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MESO- AND MACROSCOPIC MODELS TO SIMULATE THE MECHANICAL BEHAVIOR OF FIBER REINFORCED CONCRETE COMPOSITES

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Abstract. Brittle or quasi-brittle materials such as concrete or rocks typically present localized failure modes due to cracking processes which usually start from internal material defects such as micro-cracks or non-homogeneous weak zones. Recently, several studies have been carried out on the fracture mechanical behavior of concrete materials by taking into account some aspects such as material strength, presence of reinforcing fibers, aggregate size effects, presence of nano-particles, etc.

In this work both plain and fiber reinforced concrete composite (FRCC) are analyzed and modeled with two different approaches. On one hand, a continuum (smeared-crack) formulation based on non-linear microplane theory is proposed. While on the other hand, a discontinuous constitutive theory is formulated to model the cracking response of fiber reinforced mortar-mortar interfaces. The well-known "Mixture Theory" is considered in both models to describe the fiber effects on the failure behavior of FRCC. The interaction between concrete/mortar and steel fibers in terms of fiber debonding and dowel effects are similarly treated in both macroscopic and interface models.

The main objective is to comparatively evaluate the capabilities of the proposed models and numerical tools to capture the significant improvements of post-cracking behavior of fiber-reinforced concrete when different fiber contents and directions are considered. Different amount and type of metallic reinforcing fibers are examined, studying their benefits by bridging cracks and providing resistance to crack opening processes.

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1 GENERAL OVERVIEW

Fiber Reinforced Concrete Composite (FRCC) materials, obtained by mixing steel or synthetic fibers to concrete, is becoming largely used in structural engineering applications. The main benefits, obtainable by means the inclusion of fibers in concrete, are the significant residual strengths in the cracked phase dealing with a more ductile material in post-peak behavior, especially for tensile or shear stress regimes (Carpinteri and Brighenti, 2010).

Several experimental studies have recently been presented by the scientific community leading with the mechanical characterization of FRCCs at both fresh and hardened state. Several aspects such as the workability related to the fiber distributions (Ferrara and Meda, 2006), the fiber orientations in function of the compactation procedure for the specimen fabrication (Gettu et al., 2005), the fiber pull-out mechanisms (Shannag et al., 1997), the flexural behavior tested by means of the three (Buratti et al., 2011) or four (Fraternali et al., 2011) point bending test apparatus, the multiaxial compressive behavior (Fantilli et al., 2011), etc. are well documented into the scientific literature.

Against all the experimental characterization, it is therefore necessary to realistically model the pre- and post-cracking behavior of FRCC at material and structural level. Mechanical modeling and computational formulation approaching the failure processes of concrete composites still remains an open question in the frame of the solid mechanics. Failure behavior governed by the evolution of fracture process can be modeled in many different approaches. Depending on the kinematic description used to model the fracture mechanisms, three main model classes can be recognized (Jirasek and Patzak, 2001):

1. models that admit the presence of strong discontinuity across which the displacement field has a jump. Computational techniques for discrete crack analysis have been based on, a.o., the embedded strong discontinuities (E-FEM) see e.g. Dvorkin et al. (1990) or Oliver (1996), the extended finite element method (X-FEM) by Wells and Sluys (2001), lattice models (van Mier et al., 2002), particle models (Bazant et al., 1990) and zero-thickness interface models (Lopez et al., 2008);

2. models that use a continuous description of the displacement field but admit the presence of weak discontinuities (jumps) of the strain field. In these models the fracture process zone is defined by a finite band of localized strain, then the localization band is considered as a material parameter with a precise physical meaning (Bazant and Oh, 1983; Oliver et al., 1998);

3. models characterized by continuity of both the displacement and strain field. In this ambit, non-local models (Peerlings et al., 2001), strain-gradient plasticity approaches (Vrech and Etse, 2006) or regularized smeared-cracking models have been classical used for enhancing the regularization problem in softening regime.

A proposed fracture-based zero-thickness interface model (Caggiano et al., 2011b,c), belonging to the first model class of the classification, and a continuum microplane formulation under the category 3 of the above classification (Vrech et al., 2010), have been proposed and compared in this work to analyze FRCC probes.

After this Introduction, Section 2 briefly summarizes the proposed numerical models including "Mixture Theory" concepts (Truesdell and Toupin, 1960) and the main assumptions used in each constitutive proposal. Section 3 deals with the model formulations of fibers, in terms of bond-slip and dowel effects, on concrete cracks. Section 4 presents the results of both numerical

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approaches at material level. In Section 5 the meso-structural material geometries and the FE meshes to employ in both the continuous and discontinuous models are presented. Some of the obtained non-linear FE results are given. Further developments are currently ongoing in order to improve the complete FE modeling of fiber-concrete behavior, at both level.

