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# OPTIMIZATION OF GEOMETRICALLY NONLINEAR PLANAR FRAMES STRUCTURES USING GENETIC ALGORITHMS

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**Abstract.** The problem of choosing the sizes of the members in order to minimize the weight of structures while satisfying stress, displacement, stability, and other applicable constraints is complicated by the requirement of considering the non-linear structural behavior. The problem is further complicated if the members are to be chosen from a discrete set of commercially available sizes, which is often the case. Genetic Algorithms, inspired by Darwin's theory of evolution by natural selection, are powerful and versatile tools in difficult search and optimization problems. In this paper a genetic algorithm is proposed to find the optimum discrete and continuous values of the cross-sectional areas of the members that minimize the weight of planar frame structures presenting geometrically non-linear behavior. Several computational experiments are discussed involving these structures.

## 1 INTRODUCTION

Several types of structures, naturally, present a geometrically nonlinear behaviour and it is very common such as the dome and space and planar frames, for example, particularly when they are subjected to certain kind of loads. Sometimes, these structures may present instability to be avoided choosing the adequate sizing of cross-sectional areas, topology, supports, etc.

When the optimum configuration of these structures is searched by using algorithms of optimization the designer should be aware in considering the effects of geometrically nonlinear behaviour.

To define the optimum structural configuration, preliminary engineering analysis is usually made of a few conventional possibilities, pre-defined by the architect, and the engineer only chooses the sizes of the members to satisfy the applicable codes, considering economic aspects. The normal and buckling stresses arising from the axial forces in the members, and the displacements at the nodes, are the values that affect the sizes of the members and the final cost of the structure. For this kind of structure it is interesting to carry out a nonlinear analysis to obtain the axial forces and displacements.

Comparing the optimization using a linear analysis against a nonlinear analysis it is possible, sometimes, to reach lighter weight considering the nonlinear case. Also, a nonlinear analysis requires the interaction between axial forces and bending moments in members with a high slenderness coefficient in order to check the stability of the structure Saka and Ulker (1991); Ebenau et al. (2005); Kameshki and Saka (2007). If the structure demands the nonlinear analysis the designer needs to adopt an structural optimization considering this behavior to define the optimal design.

In Saka and Kameshki (1997) an algorithm is presented for the optimum design of three dimensional rigidly jointed frames which takes into account the nonlinear response due to the effect of axial forces in members. The stability functions for three-dimensional beam-columns are used to obtain the nonlinear response of the frame. The problem of maximization of the critical load or limit point of instability of shallow space trusses of constant volume is presented in Kamat et al. (1984). In Pyrz (1990) a discrete optimization of trusses considering stability constraints is discussed presenting examples of shallow truss structures when snap-through can occur. Benchmark case studies in optimization of geometrically nonlinear structures can be found in Suleman and Sedaghati (2005) where a structural optimization algorithm is developed for truss and beam structures undergoing large deflections against instability.

In Saka (2007) an algorithm takes into account the nonlinear response of the dome structure due to effect of axial forces on the flexural stiffness of members and the optimum solution of the design problem is obtained using a coupled genetic algorithm.

A technique for the optimization of stability-constrained geometrically nonlinear shallow trusses with snap-trough behavior is demonstrated using the arc length method and a strain energy density approach with a discrete formulation in Hrinda and Nguyen (2008). In Degertekin et al. (2008) algorithms are presented for the optimum design of geometrically nonlinear steel space frames using tabu search and genetic algorithm.

In this paper, a GA is proposed to minimize the weight of planar frame structures considering both linear static and nonlinear static geometrically analysis. Discrete and continuous design variables are considered corresponding to the sizing of the cross-sectional areas of the members. To solve the nonlinear equilibrium equation of the structure the iterative Newton-Raphson's Method is adopted. The algorithm used in this paper are previously presented in Lemonge et al. (2010) applied to optimization of geometrically nonlinear dome structures.

This paper is organized as follows. The structural optimization problem is presented in Section 2 The geometrically nonlinear approach is summarized in Section 3 The genetic algorithm and the constraint handling technique are discussed in Section 4 Numerical experiments are presented in Section 5 Finally, conclusions are presented in Section 6.

