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VISCOELASTIC NEUTRALIZERS FOR A NON-LINEAR CUBIC PRIMARY SYSTEM: CLASSICAL APPROACH

Vinicius V. Gonçalves[a], Mariano Febbo[b], and Carlos A. Bavastri[a]

[a]*Grupo de Vibrações e Som do Departamento de Engenharia Mecânica, Universidade Federal do Paraná, Centro Politécnico, Bairro Jardim das Américas, CP 19011, CEP 81531-980, Curitiba - PR, Brasil, bavastri@ufpr.br, http://www.pgmec.ufpr.br*

[b]*Instituto de Física del Sur, CONICET y Departamento de Física, Universidad Nacional del Sur, Avda. Alem 1253, 8000 Bahía Blanca, Argentina, mfebbo@uns.edu.ar, http://www.uns.edu.ar*

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Abstract. In this work we carry out the numerical optimization of a linear viscoelastic vibration neutralizer which is attached to a nonlinear primary system. The study considers a cubic nonlinearity in the primary system which is subjected to a harmonic external excitation of variable frequency. The optimum neutralizers will be obtained for different values of the linearized frequency of the nonlinear system and for different values of the magnitude of the harmonic excitation. Numerical tests will also be performed to show how a temperature variation affects the efficiency of the optimum viscoelastic neutralizer for the considered materials. Additionally, the main effect that the neutralizer produces in the dynamics of the compound system is also discussed.

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1 INTRODUCTION

Dynamic Vibration Neutralizers (DVN) are mechanical systems to be attached to another mechanical system, or structure, called the primary system, with vibration reduction purposes. The first theory of the DVN was proposed by Den Hartog (Den Hartog, 1956) and since then, many publications on the subject have steadily come to light, demonstrating their efficiency not only in mitigating vibrations in mechanical systems but also sound radiation in many structures and machines. Espíndola and Silva (1992) developed a general theory for the optimum design of DVNs when applied to a generic structure of any shape, with any amount and distribution of damping. That theory has been applied with great success to optimum design of systems of viscoelastic absorbers (Espíndola and Bavastri, 1995; Espíndola and Bavastri, 1997; Bavastri et al., 1998; Espíndola et al., 2005a). It is based on the concept of equivalent generalized mass and damping parameters for the absorbers, introduced in Espíndola and Silva (1992) by the first author. With this concept, it is possible to write down the equations for the movement of the compound system (primary system plus absorbers) in terms of the generalized coordinates (degrees of freedom), previously chosen to describe the configuration space of the primary system alone, in spite of the fact that the compound system has additional degrees of freedom introduced by the attached absorbers.

Viscoelastic dynamic vibration neutralizers are different from the traditional viscous neutralizers (proposed by Den Hartog) in the sense that they are easy to make and apply to a structure of any size and shape. This is in part possible thanks to modern technology of viscoelastic materials, which makes it easy to mould in any shape and to tailor it to meet almost any specifications. Several models have been proposed throughout the years to describe the behavior of viscoelastic materials (Espíndola, 1995). In recent years, the concept of fractional derivative has emerged as an alternative to model viscoelasticity through the construction of parametric models for viscoelastic materials (Bagley & Torvik, 1979; Bagley & Torvik, 1986; Torvik & Bagley, 1987; Pritz, 1996; Rossikhin & Shitikova, 1998; Lopes, 1998 and Espíndola et. al, 2005b).

The study of two-degree-of freedom systems with nonlinear characteristics (Nayfeh & Mook, 1979; Schimdt & Tondl, 1986) has captured a lot of attention in the past decades. Many of them were focused on nonlinear dynamic vibration absorbers (NDVAs) or nonlinear tuned mass dampers (NTMDs) attached to linear or nonlinear systems. It is worth mentioning the works of Roberson (Roberson, 1952), Pipes (Pipes, 1953), Soom (Soom, 1983) and Nissen (Nissen, 1985). They used several approximation methods (Ritz, Harmonic Balance) to obtain the steady-state responses with the aim of optimizing the NDVAs for vibration reduction. The works of Rice (Rice, 1986) and Shaw (Shaw, 1989) pointed out the possibility of having dynamic instabilities such us high amplitude limit cycle oscillations for certain frequencies if the damping of the primary system or the absorber is low. Natsiavas (1992) further studied the same phenomenon finding that a proper selection of the system parameters can avoid the quasi-periodic solutions leading to this high amplitude limit cycles.

In this work we present a mathematical formulation as well as a numerical scheme to optimal design a linear viscoelastic dynamic neutralizer. The neutralizer is applied to a cubic non-linear one-degree-of-freedom system to diminish its amplitude of vibration. The viscoelastic material is modeled through a four parameter fractional derivative model as can be seen in Espíndola et. al (2010) and use a parametric model presented by Pritz (1996). After an introductory section on viscoelastic vibration neutralizers, section two introduces the mathematical formulation of the problem and obtains the frequency response curves which

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are used in the optimization process. Next, a the third section briefly describes the optimization strategy. Finally, various numerical optimizations are presented for two different resilient materials to show the effectiveness of this control device.

2 VISCOELASTIC VIBRATION NEUTRALIZERS

Fig. 1a illustrates vibration absorbers of viscoelastic (hysteretic) nature applied to a structure of general shape, as shown in Espíndola et. al (2010). Fig. 1b shows a particular ordinary absorber. In between its rigid mass m_a and the structure lies a viscoelastic spring which is a piece of viscoelastic material, perhaps with some metal inserts.

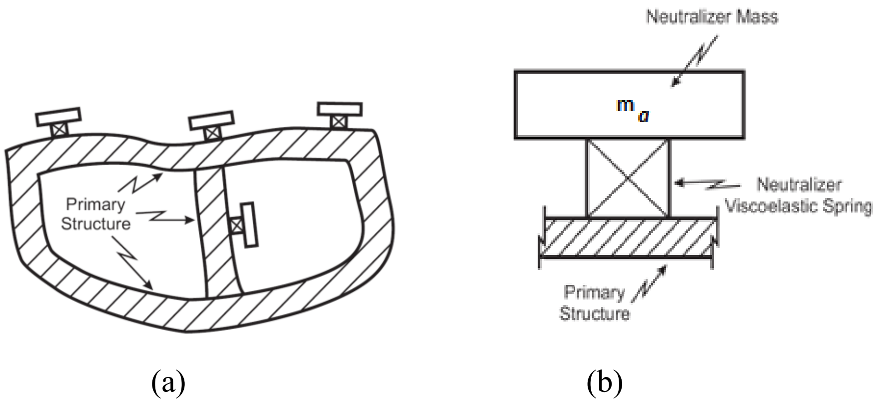

Figure 1 – (a) Primary structure with absorbers attached to it. (b) A particular absorber.

This is in fact a single degree-of-freedom viscoelastic vibration absorber. The stiffness constant (in the frequency domain) of the viscoelastic spring is given by Espíndola (1995):

$$k_s(\Omega) = \vartheta G_c(\Omega) = \vartheta G(\Omega)[1 + i\eta(\Omega)] \tag{1}$$

In Eq. 1, $G_c(\Omega)$ is the so called complex shear modulus of the viscoelastic material and ϑ is a geometric constant depending on the shape of the viscoelastic spring and of its metal inserts (Espíndola, 1995). It is in fact a complex quantity, in the sense that it has a real and an imaginary part. It is frequency dependent with Ω being the circular frequency. It is also temperature dependent, although it is not explicitly shown in the above equation.