2 MIXTURE THEORY AND PROPOSED MODELS

The constitutive formulations have been founded on the incremental flow theory of plasticity. It depends on the main additive Prandtl-Reuss decomposition of cinematic fields

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p \tag{1}$$

where ε is the total cinematic field, while ε^e and ε^p the elastic and the plastic one, respectively. Based on Eq. (1) and the generalized Hook's law, the following constitutive equation can be founded

$$\dot{\boldsymbol{\varepsilon}}^e = \dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p \Rightarrow \dot{\boldsymbol{\sigma}} = \boldsymbol{E} \odot (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p) \tag{2}$$

in which the symbol $\odot$ deals with a doble contraction : for the microplane model, while with a single contraction $\cdot$ for the interface relationship; $\dot{\boldsymbol{\sigma}}$ and $\boldsymbol{E}$ are the stress rate and the elastic stiffness operators, respectively.

Based on the consistency Kuhn-Tucker approach, the previously relation can be easily transformed in terms of $\dot{\boldsymbol{\sigma}} - \dot{\boldsymbol{\varepsilon}}$ law as follows

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E}^{ep} \odot \dot{\boldsymbol{\varepsilon}} \tag{3}$$

The basis of the mixture theory is used to achieve the constitutive tangent operator

$$\boldsymbol{E}^{ep} = \rho^m \boldsymbol{C}^{ep} + \sum_{f=0}^{n_f} \xi^f \rho^f \left[E_f^{ep} \boldsymbol{B}_f (\boldsymbol{n_N}, \boldsymbol{n}_T) + G_f^{ep} \boldsymbol{D}_f (\boldsymbol{n_N}, \boldsymbol{n}_T) \right] \tag{4}$$

where the quantities $\rho^{\#}$ deal with the volumetric fraction of each mixture component, in which m represents the matrix and f the fibers, $\boldsymbol{n}_N$ and $\boldsymbol{n}_T$ are two unit vectors identifying the parallel and orthogonal directions of each fiber respect to the global Cartesian reference system, ξ^f is a reductive coefficient given in Caggiano et al. (2011b) and n_f the considered number of fibers. Finally, $\boldsymbol{B}_f$ and $\boldsymbol{D}_f$ are directional operators based on the normal and tangential direction of a single fiber.

The material (singular) operators of the plain concrete matrix $\boldsymbol{C}^{ep}$, the fiber pull-out characterization E_f^{ep} and the dowel modeling G_f^{ep} mainly depend on the constitutive formulations given in Sections 2.1, 2.2 and 3.

2.1 Continuous Microplane Model

The proposed macroscopic model is based on the bidimensional microplane theory, according to the original proposal by Park and Kim (2003). This constitutive theory considers 2D stress and strain fields, see Etse et al. (2010).

The microplane constitutive law is based on the mixture theory by Truesdell and Toupin (1960) with the hypothesis of the composite model by Oliver et al. (2008). The yield condition in hardening/softening regime and the non-associated flow are defined by the following unified equations

$$F(\boldsymbol{t}, \kappa) = \alpha|\boldsymbol{\sigma}_T| + \sigma_N - (f'_t - K(\kappa)) = 0 \quad , \quad \alpha = \frac{f'_t}{\tau'_y} \tag{5}$$

$$Q(\boldsymbol{t}, \kappa) = \alpha|\boldsymbol{\sigma}_T| + \eta\sigma_N - (f'_t - K(\kappa)) = 0 \tag{6}$$

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being f'_y and τ'_y the tensile and shear strength, respectively and η the non-associativity parameter. The constitutive modeling, linearly depends on the normal σ_N, and tangential σ_T stresses. The evolution of the internal variable is defined in terms of the plastic parameter rate $\dot{\lambda}$ as

$$\dot{\kappa} = \frac{\partial F}{\partial K} = \dot{\lambda} \tag{7}$$

The evolution of the dissipative stress in the post-peak regime is due to micro-fracture processes at the microplane level. $\dot{K}$ is defined through the homogenization process of the fracture energy released in the discontinuous with the plastic dissipation of an equivalent continuum of the same high, similarly to the fracture energy-based plasticity model by Willam et al. (1985) and Etse and Willam (1994), as

$$\dot{K} = f'_t \left[1 - exp \left(-5 \frac{h_t}{u_r} \frac{G_f^I}{G_f^{IIa}} \dot{\varepsilon}_f \right) \right] \tag{8}$$

being $\dot{\varepsilon}_f$ the equivalent fracture strain, defined as

$$\dot{\varepsilon}_f = |\boldsymbol{m}| \dot{\kappa} \tag{9}$$

where h_t represents the characteristic length associated with the active fracture process and, more specifically, the distance or separation between microcracks. Moreover, u_r represents the maximum crack opening displacement in mode I type of failure. G_f^I and G_f^{IIa} are the fracture energies in modes I and II of failure, respectively.

