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FINITE VOLUME SIMULATION OF THE COMPRESSIBLE ORSZAG–TANG MGD PROBLEM

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Abstract. Computational magnetogasdynamics is an important tool to develop interdisciplinary technologies as aerospace design and for astrophysical studies. A model of a flow affected by electromagnetic forces must include the full set of Maxwell's equations coupled with the Navier-Stokes equations (full MGD). However, in some phenomena the idea lmagnetogasdynamics equations (ideal MGD) are an accurate description. The ideal MGD equations are simplest than the full MGD equations. The ideal MGD numerical simulations allow the reduction of expensive, and sometimes unviable, experimental parametric studies. However numerical simulations are limited by the requirement of solving accurately the hyperbolic non-linear differential equations. In addition, the ideal MGD equations are nonconvex and, as consequence, the wave structure is more complex than the Euler gasdynamics equations. In this work are presented results of the compressible, two-dimensional, time-dependent transient Orszag-Tang MGD problem. The results were obtained using a modification of the original finitevolume Harten-Yee TVD scheme, incorporating a new sonic fix for the acoustic causality points.

1 INTRODUCTION

Magnetogasdynamics (MGD) flows have applications in aerospace technologies, astrophysics, geophysics, interstellar gas masses dynamics, etc. A MGD model is based on the assumption of the plasma continuum hypothesis and thus relatively few macroscopic quantities are required to characterize the state of the system. A revision about the physical models used in aerospace applications is given in (D'Ambrosio and Giordano, 2004). The ideal MGD equations constitute a hyperbolic partial differential system. This system presents non-convex singularities and the wave structure is more complicated than for the Euler equations (Kantrowitz and Petschek, 1966). The nonlinear coupling of these waves plays an important role in determining physical phenomena and in the numerical solution (Leveque *et al.*, 1998).

In ideal MGD the numerical simulations are a very important tool, by reducing expensive, and sometimes unviable, experimental parametric studies. However, the numerical simulations always are limited by the ability to analyze and to solve accurately the hyperbolic non-linear differential equations system. To solve the ideal MGD equations system is convenient to use a conservative form because it allows obtaining the correct jump conditions of discontinuities and shocks (Leveque, 1992; Toro, 2009). The utilization of the numerical conservative scheme is desirable because ensures that mass, momentum, and energy are conserved. Several schemes has been proposed and implemented to solve the ideal MGD equations (Balbas *et al.*, 2004; Myong and Roe, 1998; Udrea, 1999). In this work, a modification of the Harten-Yee TVD technique is used (Yee *et al.*, 1985; Maglione *et al*., 2011). The Harten-Yee TVD scheme has proven to be accurate and reliable for the simulation of supersonic flows of gases (Yee, 1989; Elaskar, *et al.*, 2000; Falcinelli, *et al,* 2008). We implement this technique, with a modification that allows to numerically solve the ideal MGD flows.

Among the difficulties to reach accurate numerical solutions we have the problem of the acoustic causality points. A new wave structure is produced by the non-linear wave interaction (Courant and Friedrich, 1999). In ideal MGD there are sonic points and points where non-convexity appears, these points are called points of acoustic causality (Serna, 2009) and it is necessary to implement an entropy corrector scheme, introducing the necessary artificial viscosity.

The main objective of this work is to prove the new sonic fix capacity to solve the Orszag–Tang vortex problem. Another important objective was to test the code in solving problems with periodic boundary conditions. The proposed sonic fix had been successful implemented: for the 2D Riemann magnetogasdynamics problem proposed by Brio and Wu, and for the 2.5D Tóth magnetogasdynamics flow, (Maglione and Elaskar, 2010; Maglione *et al*., 2011). Also the numerical code was used in astrophysical applications (Maglione *et al*., 2011).

The numerical approach uses an approximate Riemann solver with a high resolution TVD technique. The eight-wave technique introduced by Powell (Powell, 1995) and the eigenvectors are normalized according to Zachary, *et al.* (1994) and Roe (1996) are implemented.

2 MAGNETOGASDYNAMICS EQUATIONS

The equations of non-dimensional transient real MGD in conservative form are given by (Goldston and Rutherford, 2003; D'Ambrosio and Giordano, 2004).