In terms of the four parameters model for a viscoelastic material, $G_c(\Omega)$ may be expressed as:

$$G_c(\Omega) = \frac{G_0 + G_\infty (ib\Omega)^\beta}{1 + (ib\Omega)^\beta} \tag{2}$$

or

$$G_c(\Omega) = \frac{G_0 + G_\infty \varphi_0 (i\alpha_T(T)\Omega)^\beta}{1 + \varphi_0 (i\alpha_T(T)\Omega)^\beta} \tag{3}$$

where G_0 and G_∞ are the so called low and upper asymptotes, respectively, β is the fractional order of the derivative appearing in the constitutive differential equation for the viscoelastic material and b is the relaxation time constant of the material. The relaxation time constant is highly sensitive to temperature and is normally expressed as $b = \alpha_T(T) b_0$, where b_0 is b computed in the so called reference absolute temperature T_0 and $\varphi_0 = {b_0}^\beta$ (Lopes,

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1998). An expression normally used to compute the shift function is $\log_{10}\alpha_T = -\theta_1(T-T_0)/(\theta_2+T-T_0)$, where θ_1 and θ_2 are constants to be determined experimentally and T is the environmental absolute temperature. This empirical expression, consistent with experience and known as the William-Landel-Ferry (WLF) equation can be found in Ferry (1980).

It is also appropriate to write the complex shear modulus, for a given temperature, in the form:

$$G_c(\Omega) = G(\Omega)[1+i\eta(\Omega)] \tag{4}$$

where $G(\Omega)$ is the real part of the complex shear modulus and $\eta(\Omega)$ is the so called loss factor. The loss factor is a measure of the ability of the material to convert energy of deformation into heat.

3 MATHEMATICAL MODEL FOR THE NEUTRALIZER ATTACHED TO A NON-LINEAR SYSTEM

The mechanical model of the system under study can be observed in Fig. 2. As it can be shown, the primary system, or the system to be controlled, is a non-linear cubic one and the control system is a dynamic viscoelastic absorber (DVA). In this paper, the DVA has a linear behavior and will be called in this work dynamic viscoelastic neutralizer (DVN). After applying Newton's second law, the equations of motion of the compound system, by a given external excitation force $f\cos(\Omega t)$, are given by:

$$m_1\ddot{x}_1 + k_1x_1 + k_{1NL}x_1^3 + c_1\dot{x}_1 - k_s(\Omega)(x_2-x_1) = f\cos(\Omega t) \tag{5}$$

$$m_a\ddot{x}_2 + k_s(\Omega)(x_2-x_1) = 0 \tag{6}$$

where m_1 and m_a are the masses of the primary system and neutralizer, respectively. The parameters k_1, k_{1NL} and c_1 denote the linear and non-linear stiffness and damping constant of the primary system and $k_s(\Omega)$ is the stiffness constant (in the frequency domain or in a harmonic excitation with frequency Ω) of the viscoelastic spring. As usual, an overdot represents time derivative.

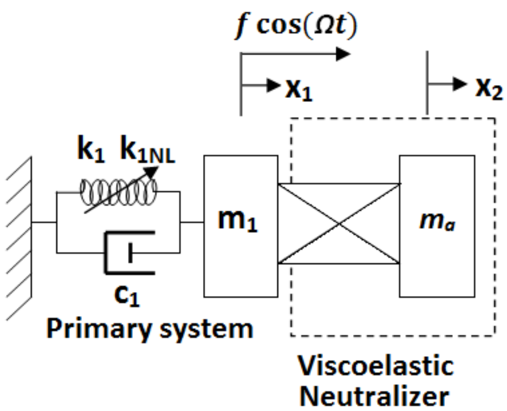

Figure 2 – Compound system under study.

After dividing eq. (5) by m_1 and eq. (6) by m_a, and introducing the following parameters

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$$\omega_{10} = \sqrt{\frac{k_1}{m_1}};\ \varepsilon = \frac{k_{1NL}}{m_1};\ \mu_{10} = \frac{c_1}{m_1};\ f_0 = \frac{f}{m_1};\ \mu = \frac{m_a}{m_1};\ x_r = x_2 - x_1,$$

the system of equations result:

$$\ddot{x}_1 + \omega_{10}^2 x_1 + \varepsilon x_1^3 + \mu_{10}\dot{x}_1 - \frac{k_R(\Omega)}{m_1}x_r - \frac{k_I(\Omega)}{m_1\Omega}\dot{x}_r = f_0\cos(\Omega t) \quad (7)$$

$$\ddot{x}_r + \frac{k_R(\Omega)}{m_a}x_r + \frac{k_I(\Omega)}{m_a\Omega}\dot{x}_r = -\ddot{x}_1 \quad (8)$$

where $k_R(\Omega)$ and $k_I(\Omega)$ are the real and imaginary part of the complex stiffness of the viscoelastic material, $k_s(\Omega)$, respectively.

Introducing the variables of interest, the following relationships can be written (Espíndola et al, 2009):

$$\frac{k_R(\Omega)}{m_1} = \frac{k_R(\Omega_a)}{m_1}R(\Omega) = \mu\Omega_a^2 R(\Omega)$$

where Ω_a is the natural frequency of the viscoelastic absorber given by

$$\Omega_a^2 = \frac{k_R(\Omega_a)}{m_a} \quad (9)$$

and

$$R(\Omega) = \frac{k_R(\Omega)}{k_R(\Omega_a)} = \frac{G_R(\Omega)}{G_R(\Omega_a)} \quad (10)$$

We must point out that $G_R(\Omega)$ is the real part of the complex shear modulus evaluated at $\Omega = \Omega_a$. Also it is possible to write,

$$\frac{k_I(\Omega)}{m_1\Omega} = \frac{k_R(\Omega)}{m_1}\frac{\eta(\Omega)}{\Omega} = \frac{\mu\Omega_a^2}{\Omega}R(\Omega)\eta(\Omega)$$

Then, eqs. (7) and (8) can be written as

$$\ddot{x}_1 + \omega_{10}^2 x_1 + \varepsilon x_1^3 + \mu_{10}\dot{x}_1 - \mu\Omega_a^2 R(\Omega)x_r - \mu\frac{\Omega_a^2}{\Omega}R(\Omega)\eta(\Omega)\dot{x}_r = f_0\cos(\Omega t) \quad (11)$$

$$\ddot{x}_r + \Omega_a^2 R(\Omega)x_r + \frac{\Omega_a^2}{\Omega}R(\Omega)\eta(\Omega)\dot{x}_r = -\ddot{x}_1 \quad (12)$$

Making $\tau = \Omega t$, it is possible to calculate: $\frac{d^2(.)}{dt^2} = \Omega^2\frac{d^2(.)}{d\tau^2}$ and $\frac{d(.)}{dt} = \Omega\frac{d(.)}{d\tau}$. Then, eqs. (11) and (12), after dividing both of them by Ω^2, result:

$$x_1'' + \bar{\omega}_{10}^2 x_1 + \bar{\varepsilon}x_1^3 + \bar{\mu}_{10}x_1' - \mu\frac{R(\Omega)}{\beta^2}x_r - \mu\frac{R(\Omega)}{\beta^2}\eta(\Omega)x_r' = \bar{f}_0\cos(\tau) \quad (13)$$

$$x_r'' + \frac{R(\Omega)}{\beta^2}x_r + \frac{R(\Omega)}{\beta^2}\eta(\Omega)x_r' = -x_1'' \quad (14)$$

where apostrophe represents differentiation with respect to τ and

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$\bar{\omega}_{10}^2 = \frac{\omega_{10}^2}{\Omega^2}$; $\beta = \frac{\Omega}{\Omega_a}$; $\bar{\mu}_{10} = \frac{\mu_{10}}{\Omega}$; $\bar{\varepsilon} = \frac{\varepsilon}{\Omega^2}$; $\bar{f_0} = \frac{f_0}{\Omega^2}$; $\bar{\Omega}_a = \frac{\Omega_a}{\Omega}$.;

Finally, eqs. (13) and (14) can be put in matrix form:

$$Mq'' + Cq' + Kq = f \tag{15}$$

where $M = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$; $C = \begin{pmatrix} \bar{\mu}_{10} & -\mu\beta^{-2}R(\Omega)\eta(\Omega) \\ 0 & \beta^{-2}R(\Omega)\eta(\Omega) \end{pmatrix}$; $K = \begin{pmatrix} \bar{\omega}_{10}^2 & -\mu\beta^{-2}R(\Omega) \\ 0 & \beta^{-2}R(\Omega) \end{pmatrix}$;
and $q = \begin{pmatrix} x_1 \\ x_r \end{pmatrix}$ and $f = \begin{pmatrix} \bar{f_0}\cos(\tau) - \bar{\varepsilon}x_1^3 \\ 0 \end{pmatrix}$.