In the special case of uniaxial tension state, the evolution of the dissipative stress can be obtained with the simplified expression

$$\dot{K} = f'_t \left[1 - exp \left(-5 \frac{h_t}{u_r} \dot{\varepsilon}_f \right) \right] \tag{10}$$

The gradient tensor $\boldsymbol{m}$ to the plastic potential can be obtained by a modification of the gradient tensor $\boldsymbol{n}_\sigma$ to the yield surface as

$$\boldsymbol{m} = \begin{bmatrix} m_{\sigma_N} \\ \boldsymbol{m}_{\sigma_T} \end{bmatrix} = \begin{bmatrix} \eta n_{\sigma_N} \\ \boldsymbol{n}_{\sigma_T} \end{bmatrix} \tag{11}$$

2.2 Discontinuous Interface Formulation

The failure behavior of FRCC, at the interface level, is formulated in terms of normal/shear stress components, $\boldsymbol{t}^t = [\sigma, \tau]$, with the corresponding relative displacements, $\boldsymbol{u}^t = [u, v]$. The model is based on the original proposal for plain concrete by Carol et al. (1997), where the three-parameter hyperbola is considered as failure criterion

$$f\left(\boldsymbol{t}^i, \kappa\right) = \sigma_T^2 - (c - \sigma_N \tan\phi)^2 + (c - \chi \tan\phi)^2 \tag{12}$$

being $\boldsymbol{t}^i$ the vector containing the stress of plain interface $[\sigma_N, \sigma_T]$, while the traction strength χ, the cohesion c and the frictional angle $\tan\phi$ are three parameters of the yielding surface, which evolutions in post-elastic regime, only depend on the internal parameter, κ. This latter mainly interprets the ratio between the work spent in fracture (mode I, II or mixed), with the corresponding values of fracture energies, G_f^I and G_f^{IIa}. The plastic flow rule is represented by a

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general non-associated law which controls the direction of cracking displacements, symbolized by $\boldsymbol{m}$, by means of the transformation operator, $\boldsymbol{A}$, applied to the normal direction, $\boldsymbol{n}$.

The "Mixture Theory" (Truesdell and Toupin, 1960) has been employed for modeling the contribution of fibers in cracking processes of FRCC. On the one hand, the bridging effect of fibers under axial stresses is considered, by explicitly taking into account the debonding with the surrounding concrete matrix. Furthermore, their dowel action is simulated as a possible restrain to the development of relative displacements of the two sides of the cracks in the transversal fiber directions. The latter contribution is relevant for steel fiber, but can be usually neglected for plastic reinforcements.

The detailed description of the model, its key features and the macroscopic verification against experimental data on plain concrete are given in Table 1 and well outlined in Caggiano et al. (2011b).

	Fracture − based energy interface model
Constitutive equation	$\dot{\mathbf{t}}^i = \mathbf{C}\cdot(\dot{\mathbf{u}} - \dot{\mathbf{u}}^{cr})$ $\dot{\mathbf{u}} = \dot{\mathbf{u}}^{el} + \dot{\mathbf{u}}^{cr}$
Yield condition	$f(\mathbf{t}^i, \kappa) = \sigma_T^2 - (c - \sigma_N \tan\phi)^2 + (c - \chi \tan\phi)^2$
Flow rule	$\dot{\mathbf{u}}^{cr} = \dot{\lambda}\mathbf{m}$ $\mathbf{m} = \mathbf{A}\cdot\mathbf{n}$
Cracking work evolution	$\dot{\kappa} = \dot{w}_{cr}$ $\dot{w}_{cr} = \sigma_N \cdot u^{cr} + \sigma_T \cdot \dot{v}^{cr} \quad \text{if} \quad \sigma_N \geq 0$ $\dot{w}_{cr} = \sigma_T \cdot \dot{v}^{cr}\left(1 - \frac{\lvert\sigma_N\rvert tan(\phi)}{\sigma_T}\right) \quad if \quad \sigma_N < 0$
Evolution laws	$p_i = [1 - (1 - r_p)\, S[\xi_{p_i}]]\, p_{0i}$
Kuhn − Tucker loading/unloading	$\dot{\lambda} \geq 0, \quad f(\mathbf{t}^i, \kappa) \leq 0, \quad \dot{\lambda}\, f(\mathbf{t}^i, \kappa) = 0$

Table 1: Interface model for plain concrete.