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho\boldsymbol{u}\\ \boldsymbol{B}\\ e\end{bmatrix}+\nabla\cdot\begin{bmatrix}\rho\boldsymbol{u}\\ \rho\boldsymbol{uu}-\boldsymbol{BB}+\boldsymbol{I}\left(p+\frac{1}{2}B^2\right)\\ \boldsymbol{uB}-\boldsymbol{Bu}\\ \left(e+p+\frac{B^2}{2}\right)\boldsymbol{u}-\left(\boldsymbol{B}\cdot\boldsymbol{u}\right)\boldsymbol{B}\end{bmatrix}=\nabla\cdot\begin{bmatrix}0\\ \frac{\boldsymbol{\tau}}{R_eA_l}\\ \frac{\boldsymbol{E}_r}{L_uA_l}\\ \frac{\boldsymbol{u}\cdot\tau}{R_eA_l}-\frac{\left[\boldsymbol{\eta}\cdot\left(\nabla\times\boldsymbol{B}\right)\right]\times\boldsymbol{B}}{L_uA_l}+\frac{\boldsymbol{k}\cdot\nabla T}{P_eA_l}\end{bmatrix} \quad (1)$$

where ρ, $\boldsymbol{u}$,e ,p, T are the density, velocity, total energy, pressure and temperature of plasma respectively. B is the magnetic field, K thermal conductivity, η electrical resistive and τ viscous stress. R_e, A_l, L_u, P_e are the Reynolds, Alfvén, Lundquist and Peclet numbers.

The ideal MGD equations accurately describe the macroscopic dynamics of perfectly conducting plasma. This system expresses conservation of mass, momentum, energy, and magnetic flux and conform a nonlinear conservative system of eight partial differential equations. The equations of non-dimensional ideal one-fluid MGD in conservative form are given by (D'Ambrosio and Giordano, 2004);

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho\boldsymbol{u}\\ \boldsymbol{B}\\ e\end{bmatrix}+\nabla\cdot\begin{bmatrix}\rho\boldsymbol{u}\\ \rho\boldsymbol{uu}-\boldsymbol{BB}+\boldsymbol{I}\left(p+\frac{1}{2}B^2\right)\\ \boldsymbol{uB}-\boldsymbol{Bu}\\ \left(e+p+\frac{1}{2}B^2\right)\boldsymbol{u}-\left(\boldsymbol{B}\cdot\boldsymbol{u}\right)\boldsymbol{B}\end{bmatrix}=\boldsymbol{0} \quad (2)$$

To close de system, is introduced perfect gas state equation, so the specific internal energy depends on temperature only. Then for the total energy results as,

$$e=\frac{p}{\gamma-1}+\rho\ \frac{\boldsymbol{u}\cdot\boldsymbol{u}}{2}+\frac{\boldsymbol{B}\cdot\boldsymbol{B}}{2} \quad (3)$$

Using a Cartesian coordinate system the Eq. (2) can be written, for two dimensions in quasi-linear form, as

$$\frac{\partial\boldsymbol{U}}{\partial t}+\left[A_c\right]\frac{\partial\boldsymbol{U}}{\partial x}+\left[B_c\right]\frac{\partial\boldsymbol{U}}{\partial y}=\boldsymbol{0} \quad (4)$$

with the state vector

$$\boldsymbol{U}=\left(\rho,\rho u_x,\rho u_y,\rho u_z,B_x,B_y,B_z,e\right)^T \quad (5)$$

where [A_c] y [B_c] are the Jacobian matrices. The evaluation of the eigenvalues and the eigenvectors is simpler using the conservative variables:

$$\boldsymbol{W}=\left(\rho,u_x,u_y,u_z,B_x,B_y,B_z,p\right)^T \quad (6)$$

To overcome the difficulties introduced by the null eigenvalue of the Jacobian matrices, the eight-wave technique introduced by Powell (1995) is used in this work. The modified Jacobian matrix [A_p] (using primitive variables) is:

$$\left[A_p\right]=\begin{bmatrix} u_x & \rho & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & u_x & 0 & 0 & 0 & \frac{B_y}{\rho} & \frac{B_z}{\rho} & \frac{1}{\rho} \\ 0 & 0 & u_x & 0 & 0 & -\frac{B_x}{\rho} & 0 & 0 \\ 0 & 0 & 0 & u_x & 0 & 0 & -\frac{B_x}{\rho} & 0 \\ 0 & 0 & 0 & 0 & u_x & 0 & 0 & 0 \\ 0 & B_y & -B_x & 0 & 0 & u_x & 0 & 0 \\ 0 & B_z & 0 & -B_x & 0 & 0 & u_x & 0 \\ 0 & \gamma p & 0 & 0 & 0 & 0 & 0 & u_x \end{bmatrix} \tag{7}$$

The eigenvectors are normalized according to Zachary *et al.* (1994) and Roe (1996). The resulting eigenvalues representing MGD waves are: “entropy wave”, “Alfvén waves”, “fast magneto-acoustic waves”, “slow magneto-acoustic waves” and “magnetic flux wave”. The expressions for these are:

-Entropy wave: $\lambda_e = u_x$

$$\boldsymbol{r}_e = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \boldsymbol{l}_e = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{c^2} \end{Bmatrix} \tag{8}$$

-Alfvén waves: $\lambda_a = u_x \pm c_a$

$$\boldsymbol{r}_{\boldsymbol{a}}^{\pm} = \frac{1}{\sqrt{2}} \begin{Bmatrix} 0 \\ 0 \\ -\beta_z \\ \beta_y \\ 0 \\ \pm\sqrt{\rho}\beta_z \\ \mp\sqrt{\rho}\beta_y \\ 0 \end{Bmatrix} \qquad \boldsymbol{l}_{\boldsymbol{a}}^{\pm} = \frac{1}{\sqrt{2}} \begin{Bmatrix} 0 \\ 0 \\ -\beta_z \\ \beta_y \\ 0 \\ \pm\frac{\beta_z}{\sqrt{\rho}} \\ \mp\frac{\beta_y}{\sqrt{\rho}} \\ 0 \end{Bmatrix} \tag{9}$$

-Fast magneto-acoustic waves: $\lambda_f = u_x \pm c_f$

$$\boldsymbol{r}_{\boldsymbol{f}}^{\pm} = \begin{Bmatrix} \rho\alpha_f \\ \pm\alpha_f c_f \\ \mp\alpha_s c_s \beta_y \,\mathrm{sgn}(B_x) \\ \mp\alpha_s c_s \beta_z \,\mathrm{sgn}(B_x) \\ 0 \\ \alpha_s\sqrt{\rho}c\beta_y \\ \alpha_s\sqrt{\rho}c\beta_z \\ \alpha_f \gamma p \end{Bmatrix} \qquad \boldsymbol{l}_{\boldsymbol{f}}^{\pm} = \begin{Bmatrix} 0 \\ \pm\dfrac{\alpha_f c_f}{2c^2} \\ \mp\dfrac{\alpha_s}{2c^2} c_s \beta_y \,\mathrm{sgn}(B_x) \\ \mp\dfrac{\alpha_s}{2c^2} c_s \beta_z \,\mathrm{sgn}(B_x) \\ 0 \\ \dfrac{\alpha_s}{2\sqrt{\rho}c}\beta_y \\ \mp\dfrac{\alpha_s}{2\sqrt{\rho}c}\beta_z \\ \dfrac{\alpha_f}{2\rho c^2} \end{Bmatrix} \tag{10}$$

