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WEIGHTS AND TRANSFORMATION PARAMETERS IN THE DATUM DEFINITION OF A GEODETIC NETWORK

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Abstract. A geodetic network is a set of physical points that provides parameters which allows us to know the shape and size of the Earth. The estimation of the unknown parameters of the geodetic network based in observations is an inverse problem which can be solved using a principle of optimization in an adjustment model.

The datum problem always arises in the adjustment model, when the unknown parameters of the geodetic network are coordinates which cannot be determined from the available observations.

Hence, the datum problem must be solved in an arbitrary and reasonable way by introducing additional information not contained in the observations.

It can be done by specifying in the adjustment model, the set of all conventions, algorithms and constants necessaries that define and realize the origin, orientation, scale and their time evolution of the Geodetic Reference System (GRS) where the coordinates are expressed, in such a way that these attributes be accessible to the users through occupation, direct or indirect observation.

It is developed here, within a Singular Gauss-Markov Model (SGMM) for the adjustment of a two-dimensional trilateration network using a coordinate based formulations, the general form of three linear conditions equations namely minimum constraints to define the datum of the geodetic network based in: a) a known “a priori” Terrestrial Reference Frame TRF (xo,yo), b) a positive definite weight matrix for the unknown parameters, which can be constructed using information about the accuracy of the TRF(xo,yo) and c) three parameters of a plane coordinate Helmert transformation : two translation and one differential rotation.

1 INTRODUCTION

A geodetic network is a set of physical points that provides parameters which allows us to know the shape and size of the Earth. The estimation of the unknown parameters of the geodetic network based in observations is an inverse problem which can be solved using a principle of optimization in an adjustment model.

The datum problem always arises in the adjustment model of a geodetic network, when the unknown parameters are coordinates which cannot be determined from the available observations, because they do not carry all the necessary information to completely realize the origin, orientation and scale of the Geodetic Reference System (GRS) where the coordinates are expressed.

Hence, the datum problem must be solved in an arbitrary and reasonable way by introducing additional information not contained in the observations. (Dermanis, A.,1998).

It can be done by specifying in the adjustment model, the set of all conventions, algorithms and constants necessaries that define and realize the origin, orientation, scale and their time evolution of the Geodetic Reference System (GRS), in such a way that these attributes be accessible to the users through occupation, direct or indirect observation.

In this work, we deal with the adjustment of a two-dimensional trilateration network using coordinate based formulations within a Gauss-Markov Model (GMM), where the point positions are defined by means of coordinates (x,y) in a GRS which is a local Terrestrial Reference Cartesian Coordinate System TRS(x,y).

The TRS(x,y) is a trirectangular trihedron right-handed oriented, its vertex is a point **P** not specified of the Earth's surface and is the origin **o** of the Cartesian coordinate system (x,y), the first and second rays are the **ox** and **oy** positive axis respectively with not specified orientations. The third ray is oriented "upward" aligned with the vertical in **P** and is orthogonal to the others two rays. The scale or length defined of the unit vectors along **ox** and **oy** of the TRS(x,y) is the meter (SI), and it is realized by the observed distances of the trilateration network.

The lack of definition in the origin and orientation of the TRS(x,y) cause a datum defect and a rank-deficient Singular Gauss-Markov Model (SGMM).

To introduce in the adjustment model a definition and a realization of the origin and orientation of the TRS(x,y) it is developed within a Singular Gauss-Markov Model (SGMM) for the adjustment of a two-dimensional trilateration network using a coordinate based formulations, the general form of three linear conditions equations namely minimum constraints based in: a) a known "a priori" Terrestrial Reference Frame TRF (xo,yo), b) a positive definite weight matrix for the unknown parameters, which can be constructed using information about the accuracy of the TRF(xo,yo) and c) three parameters of a plane coordinate Helmert transformation : two translation and one differential rotation.

2 WEIGHTS AND TRANSFORMATION PARAMETERS IN THE DATUM DEFINITION

Following Vacaflor, J.L. (2010), let us consider a free geodetic network constituted by "k" physical points P_i with coordinates (x_i, y_i), $i = 1 \ldots k$ in the $TRS(x, y)$, and related through "n" observed distances, and *not* being *defined* for any epoch, the position and orientation of the $TRS(x, y)$.

Moreover, let us consider known the coordinates (x_i^0, y_i^0), $i = 1 \ldots k$ "a priori" or "approximated" from the reference frame $TRF(x_0, y_0)$.