3 FIBER-TO-CONCRETE MODELING

Non-linear constitutive models are employed for the steel fiber characterizations. The bond-slip effect and the dowel mechanism, due to the bridging phenomenon on crack openings, are presented in this section.

3.1 Fiber Bond-Slip

The axial effect of fibers consist in bridging the two sides of cracks. The bond mechanism between fibers and concrete matrix mainly controls this effect. The constitutive model assumes that each generic fiber crosses the fracture line at its mid point, then the local bond stress, τ_a, and the axial fiber stress, σ_f, are related by the simple indefinite equilibrium equation

$$\frac{d\sigma_f[x]}{dx} = -\frac{4\tau_a[x]}{d_f} \tag{13}$$

where a bilinear softening model of contact controls the slipping failure

$$\tau_a[x] = \begin{cases} -k_E s[x] & s[x] \leq s_e \\ -\tau_{y,a} + k_S\,(s[x] - s_e) & s_e < s[x] \leq s_u \\ 0 & s[x] > s_u \end{cases} \tag{14}$$

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being d_f the fiber diameter, $s[x]$ the relative displacement between the fiber and concrete, at the considered abscissa x. The constants k_E and k_S represent the elastic and softening slopes, respectively; $\tau_{y,a}$ is the maximum shear stress and s_e and s_u are the elastic and the ultimate slips. More details have been given in Caggiano et al. (2011a).

3.2 Dowel Strength Model

The dowel constitutive law is based on defining both stiffness and strength of the generic steel fiber embedded in the concrete matrix and subjected to a transversal force/displacement at the fracture level. The well-known Winkler beam theory is used to describe the dowel force-displacement relationship, while an empirical expression proposed by Dulacska (1972) is used as maximum dowel strength, $V_{d,u}$,

$$V_{d,u} = k_{dow} d_f^2 \sqrt{|f_c||\sigma_{y,s}|} \tag{15}$$

where k_{dow} is a non-dimensional coefficient (a typical value is 1.27 as reference for RC-structures), d_f is the fiber diameter, f_c and $\sigma_{y,s}$ are the strengths of concrete and steel, respectively.

4 PLAIN AND FR-CONCRETE RESULTS AT MATERIAL LEVEL

This section presents the main features and capabilities of each model comparing some numerical tests against experimental data at material level of analysis.

4.1 Microplane-based Formulation

To evaluate the capabilities of the proposed macroscopic model based on microplane theory, some tensile tests on SFRC specimens by Li et al. (1998) are considered. The tensile response of fiber reinforced concrete are evaluated by direct uniaxial tension tests using unnotched specimens which dimensions is $350 \times 100 \times 20\, mm^3$.

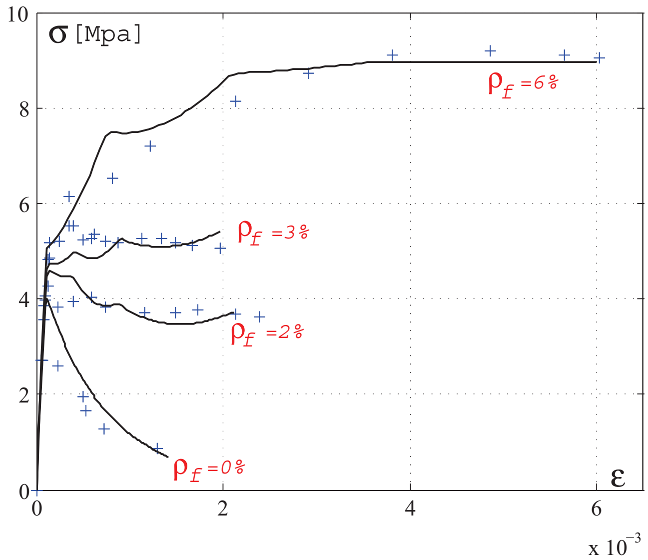

Figure 1: Tensile test data (dots) by Li et al. (1998) compared against microplane numerical simulations.