To obtain the frequency response curve (FRC) the method of averaging (Thomsen, 2010) is employed. It is assumed that the steady-state response is given by:

$$q(\tau) = u(\tau)\cos(\tau) + v(\tau)\sin(\tau) \tag{16}$$

where the time dependence of $u = [u_1(\tau) \quad u_2(\tau)]^T$ and $v = [v_1(\tau) \quad v_2(\tau)]^T$ is taken to be "slow". Then, differentiating eq. (16) with respect to time τ we obtain the velocity:

$$q'(\tau) = -u(\tau)\sin(\tau) + v(\tau)\cos(\tau) \tag{17}$$

while, according to the method

$$u'(\tau)\cos(\tau) + v(\tau)\sin(\tau) = 0 \tag{18}$$

Using eqs. (17) and (18) in evaluating q'' and substituting it in eq. (15) gives

$$(Mv' - Mu + Cv + Ku)\cos(\tau) - (Mu' + Mv + Cu - Kv)\sin(\tau) = f(u, v, \tau) \tag{19}$$

Then, eq. (18) is multiplied by $M\cos(\tau)$, eq. (19) is multiplied by - $\sin(\tau)$ and the two equations are added. The resulting equation is then integrated from 0 to 2π by assuming that u and v are constants. Finally, we arrive at

$$Mu' = -\frac{1}{2}(M - K)v' - \frac{1}{2}Cu + \begin{pmatrix} \frac{3}{8}\bar{\varepsilon}\, v_1 a_1^2 \\ 0 \end{pmatrix} \tag{20}$$

Where we have made $a_1^2 = u_1^2 + v_1^2$ and $a_2^2 = u_2^2 + v_2^2$.

Following a similar procedure, eq. (18) is multiplied by $M = \sin(\tau)$, eq. (19) is multiplied by $\cos(\tau)$ and the two equations are added. After integration from 0 to 2π by assuming again that u and v are constants, we arrive at:

$$Mv' = -\frac{1}{2}(M - K)u' - Cv + \begin{pmatrix} \frac{1}{2}(\bar{f_0} - \frac{3}{4}\bar{\varepsilon}u_1 a_1^2) \\ 0 \end{pmatrix} \tag{21}$$

Finally in order to obtain the steady-state solution of eqs. (20) and (21), one has to put the left hand sides of both equations equal to zero. Then, after a length but straightforward manipulation we arrive at the frequency response curve for the primary system which turns out to be an implicit function of a_1 and Ω:

$$a_1^2(A(\Omega)^2 + B(\Omega)^2) - \bar{f}_0^2\left[\left(1 - \beta^{-2}R(\Omega)\right)^2 + \left(\beta^{-2}R(\Omega)\eta(\Omega)\right)^2\right] = 0 \tag{22}$$

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where

$$A(\Omega) = \left[\left(1 - \bar{\omega}_{10}^2 + \mu - \frac{3}{4}\bar{\varepsilon}a_1^2\right)\beta^{-2}R(\Omega)\eta(\Omega) + \bar{\mu}_{10}(1 - \bar{\Omega}_a^2)\right]$$

and

$$B(\Omega) = \left[(1 - \bar{\omega}_{10}^2)\left(1 - \bar{\Omega}_a^2\right) - \mu\bar{\Omega}_a^2 - \bar{\mu}_{10}\beta^{-2}R(\Omega)\eta(\Omega) - \frac{3}{4}\bar{\varepsilon}a_1^2(1 - \bar{\Omega}_a^2)\right]$$

Finally, once the amplitude of the primary system has been obtained, the amplitude of the relative coordinate is given by

$$a_2^2(A^2 + B^2) - \bar{f}_0^2 = 0$$

As mentioned above, a_1 represents the amplitude of the displacement of the primary system, x_1. Hereafter, the frequency response curve (eq. 21), which accounts for the amplitude a_1 in terms of the frequency Ω will be called H(Ω). This function reduces to the modulus of the frequency response function |H(Ω)| when considering a linear single degree-of-freedom system, over all the frequency band.

4 NON-LINEAR OPTIMIZATION STRATEGY

The optimal parameters of the DVN are achieved using non linear optimization techniques. The objective function is given by

$$f_{obj}\,(x): R^n \rightarrow R = \left\|H(\Omega, x)_{\Omega_1 \leq \Omega \leq \Omega_2}\right\|_F \tag{23}$$

where $\|\cdots\|_F$ represents the Frobenius norm, Ω_1 and Ω_2 are the low and upper frequency band of interest, respectively, and x is the design vector. The proposal here is to reduce as much as possible the amplitude of the displacement of the primary system when the optimal absorber is attached to it.

The design vector, in this particular case, is defined as:

$$x = \Omega_a$$

where Ω_a is the natural frequency of the neutralizer.

In this work, it is not used the inequality or equality constrain for the design vector. The Nelder-Mead method is used as the non linear optimization technique. Both, the solution of the non linear problem, $H(\Omega, x)$, and the non linear optimization technique are implemented by self made codes in Matlab language.

After the optimization procedure, eq. (9) allows us to obtain the optimal frequency Ω_a^*, and to calculate the geometric factor of the DVN. Finally, to achieve the physical realization of the DVN one has to follow the procedure described in Espíndola et. al (2009 and 2010).

5 RESULTS

In this section, we present the results corresponding to the steady-state solution of different selected cases of nonlinear primary systems with a DVN attached to them. The steady-state solutions are determined by setting $u'_i = v'_i = 0$ on the right hand members of Eqs. (20) and (20) and solving the nonlinear system. The stability analysis is then performed by judging the eigenvalues of the Jacobian matrix of the linearized system calculated at the fixed points.

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We select two different viscoelastic materials, used as resilient elements of the neutralizer, for a given load f_0 and nonlinear parameter α. Table 1 shows the numerical values of the parameters of the fractional derivative model for the selected viscoelastic neutralizers. To considering the variation of temperature, in table 2 some complementary parameters are presented. This parameters are characteristic value for neoprene and butyl rubber founded in engineering process.

	G_0[Pa]	G_i [Pa]	B	φ_0	θ_1	θ_2	$T_T[K]$	$T_0[K]$
Neoprene	4,55e6	4,18e8	0,319	0,00274	5,09	46,5	303	273
Butyl Rubber	1,76e5	2,41e8	0,424	0,00424	9,91	119	303	273

Table 1- Numerical values of the parameters of the fractional derivative model for the viscoelastic material.

The frequency range that was used for the numerical optimization is: $\Omega_1 = 40\, rad/s$ to $\Omega_2 = 100\, rad/s$.

Constant Parameters	
T_0	273 K
T_T	303 K
c_1	0.1000 Ns/m
Ω_1	40 rad/s
Ω_2	100 rad/s
Discretization	1
$\Omega_{initialguess}$	73 rad/s
m_1	1 kg
m_a	0.1500 kg
μ	0.1500
μ_{10}	0.1000
Ω_{10}	73 rad/s

Table 2 - Constant Parameters

In the following, the considered cases are studied as indicated below. In Figs. 3, 4 and 5 we considered the response of the primary and compost system with different value of f_0. The viscoelastic material was butyl rubber and the parameter α was considered to be constant ($\alpha = 5$). Figs. 6 and 7 are equivalent to the above ones but the parameters α was changed and the parameter f_0 was fixed ($f_0 = 1000$).