## 2 THE STRUCTURAL OPTIMIZATION PROBLEM

For a given objective function $f(x)$, where $x \in R^n$ is the vector of design/decision variables, a standard structural sizing optimization problem reads: Find the set of areas $x = \{A_1, A_2, \ldots, A_N\}$ which minimizes the volume of the structure

$$f(x) = \sum_{i=1}^{N} A_i l_i, \tag{1}$$

where $l_i$ is the length of the $i$-th member of the frame and $N$ is the number of members. When shape design variables are considered the values of $l_i$ change and the weight depends not only on the values of $A_i$, but also on the joint coordinates of the structure.

The problem is usually subject to inequality constraints $g_p(x) \geq 0$, $p = 1, 2, \ldots, \bar{p}$ and sometimes equality constraints $h_q(x) = 0$, $q = 1, 2, \ldots, \bar{q}$. Also, the variables are usually subject to bounds $x_i^L \leq x_i \leq x_i^U$ but this type of constraint is trivially enforced in a GA and does not require further consideration here. The most common constraints are displacements constraints:

$$\frac{|d_j|}{d_{max}} - 1 \leq 0, \quad k = 1, 2, \ldots, p_d \tag{2}$$

where $d_j$ is the displacement at the $j$-th global degree of freedom, $d_{max}$ is the maximum allowable displacement, and $p_\sigma + p_d = \bar{p}$. Additional constraints such as a minimum natural vibration frequency or more realistic buckling stress limits can also be included.

## 3 GEOMETRICALLY NONLINEAR APPROACH

Although the material of the structures discussed in this paper present a linear elastic behavior, geometrical non-linearity needs to be considered in the analysis. In order to provide an exact structural analysis, the equilibrium equation in each joint of the structure must be written considering the final geometry of the structure. In these equations nonlinear terms involving strain and displacement must be considered and the overall equilibrium equation can be written as:

$$[K_T]\{u\} = \{P^*\} \tag{3}$$

where

$$[K_T] = [K_E] + [K_G] \tag{4}$$

and $[K_T]$ is called overall tangent stiffness matrix of the structure, $[K_E]$ is known as the overall linear elastic stiffness matrix and $[K_G]$ is the geometric stiffness matrix. The matrix $[K_G]$ depends on the elastic and geometric stiffness matrix and $\{P^*\}$ is the vector of unbalanced load. To solve the equation (4) an iterative scheme is required and here the Newton-Raphson's Method is adopted. Newtons's Method is summarized as follow:

1. Perform the linear analysis of the structure and obtain the displacements for the first load step;

2. Update the joint coordinates of the structure considering the displacements obtained in the previous step;
3. Evaluate the internal member actions;
4. Evaluate, at each node, the resultant of the internal member actions in the global axes;
5. Evaluate the unbalanced load vector that is the difference between the member actions obtained in the previous step and the load applied at the structure in this load step;
6. Assemble the stiffness matrix and check its determinant. The loss of stability is identified when a element of the main diagonal of the system of equations has a value least or equal to zero. On the other hand, the structure is analyzed considering the unbalanced load vector and new incremental displacements are obtained;
7. Update the node coordinates;
8. Repeat steps 3 to 7 until the unbalanced vector satisfies the error tolerance.

It is important to note that in the nonlinear procedure used in this paper when the loss of stability occurs the current displacements and internal actions in the members are amplified by a factor of 100. This candidate solution is strongly penalized by the penalty scheme and consequently has a low position in the rank of the population.

## 4 THE GA AND THE PENALTY SCHEME

A *rank-based* selection scheme is adopted in the binary-coded GA used here where the selection scheme operates on the current population sorted according to the values of the fitness function where better solutions have higher rank. The candidate solutions presenting lower fitness values will have a higher rank considering weight minimization problems discussed in this paper. A form of elitism is used where the best individual is always copied into the next generation along with one copy where one randomly chosen bit has been changed. The recombination of the genetic material of the selected “parent" chromosomes uses the standard uniform crossover operator applied with probability equal to $p_{cross} = 0.8$. A mutation operator is introduced with a mutation rate $p_m = 0.03$ applied to each bit in the offspring chromosomes. The whole process is repeated for a given number of generations or until certain stopping criteria are met. A pseudo-code of the binary GA used here is displayed in the Figure 1.