Experimental measures and numerical results for different fiber volume fractions are shown in Fig. 1. It is demonstrated that the proposed model for SFRC seems to provide realistic

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predictions of peak stresses, ductility and post-peak behavior of this material when different fiber contents are considered.

4.2 FRCC Interface Model

Numerical predictions carried out adopting the FRCC-interface model for uniaxial tensile tests (Li et al., 1998), but also employed in mixed-mode fracture tests (Nooru-Mohammed, 1992), in both plain and Steel FRC, are presented in this section.

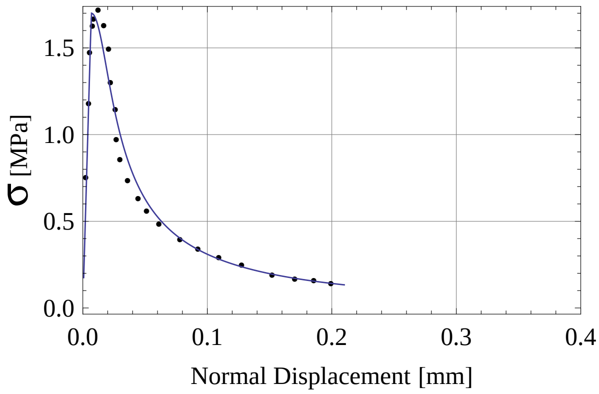

Figure 2: Nooru-Mohammed (1992) tensile test data (dots): simulation of normal stress vs. separation behavior.

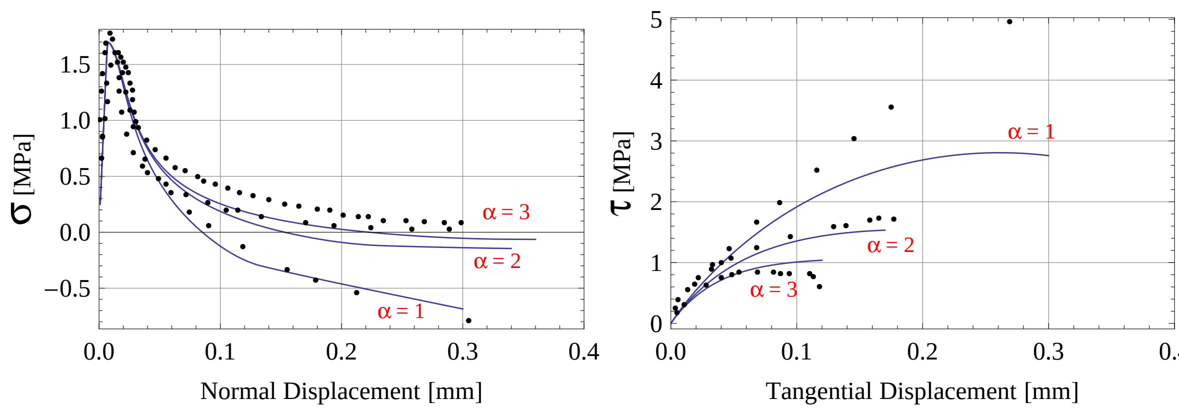

Figure 3: Nooru-Mohammed (1992) mixed-mode fracture (dots): simulation of $\sigma - u$ curve (on left) and $\tau - v$ curve (on right).

Nooru-Mohammed (1992) tests, characterized by concrete specimen of $200 \times 200 \times 50\, mm^3$ having a central notch with length and width of 25 and $5\, mm$, are used in these analyses. In particular, Fig. 2 depicts the comparison, in terms of normal stress versus displacement, of a traction test (dots) against the numerical results (line). Also, mixed-fracture tests, in which tangential (v) and normal (u) displacements are simultaneously applied with a fixed ratio ($\alpha = u/v$) to the pre-notched specimen, are reported in Fig. 3 where the comparisons of experimental vs. numerical data are given. Good agreement between experiments and numerical results can be achieved through the proposed interface model.

The proposed interface model for FRCC will be also calibrated by using experimental results performed on Steel Fiber Reinforce Concrete (SFRC) specimens tested in pure traction. The tensile tests on SFRC specimens by Li et al. (1998), also used for validating the microplane modeling of Section 4.1, are considered.