Figs. 8, 9 and 10 are equivalent to Figs. 3, 4 and 5 but neoprene is used instead of butyl rubber. In Figs. 11 and 12 the conditions were equivalent to Figs. 4, 6 and 7 but the viscoelastic material was neoprene.

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5.1 Butyl Rubber

5.1.1. For $f_0 = 100$ and $\alpha = 5$

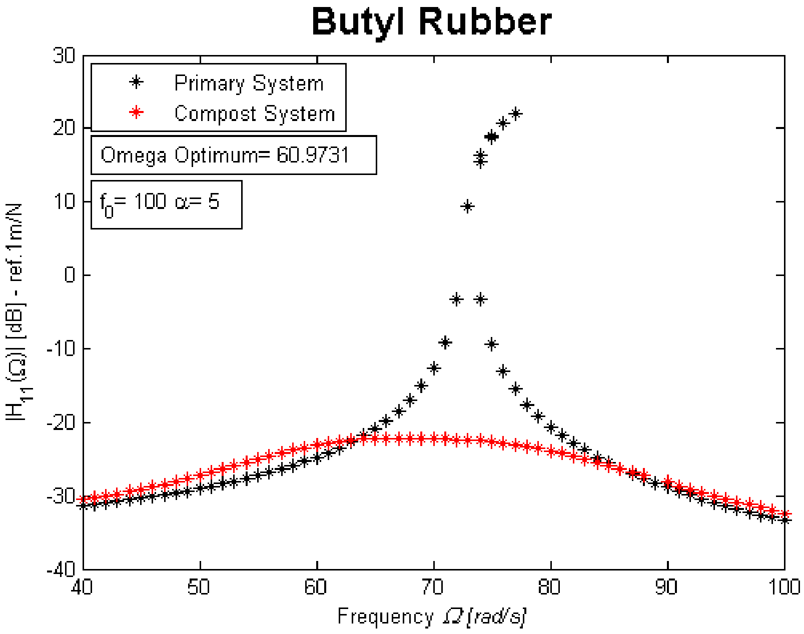

Figure 3 – FRC for Butyl Rubber for $f_0 = 100$ e $\alpha = 5$

5.1.2. For $f_0 = 1000$ and $\alpha = 5$

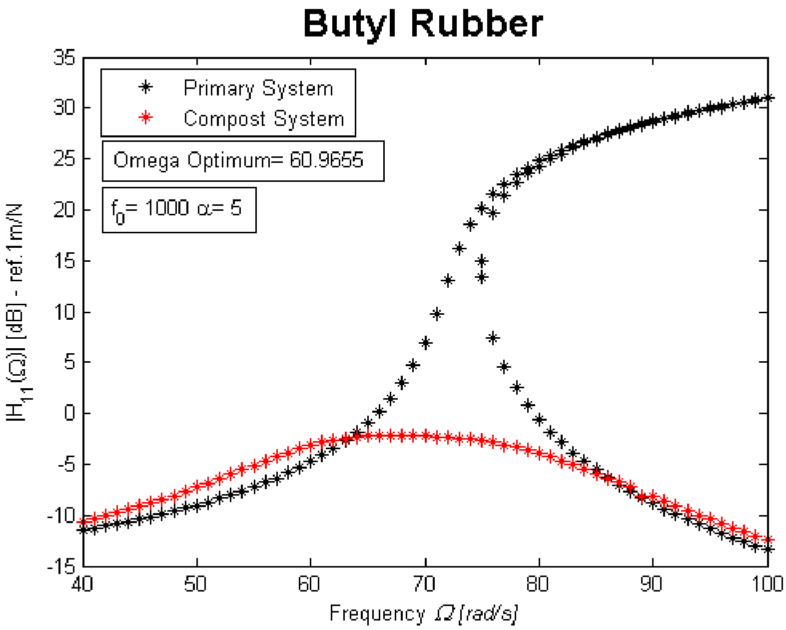

Figure 4 - FRC for butyl rubber for $f_0 = 1000$ and $\alpha = 5$

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5.1.3. For $f_0 = 10000$ and $\alpha = 5$

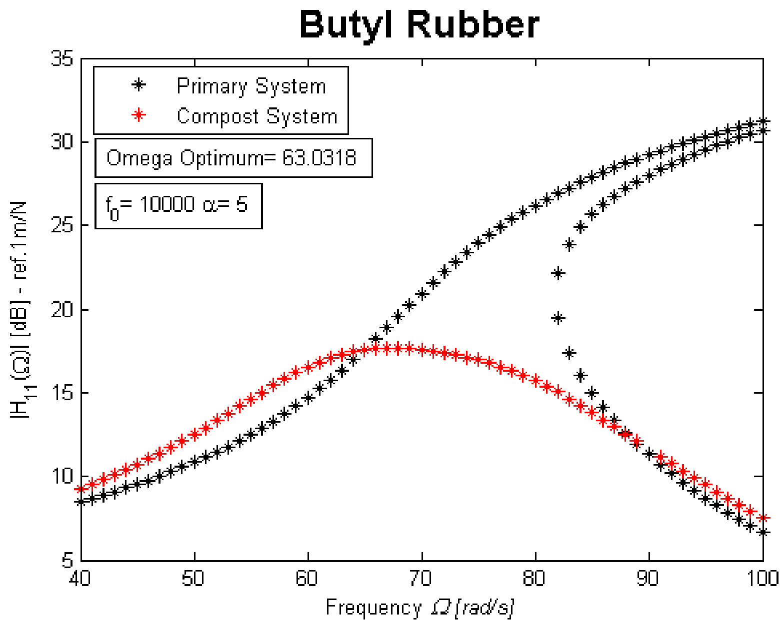

Figure 5 - FRC for butyl rubber for $f_0 = 10000$ and $\alpha = 5$

From figures 3, 4 and 5 we can observe a great reduction of the amplitude of vibration of the primary system when the DVN made of Butyl rubber is attached. This is so, for a large range of variation of the magnitude of the load. In figs. 5 and 6 a similar situation is observed for a variation of the magnitude of the nonlinear parameter α.

5.1.4. For $f_0 = 1000$ and $\alpha = 1$

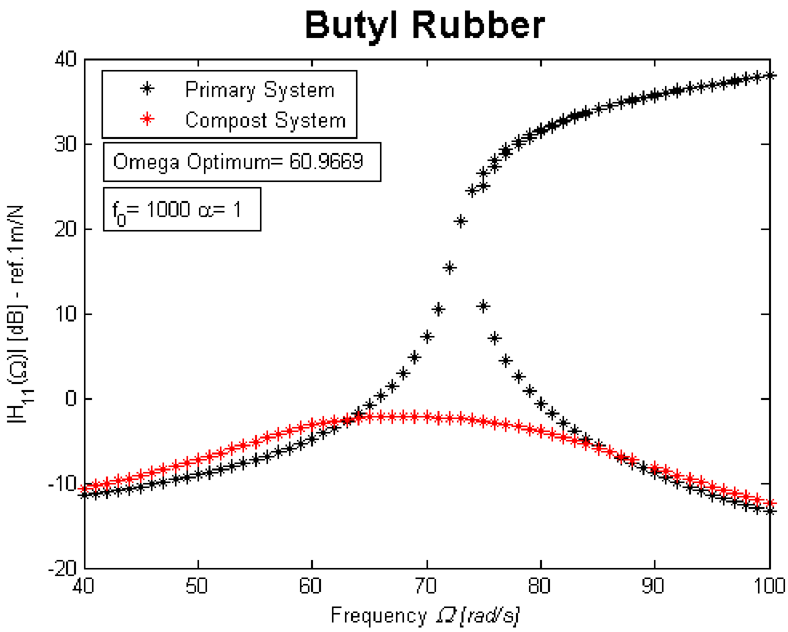

Figure 6 - FRC for butyl rubber for $f_0 = 1000$ and $\alpha = 1$

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5.1.5. For $f_0 = 1000$ and $\alpha = 10$