The adaptive penalty method introduced by Barbosa and Lemonge (2002), and applied to structural optimization problems in Lemonge and Barbosa (2004), will be applied here to enforce all the mechanical constraints considered in the numerical experiments (stresses and displacements). Defining the amount of violation of the $j$-th constraint by the candidate solution $x$ as

$$v_j(x) = \begin{cases} |h_j(x)|, & \text{for an equality constraint,} \\ \max\{0, -g_j(x)\} & \text{otherwise} \end{cases}$$

it is common to design penalty functions that grow with the vector of violations $v(x) \in R^{\mathcal{M}}$ where $\mathcal{M} = \bar{p} + \bar{q}$ is the number of constraints to be penalized. The fitness function is defined as Barbosa and Lemonge (2002)

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible,} \\ \overline{f}(x) + \sum_{j=1}^{\mathcal{M}} k_j v_j(x) & \text{otherwise} \end{cases} \quad (5)$$

**Algorithm generational GA**
  Initialize the population P
  Evaluate individuals in population P using a linear or nonlinear analysis
    **repeat**
      Copy elite to P'
      **repeat**
        Select 2 or more individuals in P
        Apply a recombination operator with probability $p_c$
        Apply mutation operator with rate $p_m$
        Insert new individuals in P'
      **until** population P' complete
      Evaluate individuals in population P' using a linear or nonlinear analysis
      P $\leftarrow$ P'
    **until** stopping criteria are met
**end**

Figure 1: A standard binary encoded genetic algorithm.

where

$$\overline{f}(x) = \begin{cases} f(x), & \text{if } f(x) > \langle f(x) \rangle, \\ \langle f(x) \rangle & \text{otherwise} \end{cases} \tag{6}$$

and $\langle f(x) \rangle$ is the average of the objective function values in the current population.

The penalty parameter is defined at each *generation* by:

$$k_j = |\langle f(x) \rangle| \frac{\langle v_j(x) \rangle}{\sum_{l=1}^{\mathcal{M}} [\langle v_l(x) \rangle]^2} \tag{7}$$

and $\langle v_l(x) \rangle$ is the violation of the $l$-th constraint averaged over the current population.

## 5 NUMERICAL EXPERIMENTS

Three experiments are discussed in this section: a 3-member, 5-member and 6-member planar frame. For each one of them, the discrete and continuous as well as the linear and nonlinear cases are analyzed where the design variables are the dimensions of the cross-sectional areas of the members, adopted here as rectangles. The members members were discretized by using six finite elements and for each experiment they are linked in one and two member groups in independent analysis.

For the discrete cases the dimensions of the sides of the cross-sectional areas are to be chosen from 128 values defined in Table 1. For the continuous case the bounds for the design variables are 0.5 in $\leq D_1, D_3 \leq$ 3.0 in (orthogonal dimensions to the plane of the frame) and 5.0 in $\leq D_2, D_4 \leq$ 30.0 in (dimensions in the plane of the frame). The Young's modulus is equal to E = 3.0 $\times$ 10 $^7$ Psi and density of 0.10 lb$/in^3$. The constraints consider the maximum displacement of any node of the frame, in any direction, equal to 0.29 in.

When the members are linked in one group only the dimensions $D_1$ and $D_2$ are used for all members. Whereas, the members are linked in two groups, the vertical members have the same dimensions of the cross-sectional areas ($D_1$ and $D_2$) and the horizontal members have the same dimensions of the cross-sectional areas ($D_3$ and $D_4$), respectively. In other words $D_1$ and $D_3$ are the width of members and $D_2$ e $D_4$ the heights of the members, respectively.

For all experiments, four independent runs were performed considering 100 individuals in the population evolved for 60 generations. The number of bits was set equal to 30 for each design variable for the continuous case. When the nonlinear analysis is performed, ten load steps and ten iterations, per load step, were adopted in the iterative Newton-Raphson's Method.

Table 1: 128 discrete values for the dimensions of the cross-sectional areas of the members.