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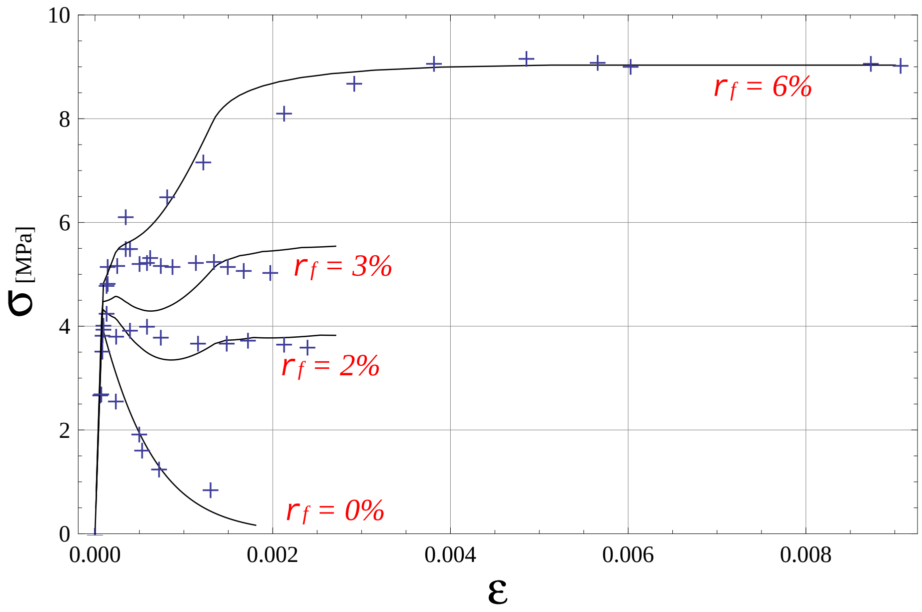

Figure 4: Experimental data (dots) by Li et al. (1998) and interface numerical simulation (lines) under uniaxial tension.

Fig. 4 shows the experimental measures against the numerical predictions employing this model, in which four volume contents of fibers are considered ($\rho_f = 0, 2, 3$ and 6%, respectively). The results show as the proposed interface model, adopting the internal parameters indicated in Caggiano et al. (2011b), leads to accurate predictions of SFRC failure behavior in direct tensile tests when several fiber contents are considered.

5 MESO-STRUCTURAL GEOMETRIES AND ANALYSES

This section outlines the key features of the generation of general 2-D composite geometries and the subsequent FE discretization at meso-mechanical scale. Some numerical tests compared with experimental results are also presented in this section.

5.1 Generation of a random meso-structure and meshes

A convex polygonal representation based on the classical Voronoi diagrams (Dirichlet, 1850) is adopted for representing the coarse aggregates present into the concrete composite. The polygonal geometry is numerically generated starting from a regular array of points, which are slightly perturbed as shown in the top of Fig. 5. Based on this set of perturbed points, the Voronoy diagrams have been generated. The concrete meso-structure is just obtained by scaling and randomly roting these polygons.

Both the polygonal particles and the surrounding matrix are meshed with finite elements. Each continuum element is assumed as linear elastic, while all nonlinearities are concentrated within zero-thickness interface elements which have been a priori inserted into the FE mesh modeling all the potential crack paths to be considered in the analysis.

Non-linear fracture-based joint law (Section 2.2) and fiber actions (in terms of both bridging and dowel effects) outlined in Section 3, are introduced in those interface elements. In particular, aggregate-matrix interfaces do not consider the fiber effects, while matrix-matrix ones take into account the contribution of fiber reinforcements crossing the interface.

In conclusion, the overall process to generate a numerical model suitable for FE mesoscale