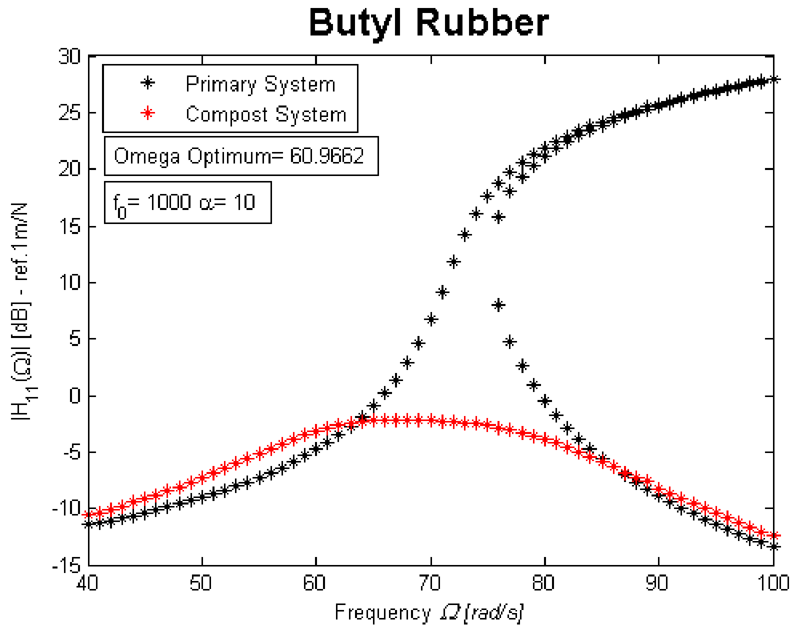

Figure 7 - FRC for butyl rubber for $f_0 = 1000$ and $\alpha = 10$

5.2 Neoprene

5.2.1. For $f_0 = 100$ and $\alpha = 5$

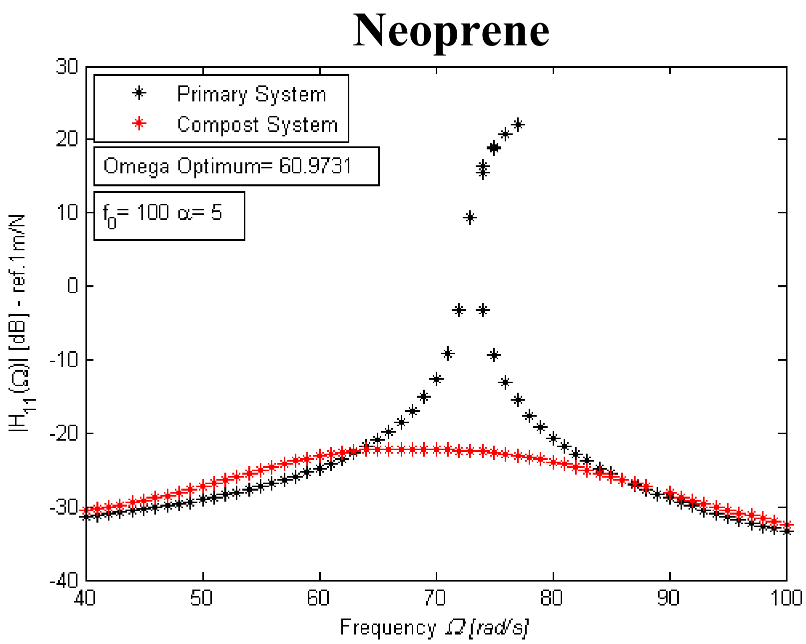

Figure 8 - FRC for neoprene $f_0 = 100$ and $\alpha = 5$

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5.2.2. For $f_0 = 1000$ and $\alpha = 5$

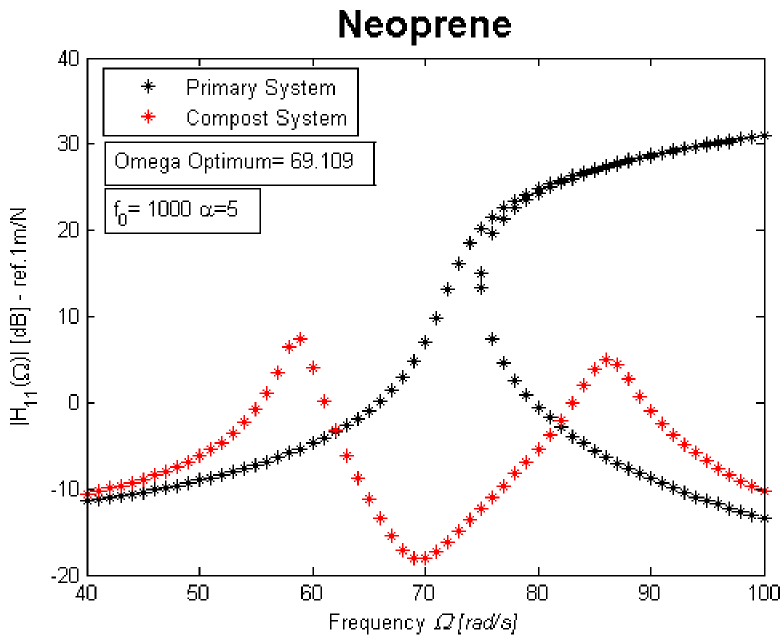

Figure 9 - FRC for neoprene for $f_0 = 1000$ and $\alpha = 5$

5.2.3. For $f_0 = 10000$ and $\alpha = 5$

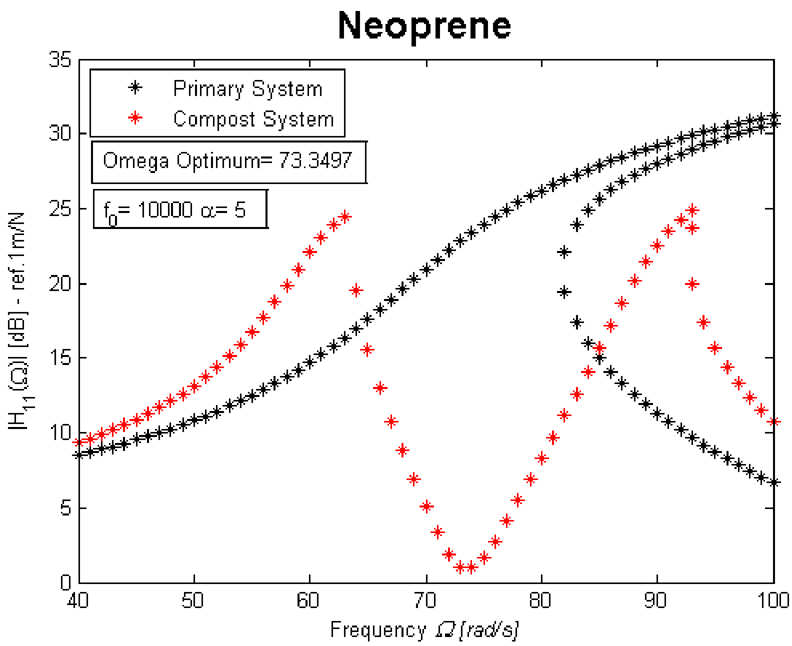

Figure 10 - FRC for neoprene for $f_0 = 10000$ and $\alpha = 5$

When neoprene is used as a resilient material for the DVN the situation is different. Whereas the reduction is large for a value of $f_0 = 100$ (fig. 8), the DVN looses effectiveness when the magnitude of the load is increased. This is observed for $f_0 = 1000$ (fig. 9) and $f_0 = 10000$ (fig. 10). In this last case the response of the compost system looses stability for

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small range of frequency values and presents the typical "jump" of nonlinear systems.