The lack of definition in the origin and orientation of the TRS(x,y) cause a datum defect and a rank-deficient Singular Gauss-Markov Model (SGMM) for the adjustment of the network (Schaffrin,1985) :

$$y - e = A\xi \quad , r(A) =: q < m < n \ , \ d =: 3 = m - q \, , \ e \sim (0, \sigma_0^2 P^{-1} =: D\{y\}) \qquad (1)$$

with,

$n =$ Number of observations; $m =$ Number of unknown parameters.
$r =$ Rank; $d =$ Number of datum defect; $D =$ Dispersion ; $P_{nxn} =$ Symmetric positive-definite weight matrix ; $o =$ Order ; $\sigma_0^2 =$ Unknown (observational) variance component; E= Expectation.

$y_{nx1} =$ Vector of observations (increments)

$$y_{nx1} = [y_{ij}] = [(s_{12}^{obs} - s_{12}^0), (s_{13}^{obs} - s_{13}^0), ..., (s_{ij}^{obs} - s_{ij}^0), ..., (s_{k-1,k}^{obs} - s_{k-1,k}^{o})]^T$$

$e_{nx1} = [e_{ij}] =$ Vector of random errors (unknown)

$$E\{e_{nx1}\} = 0$$

$$s_{ij}^0 = \sqrt{(\Delta x_{ij}^0)^2 + (\Delta y_{ij}^0)^2} \, , \ i = 1..k \ , \ j = 1..k \ , \ i < j$$

$$\Delta x_{ij}^0 = x_j^0 - x_i^0 \ ; \ \Delta y_{ij}^0 = y_j^0 - y_i^0$$

$A_{nxm} =$ Design or coefficient matrix (“Jacobian”)

$$A_{nxm} = \begin{bmatrix} \alpha_{12} \\ \cdots \\ \alpha_{ij} \\ \cdots \\ \alpha_{k-1,k} \end{bmatrix} \ ; \ \alpha_{ij_{1xm}} = [0, \ldots, -\Delta x_{ij}^0, -\Delta y_{ij}^0, \ldots, \Delta x_{ij}^0, \Delta y_{ij}^0, \ldots, 0].(1/s_{ij}^0)$$

$\xi_{mx1} =$ Vector of unknown parameters (coordinate increments).

$$\xi_{mx1} = X_{mx1} - X_{mx1}^0$$

$X_{mx1} =$ Vector of unknown coordinates of the points P_i of the TRF(x,y) expressed in the $TRS(x, y)$.

$$X_{mx1} = [x_1, y_1 ... \, x_k, y_k]^T$$

$X_{mx1}^0 =$ Vector of known coordinates of P_i of the “a priori” or “approximated” $TRF(x_0, y_0)$.

$$X_{mx1}^0 = [x_1^0, y_1^0 ... \, x_k^0, y_k^0]^T$$

$$\xi_{mx1} = [dx_1 \quad dy_1 \quad \ldots \quad dx_k \quad dy_k]^T ; \ dx_i = x_i - x_i^0 \ ; \ dy_i = y_i - y_i^0 \, , \ i = 1..k \, , m = 2k$$

To complete the datum definition of the network in (1), it is necessary to introduce as minimum three independent condition equations to *define* and *realize* in a given epoch the origin and orientation of the $TRS(x, y)$.

In Vacaflor, J.L. (2010), it was shown that this goal can be reached, if in (1), is introduced a minimum datum constraints which arises from the general form:

$$K_{3xm}\xi_{mx1} = K_{3xm}E^T_{mx3}PT^*_{3x1} \quad , \; o(K) = 3xm \; , \; r(K) = 3 \tag{2}$$

$$R(A^T) \oplus R(K^T) = \Re^m$$

$$AE^T = 0 \; , \; o(E) = dxm \; , \; r(E) = d$$

$$\Rightarrow R(A^T) \overset{\perp}{\oplus} R(E^T) = \Re^m$$

with,

$$\xi = E^T PT^* \tag{3}$$

$$PT^* = [t^*_x, t^*_y, d\delta^*]^T \tag{4}$$

$PT^* =$ Conventionally adopted numerical values of the transformation parameters: two translations t^*_x, t^*_y and one differential rotation $d\delta^*$ (of the $TRS(x, y)$) to *define* the position and orientation of the $TRS(x, y)$.

$$E_{3xm} = \begin{bmatrix} 1 & 0 & \ldots & 1 & 0 \\ 0 & 1 & \ldots & 0 & 1 \\ -y^0_1 & x^0_1 & \ldots & -y^0_k & x^0_k \end{bmatrix} \tag{5}$$