-Slow magneto-acoustic waves: $\lambda_s = u_x \pm c_s$

$$\boldsymbol{r}_{\boldsymbol{s}}^{\pm} = \begin{Bmatrix} \rho\alpha_s \\ \pm\alpha_s c_s \\ \pm\alpha_f c_f \beta_y \,\mathrm{sgn}(B_x) \\ \pm\alpha_f c_f \beta_z \,\mathrm{sgn}(B_x) \\ 0 \\ -\alpha_f\sqrt{\rho}c\beta_y \\ -\alpha_f\sqrt{\rho}c\beta_z \\ \alpha_s \gamma p \end{Bmatrix} \qquad \boldsymbol{l}_{\boldsymbol{s}}^{\pm} = \begin{Bmatrix} 0 \\ \pm\dfrac{\alpha_s c_s}{2c^2} \\ \pm\dfrac{\alpha_f}{2c^2} c_f \beta_y \,\mathrm{sgn}(B_x) \\ \pm\dfrac{\alpha_f}{2c^2} c_f \beta_z \,\mathrm{sgn}(B_x) \\ 0 \\ -\dfrac{\alpha_f}{2\sqrt{\rho}c}\beta_y \\ \dfrac{\alpha_f}{2\sqrt{\rho}c}\beta_z \\ \dfrac{\alpha_s}{2\rho c^2} \end{Bmatrix} \tag{11}$$

-Magnetic flux wave: $\lambda_d = u_x$

$$\boldsymbol{r_d} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \boldsymbol{l_d} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{12}$$

Where: $c_{A,n} = \frac{|B_x|}{\sqrt{\rho}}$;

$$c_{f,s}^2 = \frac{1}{2}\left(\frac{\gamma p + B^2}{\rho} \pm \sqrt{\left(\frac{\gamma p + B^2}{\rho}\right)^2 - 4\frac{\gamma p B_x^2}{\rho^2}}\right)$$

$$\beta_y = \begin{cases} \frac{B_y}{B_\perp} & B_\perp \neq 0 \\ \frac{1}{\sqrt{2}} & B_\perp = 0 \end{cases}$$

$$\beta_z = \begin{cases} \frac{B_z}{B_\perp} & B_\perp \neq 0 \\ \frac{1}{\sqrt{2}} & B_\perp = 0 \end{cases}$$

$$\beta_\perp = \sqrt{B_y^2 + B_z^2}$$

The Alfvén, entropy wave and magnetic flux waves, are linearly degenerate; hence the flow velocity is constant throughout the wave. The magneto-acoustic waves are nonlinear and can be shock or rarefaction waves. However, under particular relations between the magnetic field and the sound velocity theses waves may be locally non-convex (Serna, 2009).

3 FINITE VOLUME FORMULATION

To obtain the numerical solution of the system described by Eq.(2), a finite volume scheme has been implemented using a structured mesh, together an approximate Riemann solver to calculate the fluxes with an explicit finite-differences scheme for the evaluation of the time evolution.

The numerical flows are evaluated by means of the Harten-Yee TVD technique, which allows the capturing of discontinuities, simultaneously achieving a second order approach (Yee, 1989).

The explicit TVD-finite volume scheme can be expressed as, see Fig. (1),

$$\boldsymbol{U}_{ij}^{n+1} = \boldsymbol{U}_{ij}^{n} - \Delta t \left[\frac{\overline{\boldsymbol{F}}_{i+\frac{1}{2};j}^{n} - \overline{\boldsymbol{F}}_{i-\frac{1}{2};j}^{n}}{\Delta x} + \frac{\overline{G}_{i;j+\frac{1}{2}}^{n} - \overline{G}_{i;j-\frac{1}{2}}^{n}}{\Delta y} \right] \tag{13}$$

where the function that determines the second-order numerical flux is defined as

$$\overline{\boldsymbol{F}}_{i+\frac{1}{2};j}^{n} = \frac{1}{2}\left(\boldsymbol{F}_{i+\mathbf{1}}^{n} + \boldsymbol{F}_{i}^{n} + \left(\sum_{m} \boldsymbol{R}_{i+\frac{1}{2}}^{m} \Phi_{i+\frac{1}{2}}^{m} \right)^{(n)} \right) \tag{14}$$