| Dimension | in | Dimension | in | Dimension | in | Dimension | in |
|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 33 | 6.9 | 65 | 13.3 | 97 | 20.1 |
| 2 | 0.7 | 34 | 7.1 | 66 | 13.5 | 98 | 20.5 |
| 3 | 0.9 | 35 | 7.3 | 67 | 13.7 | 99 | 20.9 |
| 4 | 1.1 | 36 | 7.5 | 68 | 13.9 | 100 | 21.3 |
| 5 | 1.3 | 37 | 7.7 | 69 | 14.1 | 101 | 21.7 |
| 6 | 1.5 | 38 | 7.9 | 70 | 14.3 | 102 | 22.1 |
| 7 | 1.7 | 39 | 8.1 | 71 | 14.5 | 103 | 22.5 |
| 8 | 1.9 | 40 | 8.3 | 72 | 14.7 | 104 | 22.9 |
| 9 | 2.1 | 41 | 8.5 | 73 | 14.9 | 105 | 23.3 |
| 10 | 2.3 | 42 | 8.7 | 74 | 15.1 | 106 | 23.7 |
| 11 | 2.5 | 43 | 8.9 | 75 | 15.3 | 107 | 24.1 |
| 12 | 2.7 | 44 | 9.1 | 76 | 15.5 | 108 | 24.5 |
| 13 | 2.9 | 45 | 9.3 | 77 | 15.7 | 109 | 24.9 |
| 14 | 3.1 | 46 | 9.5 | 78 | 15.9 | 110 | 25.3 |
| 15 | 3.3 | 47 | 9.7 | 79 | 16.1 | 111 | 25.7 |
| 16 | 3.5 | 48 | 9.9 | 80 | 16.3 | 112 | 26.1 |
| 17 | 3.7 | 49 | 10.1 | 81 | 16.5 | 113 | 26.5 |
| 18 | 3.9 | 50 | 10.3 | 82 | 16.7 | 114 | 26.9 |
| 19 | 4.1 | 51 | 10.5 | 83 | 16.9 | 115 | 27.3 |
| 20 | 4.3 | 52 | 10.7 | 84 | 17.1 | 116 | 27.7 |
| 21 | 4.5 | 53 | 10.9 | 85 | 17.3 | 117 | 28.1 |
| 22 | 4.7 | 54 | 11.1 | 86 | 17.5 | 118 | 28.5 |
| 23 | 4.9 | 55 | 11.3 | 87 | 17.7 | 119 | 28.9 |
| 24 | 5.1 | 56 | 11.5 | 88 | 17.9 | 120 | 29.3 |
| 25 | 5.3 | 57 | 11.7 | 89 | 18.1 | 121 | 29.7 |
| 26 | 5.5 | 58 | 11.9 | 90 | 18.3 | 122 | 30.1 |
| 27 | 5.7 | 59 | 12.1 | 91 | 18.5 | 123 | 30.5 |
| 28 | 5.9 | 60 | 12.3 | 92 | 18.7 | 124 | 30.9 |
| 29 | 6.1 | 61 | 12.5 | 93 | 18.9 | 125 | 31.3 |
| 30 | 6.3 | 62 | 12.7 | 94 | 19.1 | 126 | 31.7 |
| 31 | 6.5 | 63 | 12.9 | 95 | 19.3 | 127 | 32.1 |
| 32 | 6.7 | 64 | 13.1 | 96 | 19.5 | 128 | 32.5 |

### 5.1 The 3-member frame

The 3-member plane frame depicted in Figure 2 is subjected to a weight minimization considering a load of 350000 lbs applied in the vertical direction and 1000 lbs in the horizontal direction at node 1.

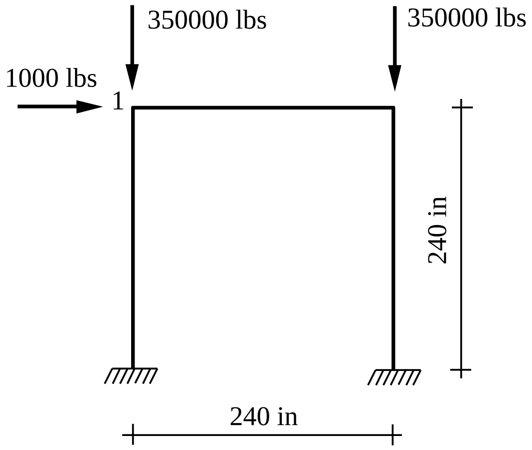


Figure 2: The 3-member planar frame.

### 5.2 The 5-member frame

The 5-member planar frame depicted in Figure 3 is subject to a weight minimization considering vertical loads of 350000.0 lbs of magnitude applied in the vertical direction at the nodes 1,7 and 13 and a load of 1000 lbs applied in the horizontal direction at the node 1.