When,

$$K := ES \quad , o(S) = mxm \; , \; r = r(S) \geq m - q \; , \; m = 2k \tag{6}$$

$$S := Diag(s_{dx_1}, s_{dy_1}, \ldots, s_{dx_i}, s_{dy_i}, \ldots s_{dx_k}, s_{dy_k}) \, , \; i = 1 \ldots k \tag{7}$$

S: Selection matrix of coordinate differences

Introducing (6) in (2), leads to:

$$E_{3xm}S_{mxm}\xi_{mx1} = E_{3xm}S_{mxm}E^T_{mx3}PT^*_{3x1} \tag{8}$$

By assigning of the values “0” or “1” respectively to the diagonal elements of S, it is selected or excluded in the first member of (8) the coordinate differences $dx_i = x_i - x^0_i$, $dy_i = y_i - y^0_i$, $i = 1 \ldots k$ and their corresponding numerical values are provide in the second member of (8) respectively. For example, if $s_{dx_1} = 1 \Rightarrow dx_1 = x_1 - x^0_1$ is selected, and if $s_{dx_1} = 0 \Rightarrow dx_1 = x_1 - x^0_1$ is exclude.

Once the structure of S is established, the minimum constraints (8) *define* and *realize* for a given epoch the origin and orientation of the $TRS(x, y)$.

Now, rearranging the order of the unknown parameter vector (coordinate differences) if necessary, and letting lowercase **s** represents the number of selected coordinate differences, the selection matrix can be written as (Snow, K.B., 2002, p.10):

$$S_{mxm} := \begin{bmatrix} I_s & 0 \\ 0 & 0 \end{bmatrix} \quad , \quad s \geq m - q \tag{9}$$

The sub-matrix I_s in (9) can be replaced by a weight matrix representing the weights of the selected coordinate differences.

Let us introduce the matrix P_0, a positive definite weight matrix of the unknown parameters ξ_{mx1}. It is equal to the weight matrix of X^0_{mx1} designated as P^0, since $\xi_{mx1} = X_{mx1} - X^0_{mx1}$, where X_{mx1} is a fixed (non random) vector of unknown coordinates of the points P_i.
Then,

$$P_0 = P^0_{mxm} \tag{10}$$

Hence, SP_0S would contain in lieu of I_s, the respective submatrix of the matrix P_0 corresponding to the selected unknown parameters and hence reduced in size sxs.
Therefore, the new class of minimum datum constraints which arises from (2) when:

$$K := ESP_0S \quad , \ o(S) = mxm \ , \ r = r(S) \geq m - q \ , \ m = 2k \tag{11}$$
$$o(P_0) = mxm, \ r(P_0) = m$$

is introduced in (2), is:

$$E_{3xm} S_{mxm} P_{0_{mxm}} S_{mxm} \xi_{mx1} = E_{3xm} S_{mxm} P_{0_{mxm}} S_{mxm} E^T_{mx3} PT^*_{3x1} \tag{12}$$

The first member of (12) contains the selected weighted coordinate differences, $p_{dx_i} dx_i = p_{dx_i}(x_i - x_i^0)$, $p_{dy_i} dy_i = p_{dy_i}(y_i - y_i^0)$, $i = 1...k$ and their corresponding numerical values are provide in the second member of (12) respectively.

Hence, from (12) can be determined the coordinates of the datum points involved in the realization of the origin and orientation of the TRS(x,y).

Summarizing, once the structure of S is established, the minimum constraints (12) define the datum of the geodetic network of trilateration:

* Define for a given epoch, the position and orientation of the $TRS(x, y)$ by assigning conventionally numerical values to the transformation parameters PT^*_{3x1}.
* Realize the position and orientation of the $TRS(x, y)$ by means of a numerical evaluation of the selected weighted coordinate differences $p_{dx_i} dx_i = p_{dx_i}(x_i - x_i^0)$, $p_{dy_i} dy_i = p_{dy_i}(y_i - y_i^0)$, $i = 1...k$.

3 CONCLUSIONS

In this work, it is developed within an stochastic linear model for the adjustment of a two-dimensional free trilateration network of the type SGMM with datum defect, three linear condition equations to the unknown parameters (increments or coordinate differences) namely minimum constraints which define the datum of the network in a given epoch regarding to the position and orientation of the TRS(x,y).

These minimum constraints are based in: a) a known "a priori" Terrestrial Reference Frame TRF (xo,yo), b) a positive definite weight matrix P_0 of the unknown parameters ξ_{mx1} which is equal to the weight matrix P^0 of the vector of known coordinates X^0_{mx1} of P_i of the "a priori" $TRF(x_0, y_0)$ and c) three parameters of a plane coordinate Helmert transformation : two translation and one differential rotation.

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