The limiter function used is one of minmod type,

$$\Phi_{i+\frac{1}{2}}^{m} = \left(g_{i+1}^{m} + g_{i}^{m} \right) - \sigma\left(\lambda_{i+\frac{1}{2}}^{m} + \gamma_{i+\frac{1}{2}}^{m} \right) \alpha_{i+\frac{1}{2}}^{m} \tag{15}$$

$$g_{i}^{m} = \mathrm{s\,gn}\left(\lambda_{i+\frac{1}{2}}^{m} \right) \max \left\{ \begin{matrix} 0 \\ \min \left[\begin{matrix} \sigma_{i+\frac{1}{2}}^{m} \left| \alpha_{i-\frac{1}{2}}^{m} \right| \\ \sigma_{i-\frac{1}{2}}^{m} \dfrac{\mathrm{sgn}\left(\lambda_{i+\frac{1}{2}}^{m} \right)}{2} \alpha_{i-\frac{1}{2}}^{m} \end{matrix} \right] \end{matrix} \right\}; \quad \sigma_{i+\frac{1}{2}}^{m} = \sigma\left(\lambda_{i+\frac{1}{2}}^{m} \right) \tag{16}$$

$$\gamma_{i+\frac{1}{2}}^{m} = \begin{cases} \dfrac{1}{\alpha_{i+\frac{1}{2}}^{m}} \left(g_{i+1}^{m} - g_{i}^{m} \right) & \alpha_{i+\frac{1}{2}}^{m} \neq 0 \\ 0 & \alpha_{i+\frac{1}{2}}^{m} = 0 \end{cases} \tag{17}$$

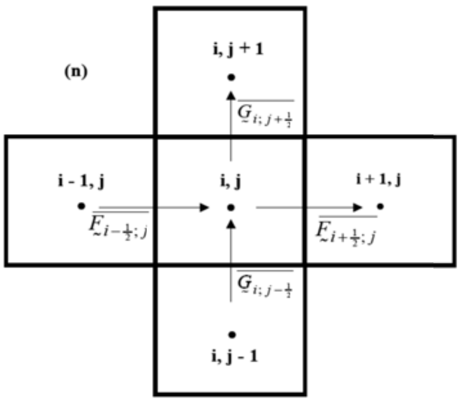

Fig. 1 Adjacent cells of the two-dimensional domain.

Approximate Roe-type Riemann solver produces only shock waves so a physically correct smooth rarefaction wave is replaced by a rarefaction shock wave that violates the entropy condition. An alternative to correct this non-physical solution is using a “sonic entropy fix” that smoothes out eigenvalues in the vicinity around zero. Harten (1982) suggested an entropy fix for Roe’s method, which has widespread use:

$$\psi(z)=\begin{cases}|z| & |z|\ge\delta\\ \dfrac{1}{2\delta}\left(z^2+\delta^2\right) & |z|<\delta\end{cases} \tag{18}$$

The function ψ in Eq. (18) is an entropy correction to z, whereas δ is generally a small and constant value that needs to be calibrated for each problem. A proper choice of the entropy parameter δ for higher Mach number flows not only helps in preventing nonphysical solutions but can act, in some sense, as a control in the convergence rate and in the sharpness of shocks (Yee, 1989).

For time-accurate calculations in explicit numerical algorithms

$$\sigma(z)=\frac{1}{2}\left[\psi(z)-\frac{\Delta t}{\Delta x}z^2\right] \tag{19}$$

and the wave strength of the m-th wave is

$$\alpha^m=\boldsymbol{L}^m\cdot\left(\boldsymbol{W}_{i+1}-\boldsymbol{W}_i\right) \tag{20}$$

where $\boldsymbol{L}^m$ is the left eigenvector for the m-th wave and $\boldsymbol{W}$ represents the primitive variable vector.

4 NEW FIX SONIC

If we apply to Eq. (2) the traditional Harten-Yee scheme, developed for gas dynamics equations, the sonic fix, given by Eq. (15), acts only on sonic point, but it does not act on non-convex point; because the gasdynamics flows do not present non-convex points.

To obtain “proper” numerical results for the Brio and Wu two dimensional MGD problem, the entropy correction of Harten scheme, Eq. (18), needs to be calibrated with relatively big values of δ (Maglione *et al.*, 2003). For gasdynamics hypersonic flows, a variable δ depending on the spectral radius of the Jacobian matrices of fluxes is very helpful in terms of stability and convergence rate (Yee, 1989). However, numerical tests show that this technique does not provide satisfactory results on the coplanar Riemann MGD problem. The use of a constant value, for 2D simulations, equal to the average in absolute value of the eigenvalues of the Jacobian matrices of fluxes show satisfactory results for short time only (Maglione *et al.*, 2007), also this technique introduces too much numerical viscosity around a large vicinity of the sonic point. As a result of this scheme the solutions are not particularly satisfactory for long computation time.