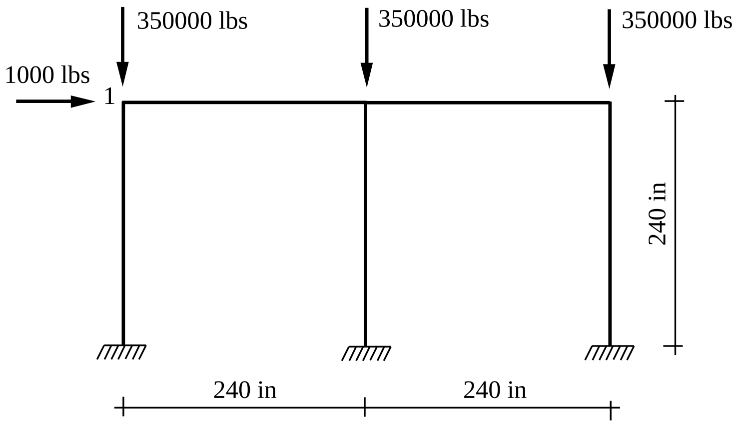


Figure 3: The 5-member planar frame.

### 5.3 The 6-member frame

The structure depicted in Figure 4 is a 6-member planar frame. The structure is subjected to a weight minimization considering vertical loads of 350000.0 lbs of magnitude applied in the vertical direction at the nodes 1 and 7 and a load of 1000 lbs in the horizontal direction at the nodes 1 and 18.

### 5.4 Results

Tables 2,3, 4 and 5 present the best solution found for the discrete and continuous cases with one and two member groups for all experiments. In these tables "$dv$" means design variable and "LIN" and "NL", linear and nonlinear analysis, respectively. All solutions shown in these Tables are rigorously feasible where the displacements are into the allowable required limits for these constraints.

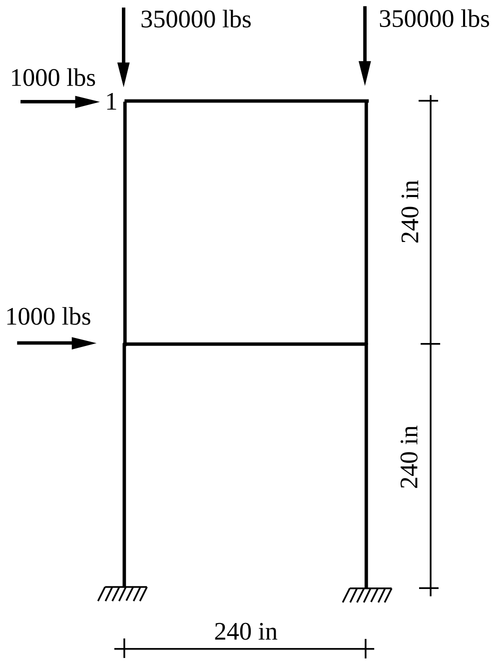


Figure 4: The 6-member planar frame.

Table 2: Final weights in lbs for the discrete cases – members linked in one group.

| | 3-member | | 5-member | | 6-member | |
|---|---|---|---|---|---|---|
| dv | LIN | NL | LIN | NL | LIN | NL |
| $D_1$ | 0.5 | 0.5 | 0.5 | 0.5 | 0.9 | 0.9 |
| $D_2$ | 20.5 | 19.5 | 19.9 | 20.1 | 24.1 | 28.1 |
| $W$ | 723.6 | 709.2 | 1194.0 | 1206.0 | 3123.4 | 3641.8 |

Table 3: Final weights in lbs for the discrete cases – members linked in two groups.

| | 3-member | | 5-member | | 6-member | |
|---|---|---|---|---|---|---|
| dv | LIN | NL | LIN | NL | LIN | NL |
| $D_1$ | 0.9 | 0.9 | 0.5 | 0.9 | 0.9 | 0.9 |
| $D_2$ | 11.9 | 11.7 | 21.3 | 12.5 | 26.5 | 25.3 |
| $D_3$ | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| $D_4$ | 10.5 | 5.1 | 0.9 | 14.1 | 15.1 | 27.3 |
| $W$ | 640.1 | 575.3 | 788.4 | 1148.4 | 2652.0 | 3812.6 |

Table 4: Final weights in lbs for the continuous cases – members linked in one group.

| | 3-member | | 5-member | | 6-member | |
|---|---|---|---|---|---|---|
| dv | LIN | NL | LIN | NL | LIN | NL |
| $D_1$ | 0.617 | 0.519 | 0.563 | 0.506 | 0.900 | 0.900 |
| $D_2$ | 15.951 | 18.786 | 17.507 | 19.478 | 23.433 | 27.442 |
| $W$ | 709.200 | 702.000 | 1182.000 | 1182.000 | 3036.877 | 3556.555 |

