black lines show upper and lower bounds values of the novel decomposition approach presented in this work (Bundle). The performance obtained considering the Modified Stopping Criterion (MSC) is represented with a black cross.

Figure 5 shows the evolution of stabilization coefficient values. Considering that in this case the minimum level of hydroelectric power is 0, neither variable $uh_{t,j}$ is considered nor its associated to a stabilization coefficient.

These coefficients are defined to penalize the distance to the last found stability center. Its starting values are close to zero and, as the iterations run, these values are exponentially increased. The effect of this behavior in the algorithm is to further penalize the distance whenever the solution is better.

Penalization variables used in the subproblem in conjunction with feasibility cuts, serve to send signals to the master problem, in such a way that solutions generated consider the constraints associated with the power network. Although the magnitude of these values must be

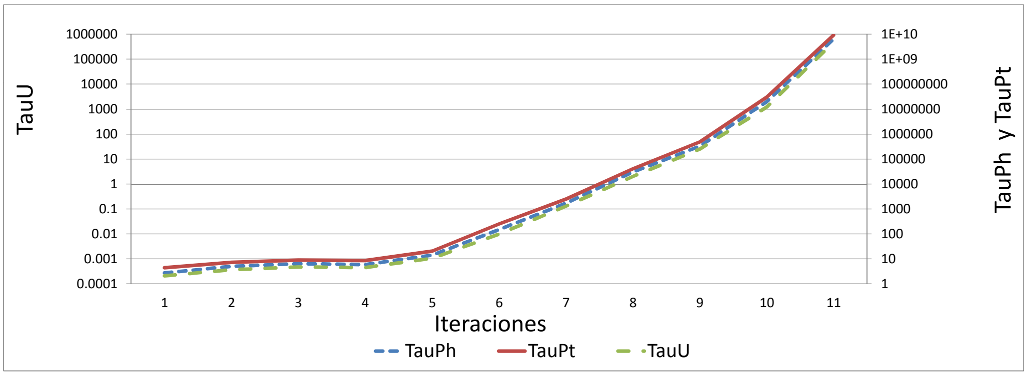

Figura 5: Stabilization coefficients evolution

zero at the optimal solution, for numerical reasons, it is difficult to reach this value. From a practical standpoint, these values are considered residues and their presence has no effect on the final dispatch. A greater error source in the final dispatch is the demand forecast which easily could be between 2 % and 3 % of the total demand value.

6. CONCLUSION

In this work, a detailed version of the STHTC considering environmental concerns was mathematically formulated. The resolution of this problem defines the unit-commitment and economical dispatch of thermal and hydraulic unit avoiding post-dispatch corrections. This formulation includes an AC modeling of transmission network, thus the optimization problem to solve is integer-mixed non-convex, nonlinear and high dimensioned.

The integration of the hydrothermal coordination problem with the resolution of an AC optimal power flow for each period on a single problem avoids the startup of units which have not been scheduled. This last fact would happen if problems are considered separately or with a DC modeling of the transmission network, which may lead to a non-optimal operation state of the power system.

The approach applied in this work consists in splitting the original problem into a quadratic master problem with mixed integer binary variables and a nonlinear subproblem with continuous variables. The former defines the state and the active power dispatched by each unit, whereas the latter determines the reactive power to meet the electrical constraints through a modified AC optimal power flow. The mechanism applied to separate it is the most appropriated one for the type of systems in which the methodology was tested. For the resolution of the optimization problem a novel approach is developed. It combines Generalized Benders Decomposition with Bundle methods presented in (Lemarechal y Sagastizabal, 1997). This proposed method resembles a stabilized version of the cutting planes method, which drastically reduces the well-known tailing-off effect that Benders methods have.

These approaches were applied to the IEEE 24-bus test case and to an 87-bus real system comparing resolution performance with the methodology presented in (Sifuentes y Vargas, 2007), which is one of the latest advances in this field. As it was mentioned in section 5 a remarkably decrease in resolution time was achieved without loosing solution quality.

Regarding future works, a different combination of solving strategies in order to modify the problem to be solved according to the progress of the iterations should be applied. For example, in the first iterations a relaxed version of the master problem can be solved only considering DC constraints in the subproblem, in order to improve resolution time, incorporating the resolution of the original master problem and subproblem only in the last iterations. Further improving the performance of the algorithm requires the application of the Bundle compression technique presented in (Bonnans et al., 2006). Considering the fact that subproblems can be resolved uncoupled, also parallel processing should be allowed to reduce total resolution time.

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