5.2.4. For $f_0 = 1000$ and $\alpha = 1$

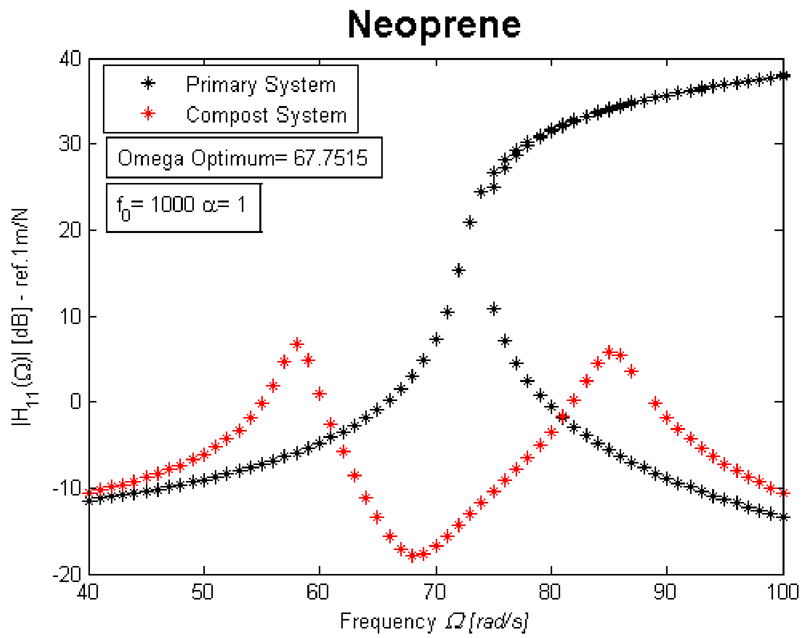

Figure 21 – FRC for neoprene for $f_0 = 1000$ and $\alpha = 1$

5.2.5. For $f_0 = 1000$ and $\alpha = 10$

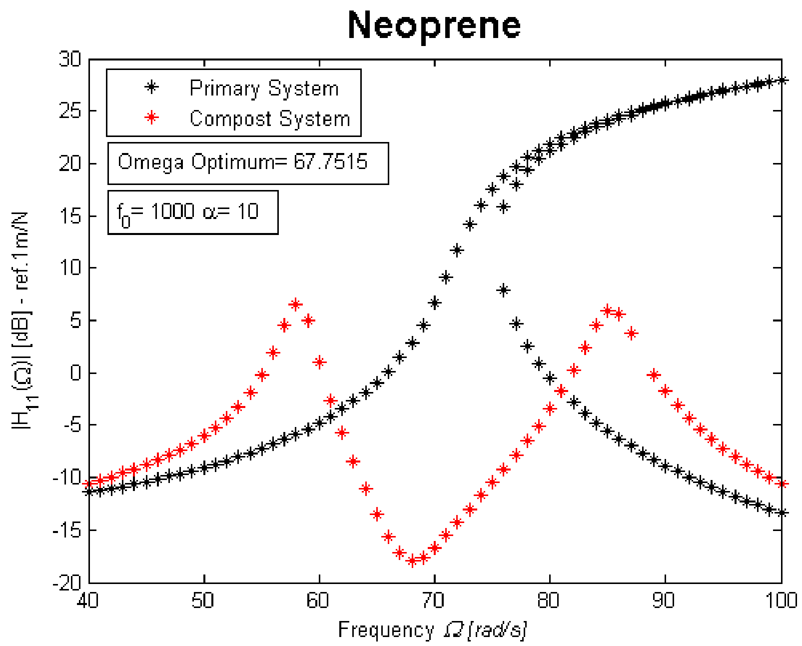

Figure 32 - FRC for neoprene for $f_0 = 1000$ and $\alpha = 10$

The cases when the magnitude of the load is fixed and α varies are shown figs. 9, 10. There, DVN causes a smaller reduction of the amplitude of vibration of the primary system when compared with the DVN made of butyl rubber. However, the solution never looses

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stability for the frequency band under study.

5.3 Simulations for different temperatures

In this section, the viscoelastic dynamic neutralizers, which were designed on the optimal form in the last section, were exposed to a variable temperature environment. In this sense we want to observe the effect of this variation on the effectiveness of the DVN to control the amplitude of the primary system for different working temperatures (T_T). Due to the variation of temperature the elasticity or shear dynamic modulus of the viscoelastic materials change, principally when we are working in the transition region where the properties of the material are more temperature dependent. This fact produces a change of stiffness and consequently a change of the natural frequency of the neutralizer, detuning the passive control system. It were selected two different working temperatures, $T_T = 323\ K$ and $T_T = 273\ K$.

With $\Omega_a^{\prime}$ we name the new natural frequency of the absorber, which is obtained in a recursive manner from the working temperature T_T, assuming that the neutralizer has been already built. This means that its geometric parameters (ϑ) and its mass have been fixed.

5.3.1 Butyl rubber: $\mathbf{f_0 = 1000}$, $\boldsymbol{\alpha = 5}$ and $\boldsymbol{T_T = 273\ K}$

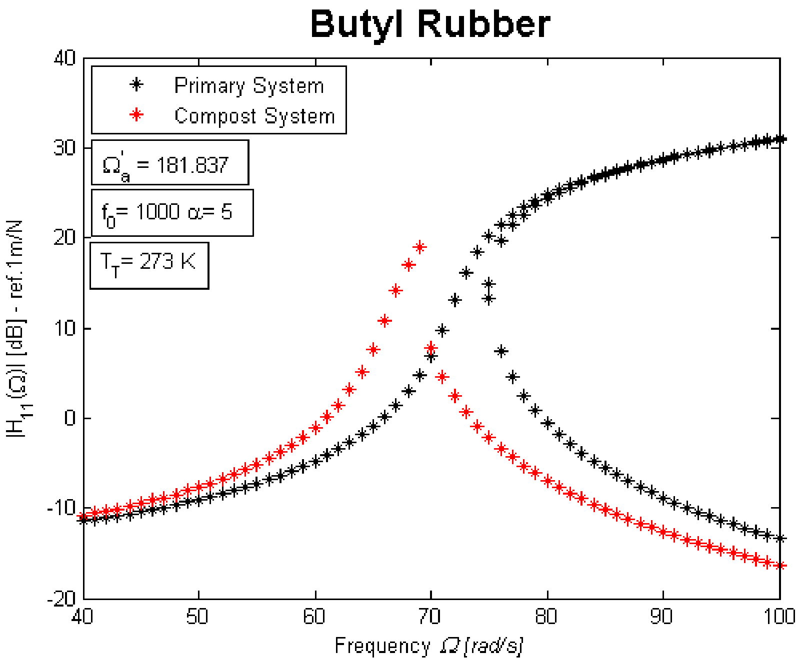

Figure 43 – FRC for butyl rubber for $f_0 = 1000$, $\alpha = 5$ and $T_T = 273\ K$

In Figs. 13 and 14 we can see the detuning of the neutralizer for different working temperatures, when we use butyl rubber and $f_0 = 1000$, $\alpha = 5$. Here, the figures show that the DVN still makes a remarkable reduction of the amplitude of vibration of the primary system. Figs. 15 and 16 are similar to Figs. 13 and 14 with the only difference that $f_0 = 10000$. In these cases, we observe that the detuning strongly affects the effectiveness of the absorber and we observe the jump phenomenon.