Table 5: Final weights in lbs for the continuous cases - members linked in two groups.

| | 3-member | | 5-member | | 6-member | |
|---|---|---|---|---|---|---|
| dv | LIN | NL | LIN | NL | LIN | NL |
| $D_1$ | 0.505 | 0.665 | 0.557 | 0.901 | 0.900 | 0.900 |
| $D_2$ | 20.805 | 15.779 | 18.852 | 12.453 | 26.232 | 29.630 |
| $D_3$ | 0.502 | 0.500 | 0.500 | 0.500 | 0.506 | 0.700 |
| $D_4$ | 5.001 | 5.006 | 0.500 | 14.029 | 14.742 | 20.806 |
| $W$ | 564.365 | 564.090 | 768.013 | 1144.560 | 2624.797 | 3259.554 |

For the 3-member planar frame one can observe that the optimization in all cases considering nonlinear analysis reached lighter weights than the others considering linear analysis. As an example, the discrete case considering two member groups, the weight of the optimized frame was 575.3 lbs against 640.1 lbs considering linear analysis. The difference between the final weigths was 10.12 %. The 3-member planar frame presented very similar final weights when the continuous case with two member groups is considered (564.365 ls (LIN) and 564.090 lbs (NL)). On the other hand, the 5-member and the 6-member planar frames present significant differences between their final wights: 768.013 lbs (LIN) against 1144.560 lbs (NL) (33.01 %), for the 5-member and, 2624.797 lbs (LIN) against 3259.554 lbs (LN) (19.47 %)

When one member group is adopted for the 5-member planar frame, in the discrete case, the best weight found corresponds to the optimization using linear analysis (1194.0 lbs against 1206.0 lbs). For the two member groups the best final weight, again, was obtained considering the linear analysis (788.4 lbs against 1148.4 lbs). In this case the difference between the final weights was significant and equal to 31.35 %. When the continuous case is adopted with one member group, the final weights were exactly the same (1182.0 lbs) but the dimensions $D_1$ and $D_2$ of the cross-sectional areas were distincts as observed from the Table 4. Finally, when the members are inked in two groups, for the continuous cases, the final weights were equal to 768.013 lbs in the linear and 1144.560 lbs in the nonlinear analysis, respectively. In this case the difference between the final weights is equal to 32.90 %.

For the discrete case, using one group, the optimization of the 6-member planar frame reached a final weight equal to 3123.4 lbs in the linear against 3641.8 ls in the nonlinear analysis, respectively, corresponding to a difference between the weights equal to 14.23 %. When 2 groups were considered, the final weights were 2652.0 lbs and 3812.6 lbs, for linear and nonlinear analysis respectively, with 30.44 % of difference between these values. Analogously, the final weights for discrete and continuous cases, when the members are linked in one and two groups, provided differences equals to 14.61 % and 19.47 %.

One can observe from the results of the analysis the importance of carry out the linear and nonlinear procedures since these analysis can lead to different final weights or it is possible to reach similar final weights but distinct cross-sectional characteristics of the members. The designer have to be attempt in order to choice the adequate analysis to be conducted in the structural optimization.

The performance of GA for each experiment is provided in Table 6 and Table 7 where the first part of the tables correspond to the cases where the members are linked in one group and the second part in two groups, respectively.

The evolution of the best solutions during the optimization process is presented in the graphics of the Figures 5 to 10.

Table 6: Performance of the GA for each experiment continuous.

| | | best | average | median | std. dev. | worst |
|---|---|---|---|---|---|---|
| 3-member | LIN | 709.200 | 709.200 | 709.200 | 0.000 | 709.200 |
| | NL | 702.000 | 708.269 | 707.338 | 5.729 | 716.400 |
| 5-member | LIN | 1182.000 | 1185.037 | 1188.075 | 5.175 | 1194.000 |
| | NL | 1182.000 | 1182.000 | 1182.000 | 0.000 | 1182.000 |
| 6-member | LIN | 3036.877 | 3037.102 | 3037.071 | 0.150 | 3037.390 |
| | NL | 3556.555 | 3556.625 | 3556.618 | 0.067 | 3556.707 |
| 3-member | LIN | 564.364 | 605.342 | 589.124 | 43.197 | 678.755 |
| | NL | 564.090 | 566.677 | 564.339 | 2.996 | 570.939 |
| 5-member | LIN | 768.013 | 812.349 | 789.972 | 54.393 | 901.437 |
| | NL | 1144.562 | 1148.609 | 1152.053 | 4.357 | 1155.777 |
| 6-member | LIN | 2624.797 | 2659.573 | 2657.029 | 26.529 | 2699.434 |
| | NL | 3259.554 | 3296.676 | 3277.668 | 42.497 | 3371.813 |