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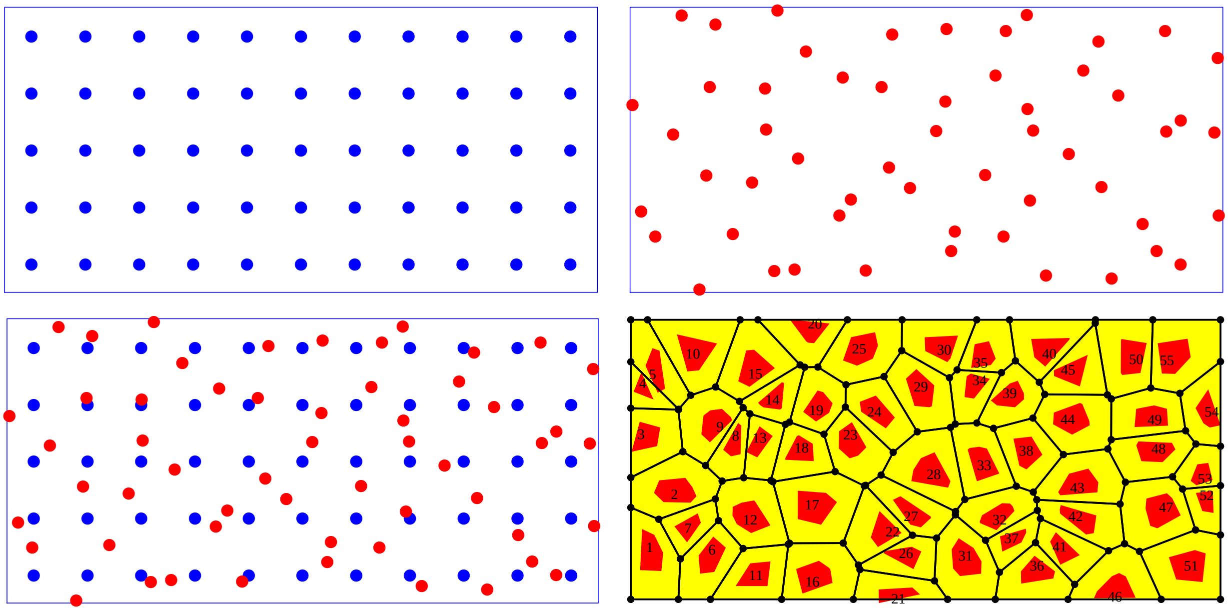

Figure 5: (a) Initial regular point distributions, (b) randomly perturbed positions, (c) superposition and (d) Voronoi/Delaunay tessellation (Klein, 1989).

analyses follows the following two main steps:

1. generating the basic meso-geometry of the composite material;
2. defining the FE-mesh of the meso-geometry with the introduction of the zero-thickness interface elements. Fig. 6 shows a typical $2-D$ meso-structure of a FRCC specimen.

5.2 Further numerical examples

The experimental data by Carpinteri and Brighenti (2010) on two type of experimental tests (i.e., plain concrete and $\rho_f = 0.5\%$ SFRC) are considered in this section for validating the numerical proposal. Particularly, the simulation of $10 \times 40\, cm^2$ pre-cracked concrete specimens tested under three-point bending have been treated.

Good accuracy of the simulated test results for all the specimens subjected to the 3-point bending conditions has been obtained as show in Fig. 8. The FE meso-mechanical response expressed in terms of force-deflection curves, previously calibrated in Caggiano et al. (2011a), leads to accurate predictions of SFRC failure behavior loaded under three point bending conditions.

Also, the final crack path of the proposed mesoscopic formulation, compared to the experiments, is quite realistic. In fact, in both experimental (Carpinteri and Brighenti, 2010) and numerical analyses, the failure process led to only one macro-crack, starting at the top of the notch which evolves in an almost vertical mode, especially throughout the aggregate-to-matrix interfaces, representing the weakest link in concrete composites.

6 CONCLUSIONS

Two models have been developed for failure analyses of FRCCs. On the one hand a continuum-based approach and microplane consideration at material level is proposed. On the other hand a discrete approach and interface model is presented. Both models consider the well-known "Mixture Theory" to simulate the combined bridging action of fibers in concrete/mortar cracks.

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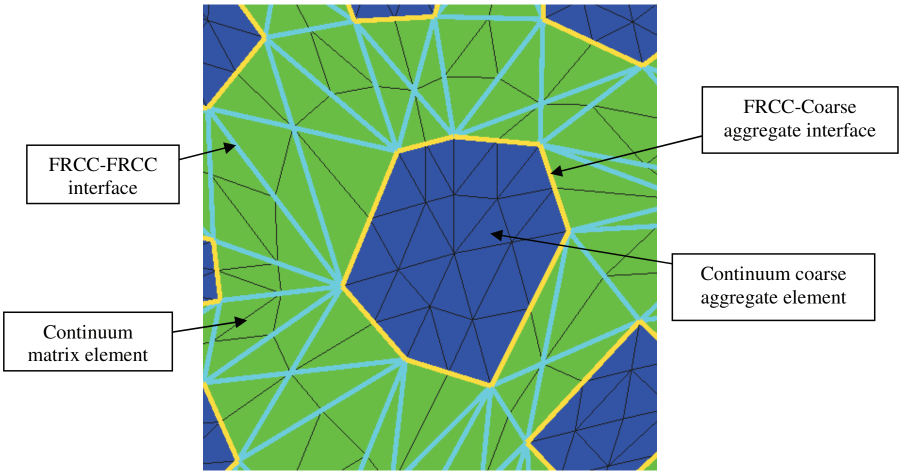

Figure 6: Detail of the FE discretization: matrix, coarse aggregates and interfaces between them.