In order to obtain a method that does not need δ calibration for each MGD problem, it is convenient to improve the Van Leer technique (Van Leer *et al.*, 1989), vastly applied for gases.

$$\delta_{GD}=\max\left[\left|\lambda^m_{i+\frac{1}{2}}-\lambda^m_{i-\frac{1}{2}}\right|,0\right] \tag{21}$$

$$\delta_k^{MGD}=\begin{cases}\max\left[\left|\lambda^m_{i+\frac{1}{2}}-\lambda^m_{i-\frac{1}{2}}\right|\right] & \text{If } \lambda^m_{i+\frac{1}{2}} \text{ cuts across zero}\\ \min\left|\lambda^m_{i+\frac{1}{2}}\right| & \text{Otherwise}\end{cases} \qquad k=1,..,8 \tag{22}$$

For increasing the accuracy of the previous schemes and to avoid the spurious oscillations, a new entropy correction function was proposed (Maglione and Elaskar, 2010; Maglione *et al*., 2011). The new entropy correction function introduces high numerical viscosity only restricted to the proximity of the acoustic points,

$$\psi(z)=\begin{cases} |z| & \text{others} \\ \dfrac{z^2}{\delta^2}+\dfrac{\delta-2}{\delta}|z|+1 & \text{acoustic points} \end{cases} \tag{23}$$

A comparison between Harten´s original sonic entropy fix, Eq. (18) and the new proposed fix Eq. (23), is shown in Fig. (2). The new function is a continuously differentiable approximation to $|z|$, fulfilling,

$$\begin{aligned} \psi\left(\delta^-\right)&=\psi\left(\delta^+\right) \\ \psi(0)&=1 \\ \psi'\left(\delta^-\right)&=\psi'\left(\delta^+\right) \end{aligned} \tag{24}$$

The necessity to introduce a new sonic fix for 2-D MGD flow and not for the 1-D MGD occurs because the number of the eigenvalues crossing over zero, when the modified Van Leer's technique is used, is increasing for the two-dimensional test with respect the one-dimensional case. This effects it is specially note for the compound wave (Maglione *et al.*, 2011).

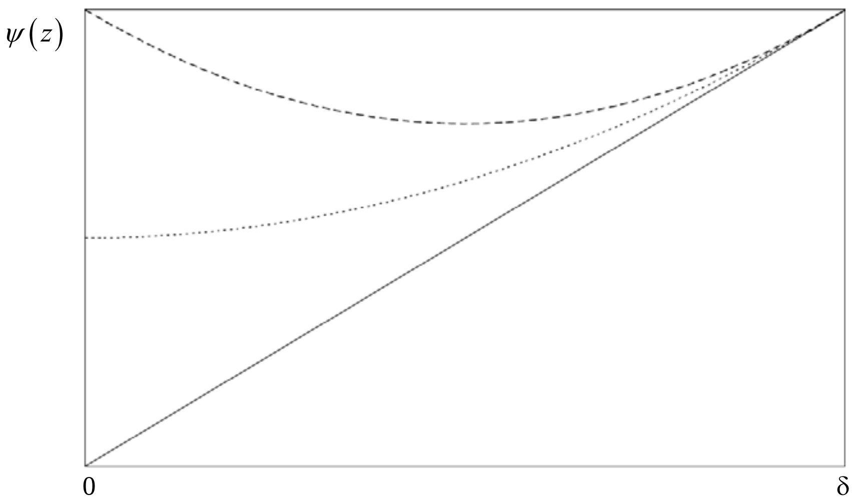

Fig. 2 Comparison between the new sonic fix and Harten´s original (Dotted line: Original sonic fix, Long Dash line: Proposed Sonic fix).