Table 7: Performance of the GA for each experiment discretes.

| | | best | average | median | std. dev. | worst |
|---|---|---|---|---|---|---|
| 3-member | LIN | 723.60 | 723.60 | 723.60 | 0.00 | 723.60 |
| | NL | 709.20 | 711.72 | 709.20 | 4.36 | 719.28 |
| 5-member | LIN | 1194.00 | 1262.40 | 1285.20 | 39.49 | 1285.20 |
| | NL | 1206.00 | 1212.00 | 1206.00 | 10.39 | 1230.00 |
| 6-member | LIN | 3123.36 | 3123.36 | 3123.36 | 0.00 | 3123.36 |
| | NL | 3641.76 | 3641.76 | 3641.76 | 0.00 | 3641.76 |
| 3-member | LIN | 640.08 | 642.24 | 640.08 | 3.74 | 648.08 |
| | NL | 575.28 | 591.36 | 581.76 | 21.04 | 626.64 |
| 5-member | LIN | 788.40 | 851.28 | 850.56 | 62.88 | 915.60 |
| | NL | 1148.40 | 1148.00 | 1148.40 | 0.00 | 1148.40 |
| 6-member | LIN | 2652.00 | 2657.28 | 2657.28 | 5.28 | 2662.56 |
| | NL | 3812.64 | 3858.24 | 3871.68 | 26.68 | 3876.96 |

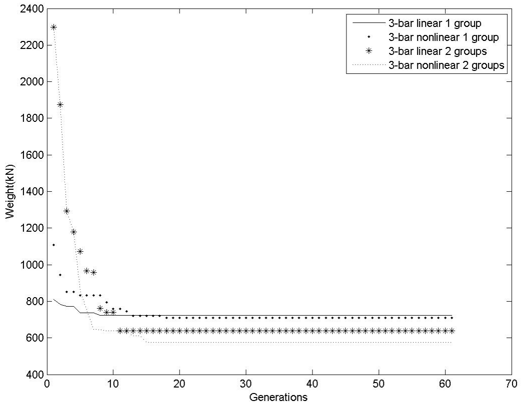


Figure 5: Evolution of the best discrete solution of the 3-member frame.

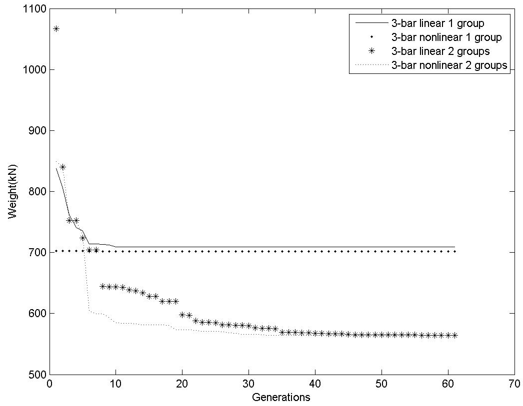


Figure 6: Evolution of the best continuous solution of the 3-member frame.

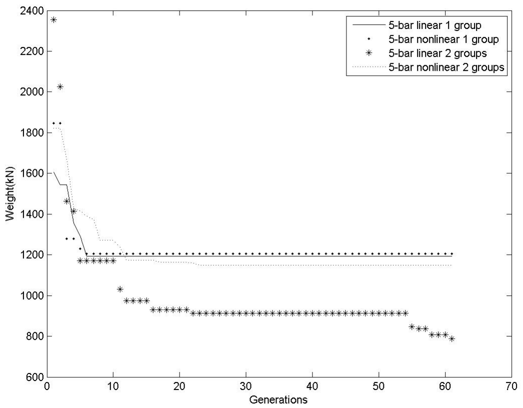


Figure 7: Evolution of the best discrete solution of the 5-member frame.