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5.3.2 Butyl rubber: $\mathbf{f_0} = \mathbf{1000}$, $\boldsymbol{\alpha} = \mathbf{5}$ and $\boldsymbol{T_T} = \mathbf{323}\ \boldsymbol{K}$

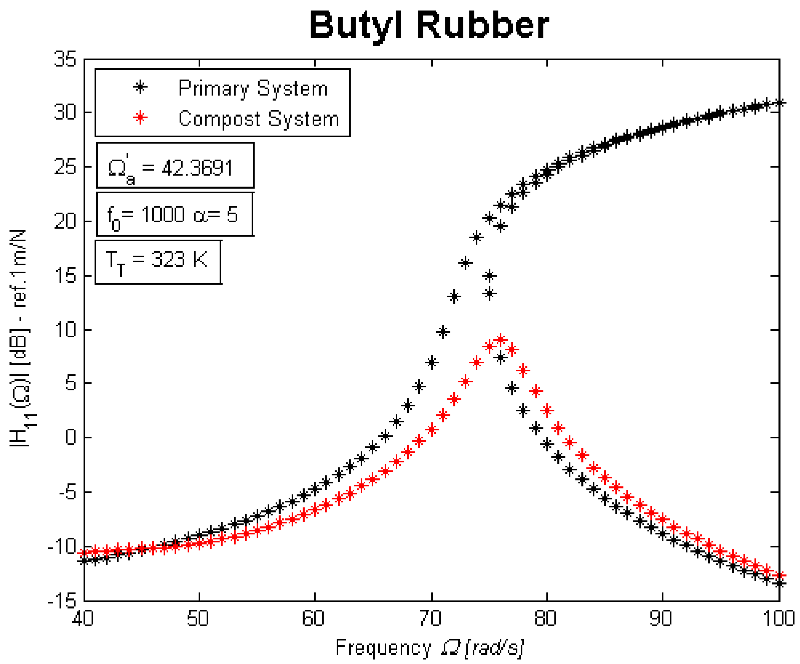

Figure 54 - FRC for butyl rubber for $f_0 = 1000$, $\alpha = 5$ and $T_T = 323\ K$

5.3.3 Butyl rubber: $\mathbf{f_0} = \mathbf{10000}$, $\boldsymbol{\alpha} = \mathbf{5}$ and $\boldsymbol{T_T} = \mathbf{273}\ \boldsymbol{K}$

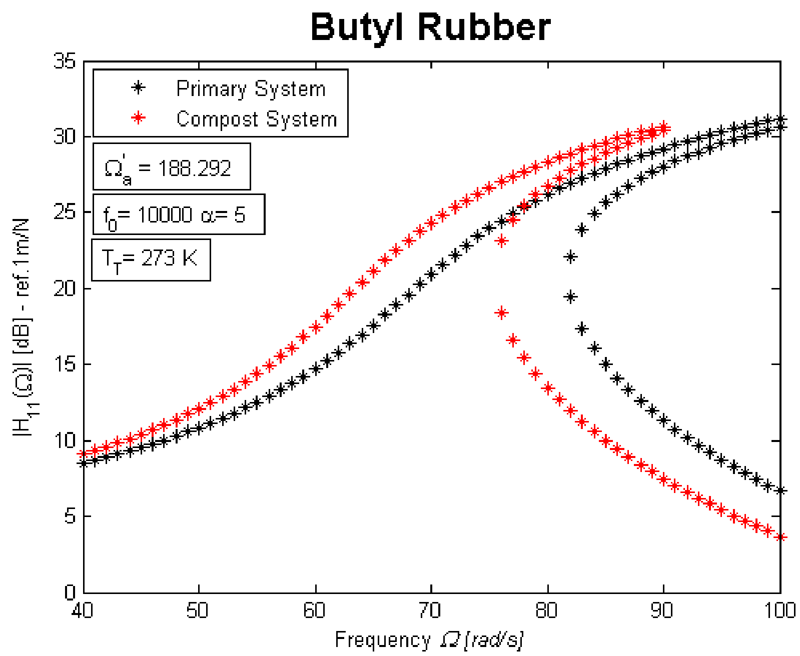

Figure 65 - FRF for butyl rubber $f_0 = 10000$, $\alpha = 5$ and $T_T = 273\ K$

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5.3.4 Butyl rubber: $\mathbf{f_0 = 10000}$, $\boldsymbol{\alpha} = \mathbf{5}$ and $\boldsymbol{T_T = 323\ K}$

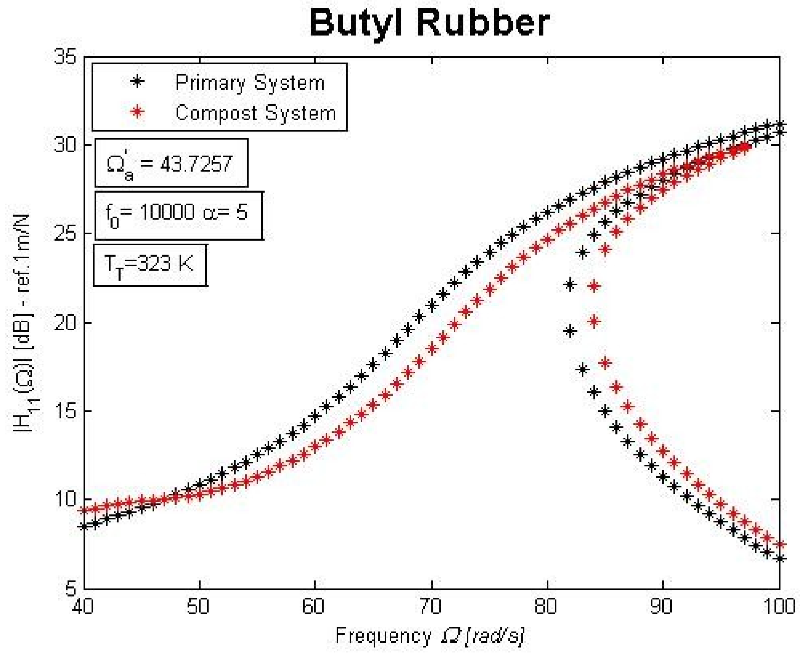

Figure 76 - FRC for butyl rubber for $f_0 = 10000$, $\alpha = 5$ and $T_T = 323\ K$

Figures 17 and 18 show the detuning of the neutralizer for different working temperatures, this time for neoprene and $f_0 = 1000$, $\alpha = 5$. We see that in these cases the detuning is not so strong as in the case of Butyl rubber. Finally, figs. 19 and 20 show the results for neoprene with $f_0 = 10000$ and $\alpha = 5$. Despite of the fact that the detuning is still not large, the reduction of the amplitude of vibration of the primary system is poor and it also appears a jump in the solution.

5.3.5 Neoprene: $\mathbf{f_0 = 1000}$, $\boldsymbol{\alpha} = \mathbf{5}$ and $\boldsymbol{T_T = 273\ K}$

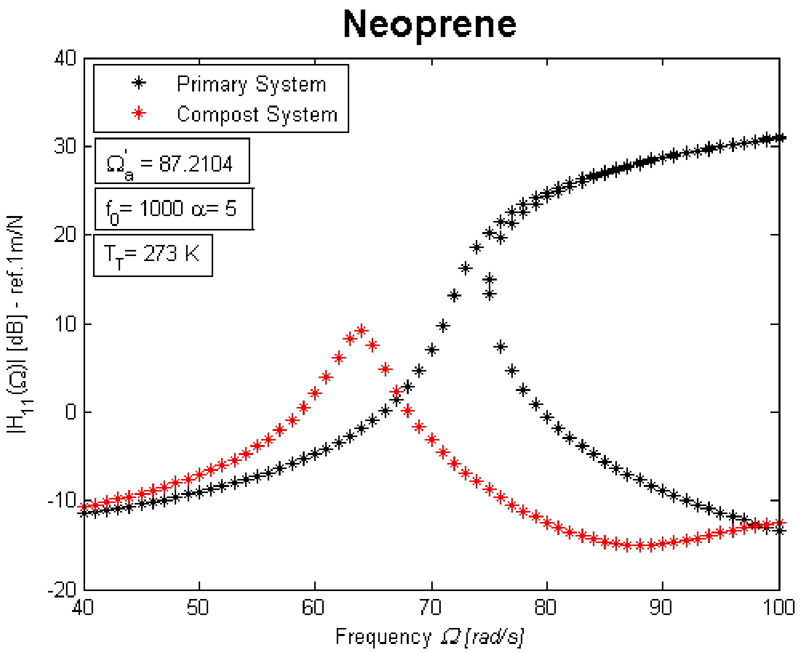

Figure 87 - FRC for neoprene for $f_0 = 1000$, $\alpha = 5$ $T_T = 273\ K$

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5.3.6 Neoprene: $f_0 = 1000$, $\alpha = 5$ and $T_T = 323\ K$