5 ORSZAG–TANG PROBLEM

The Orszag–Tang vortex problem is a very important two-dimensional numerical test for MGD codes (Serna, 2009; Tóth, 2000). The main reason for choosing this problem is that it requires the implementation of periodic boundary conditions. This feature had not been implemented earlier in the numerical code. This test problem was proposed for the first time in (Zachary *et al*., 1994) and has become a standard benchmark for MGD numerical schemes. The evolution of this complex MGD flow contains interactions between several shock waves traveling at different speeds and the formation of intermediate shocks. In the Orszag–Tang vortex problem the flow starts from smooth initial data but gradually becomes very complex.

The computational domain is a square with:

$$(x, y) \in [0, 2\pi] \times [0, 2\pi] \quad (25)$$

The two-dimensional MGD system (4) is solved with a uniform grid of 200 x 200 cells using the high resolution scheme (13), (14). Periodic boundary conditions are imposed in both *x*- and *y*-directions following the methodology presented by (Leveque, 2002) for high-resolution methods.

The initial primitive variables (6) are defined as:

$$\rho(x, y, 0) = \gamma^2, \quad p(x, y, 0) = \gamma$$

$$u_x(x, y, 0) = -\sin(y), \quad u_y(x, y, 0) = \sin(x), \quad u_z(x, y, 0) = 0 \quad (26)$$

$$B_x(x, y, 0) = -\sin(y), \quad B_y(x, y, 0) = \sin(2x), \quad B_z(x, y, 0) = 0$$

with γ = 5/3. Figs. 3-5 show the numerical approximation of the Orszag–Tang vortex system at t = 1.533 for the x component of the velocity, density and magnetic field module, respectively.

The problem is symmetric under a rotation of 180º, providing a symmetry test for the numerical code developed.

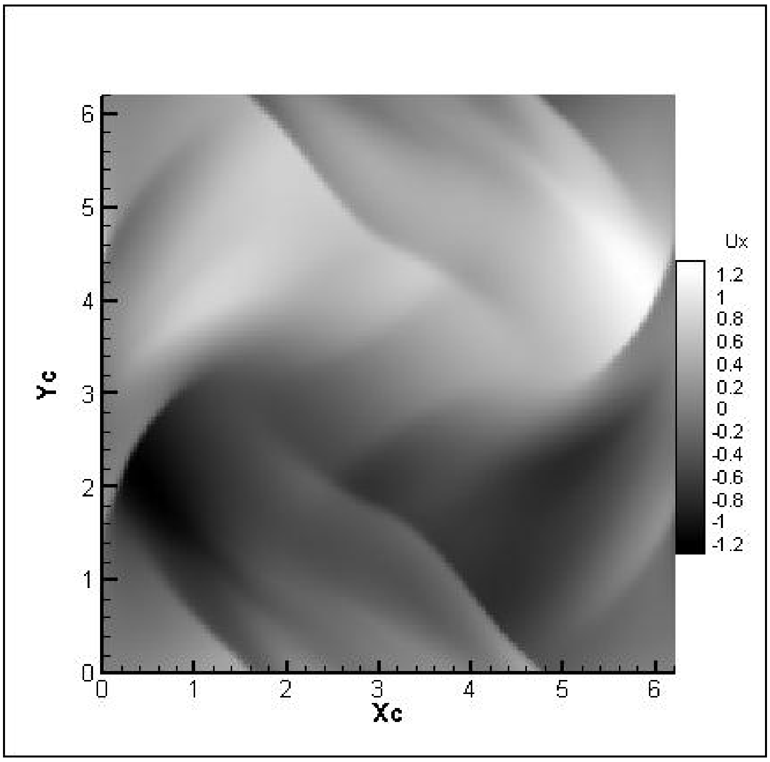

Fig. 3. Numerical approximation for the *x* component of the velocity in the Orszag–Tang vortex.