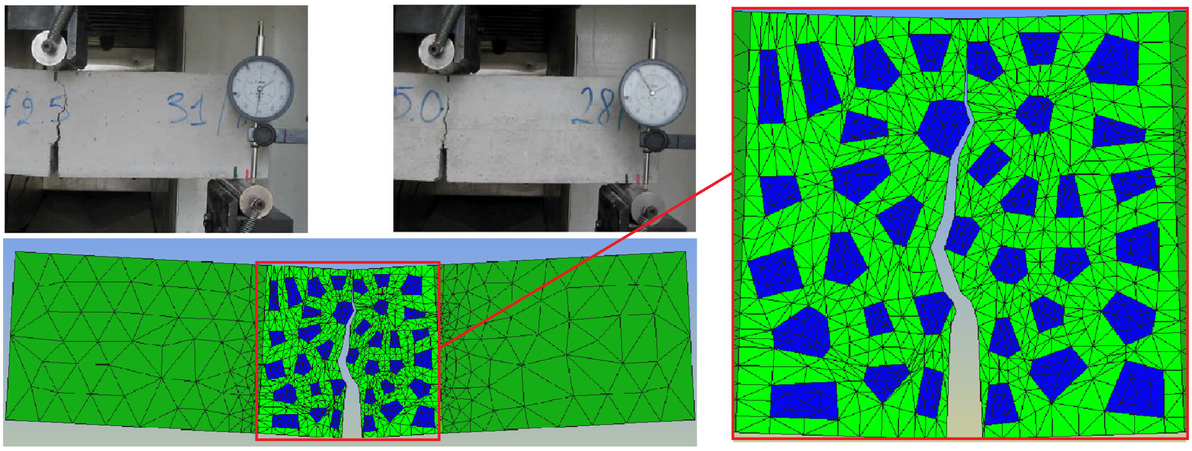

Figure 7: Carpinteri and Brighenti (2010) test configuration at failure against numerical results.

Continuum model uses normal and tangential strain decomposition at microplanes. Discontinuous interface approach is particularly suitable for mesoscale failure analyses. Both constitutive formulations have been implemented in a “in-home” finite element code. The numerical simulations, presented in this work, may conclude that the constitutive proposals mainly capture the fundamental behaviors of fibrous concretes. Very good agreement with the experimental data in terms of peak strength and post-crack predictions are achieved.

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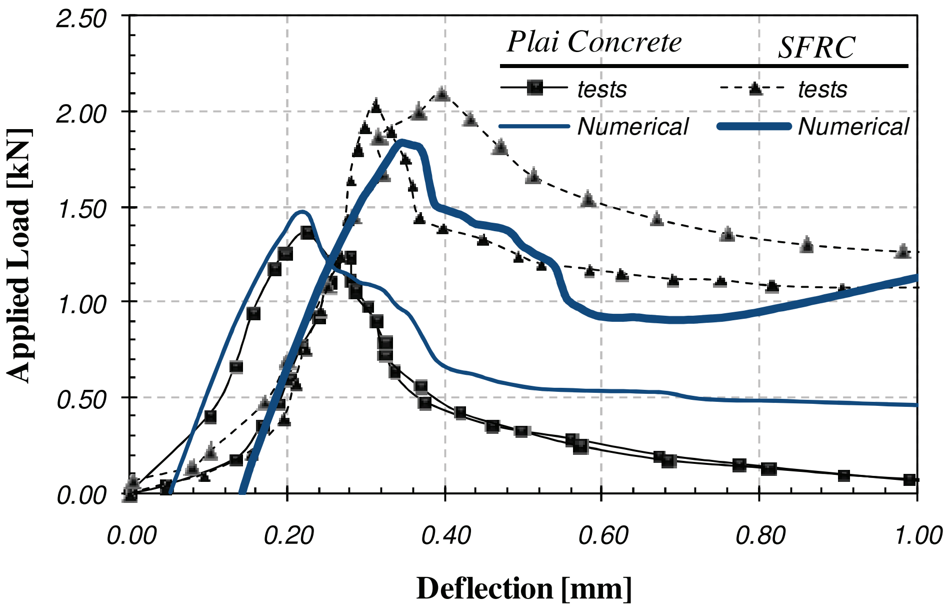

Figure 8: Carpinteri and Brighenti (2010) test configuration at failure against numerical results.

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