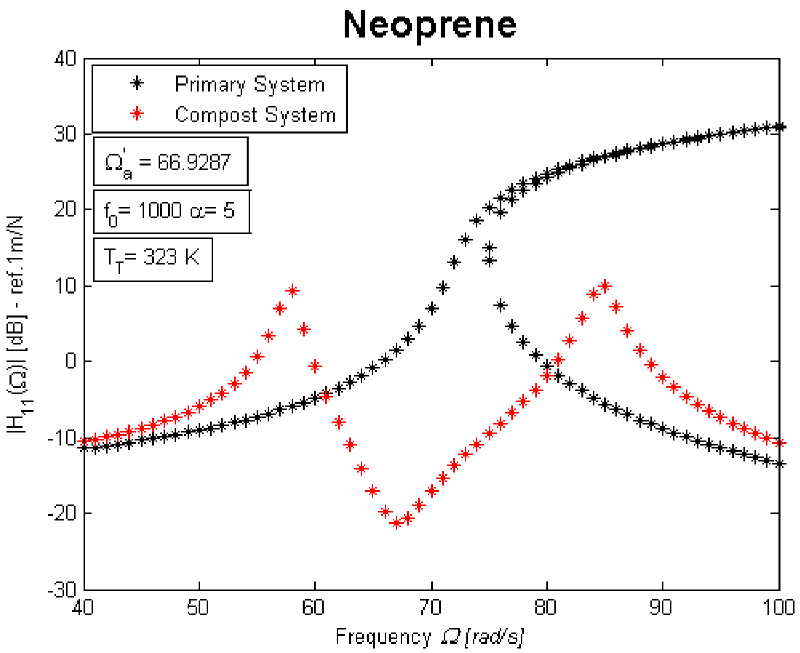

Figure 98 - FRC for neoprene for $f_0 = 1000$, $\alpha = 5$ $T_T = 323\ K$

5.3.7 Neoprene: $f_0 = 10000$, $\alpha = 5$ and $T_T = 273\ K$

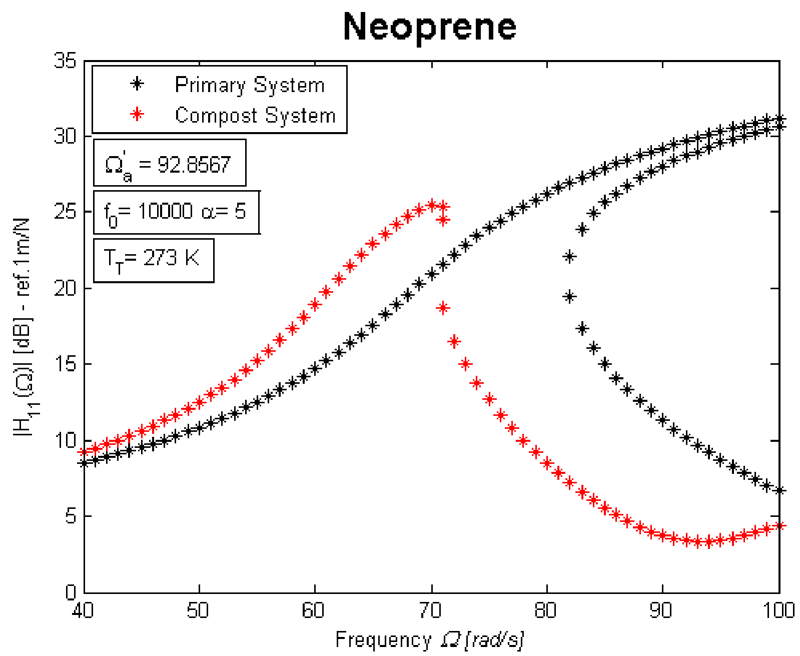

Figure 19 - FRC for neoprene for $f_0 = 10000$, $\alpha = 5$ $T_T = 273\ K$

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5.3.8 Neoprene: $f_0 = 10000$, $\alpha = 5$ and $T_T = 323\ K$

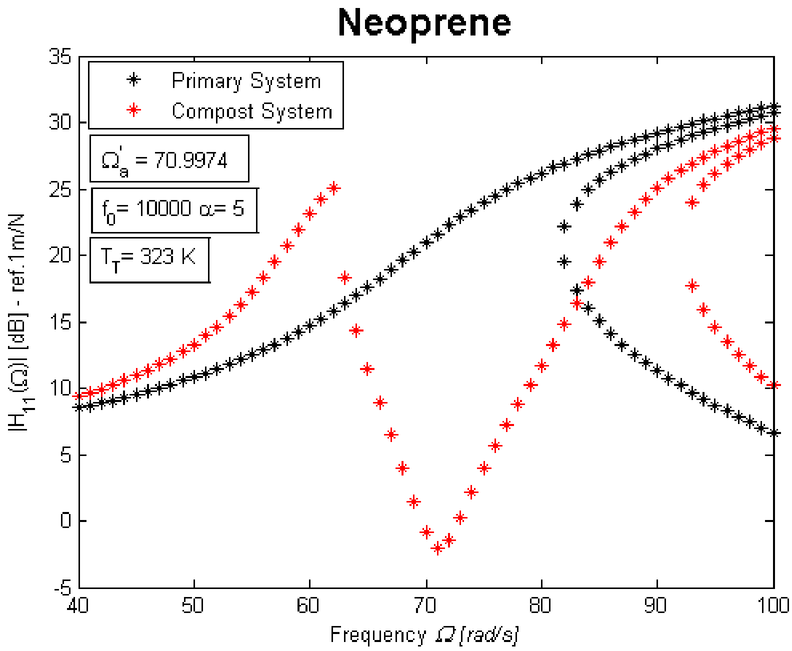

Figure 100 - FRC for neoprene for $f_0 = 10000$, $\alpha = 5$ $T_T = 323\ K$

6 CONCLUSIONS

The present work implements the analytical formulation and numerical optimization of a dynamic viscoelastic neutralizer which is attached to a cubic non-linear one degree-of-freedom system. The viscoelastic material of the neutralizer is modeled using a fractional derivative four parameter model. The implementation of the optimization is carried out firstly by obtaining the steady-state solution and secondly by looking for the optimum parameters of the viscoelastic neutralizer. A nonlinear optimization technique is implemented for obtaining the optimal frequency of the viscoelastic neutralizer to reduce the response of the primary system in a broad frequency band.

Two different real viscoelastic materials were used: butyl rubber, and neoprene. Both of them present excellent alternatives for vibration reduction purposes with a large attenuation of the amplitude of vibration of the primary system under the studied conditions.

The optimization process shows that both types of optimum viscoelastic neutralizers perform such a large attenuation that makes the system to behave as a linear two degree-of-freedom system: heavily damped for the case of butyl rubber and lightly damped when neoprene is used. In this last case, only for an extremely large magnitude of the load, neoprene is incapable to present a linear behavior and shows the typical behavior of a nonlinear system (jump phenomena).

Variations of the working temperature were performed. For both optimum neutralizers the temperature changes introduce a detuning of the optimum frequency of the neutralizers and therefore reduce the control efficiency. Depending on the viscoelastic material that was used the detuning may be large (butyl rubber) or small (neoprene). This is due to fact that for neoprene, the transition region is at very low temperatures and therefore, the influence of temperature changes on the efficiency of control was less than for butyl rubber. In both cases,

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a large value of $f_0 = 10000$ causes that the system loses stability and presents the typical jump phenomena.

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