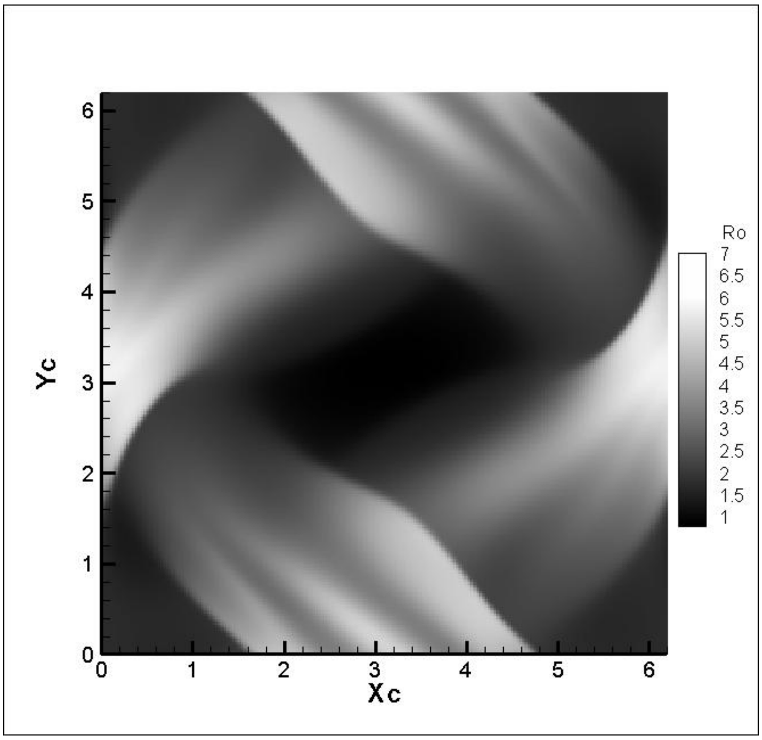

Fig. 4. Numerical approximation for the density in the Orszag–Tang vortex.

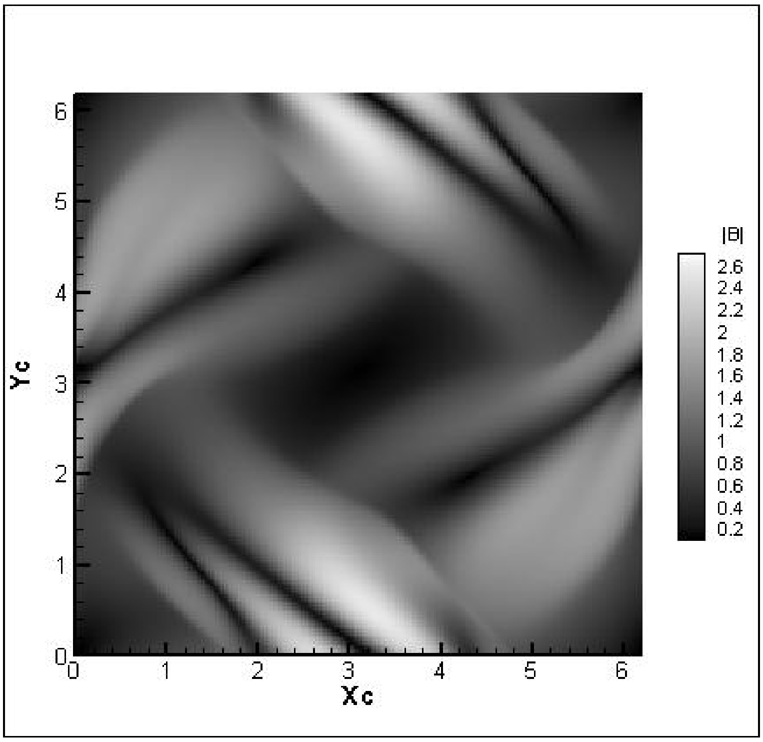

Fig. 5. Numerical approximation for the magnetic field module in the Orszag–Tang vortex.

6 CONCLUSION

In this paper the new sonic fix introduced for the TVD Harten-Yee's scheme is used to solve a high nonlinear flow and a very important two-dimensional numerical test for MGD codes: The Orszag–Tang vortex problem. Also we confirm advantages found with the new sonic fix in previous applications (Maglione *et al*., 2011). The advantages are that the sonic fix does not require a particular calibration (e.g. a function of the eigenvalues of the jacobian matrix). The results obtained are in agreement with the ones described in (VITA, 2012; Serna, 2009; Tóth, 2000).

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