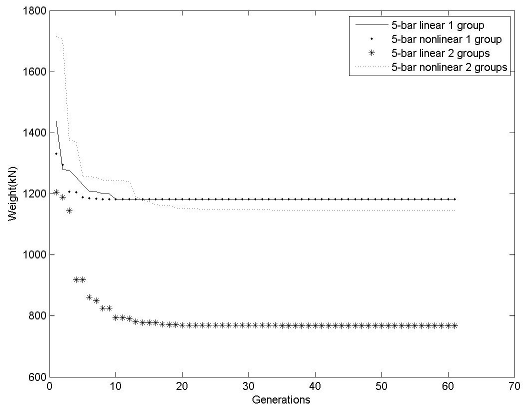


Figure 8: Evolution of the best continuous solution of the 5-member frame.

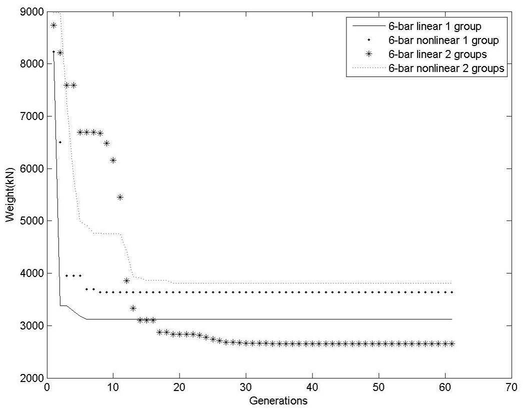


Figure 9: Evolution of the best discrete solution of the 6-member frame.

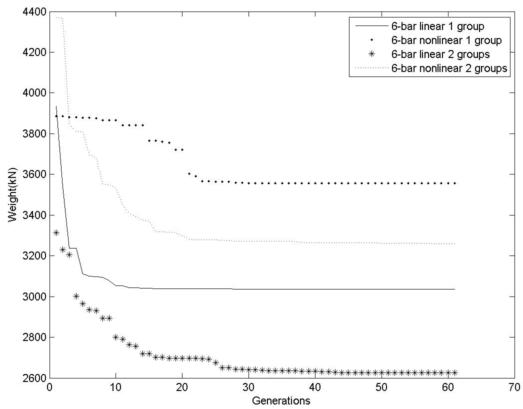


Figure 10: Evolution of the best continuous solution of the 6-member frame.

Figures from 11 to 14 show the displacement of the node 1 in the horizontal direction for each load step for the best solutions found for discrete and continuous cases, respectively, considering the members linked in one and two groups. It is very important to observe that the aspects of the curves presented by the analysis of the 3-member plane frame show a gain of stiffness of the structure. This fact justify the ligther weight obtained for the strucuture at the end of optimization considering nonlinear analysis in comparison with the linear analisys (709.2 lbs (NL) against 723.6 lbs (LIN), using one member group and 575.3 lbs (NL) against 640.1 lbs (NL), considering two member groups). For the 5- and 6-member plane frame the nonlinear analysis presented heavier final weights considering nonlinear analysis than those obtained with the linear analysis. The structures present a loss of stiffness and it can be observed from the graphics of the Figures 15 to 22. This loss of stiffness strongly induces an increase of mass in the members (areas of the cross-sections), of these structures in order to avoid displacements greater than the limits imposed by the structural optimiation problem. Besides, it is possible to observe distincts mechanical behaviour between the structures analyzed by linear and nonlinear analysis. In this way, these structures became heavier in comparison with others optimized using linear analysis.

Observing all graphics, it is easy to note a more pronounced nonlinear behavior of the 3-member and 5-planar frames whereas in the 5-member planar frame this behavior is less perceptible.

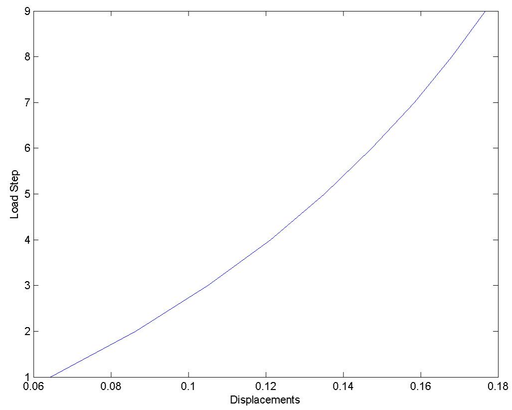


Figure 11: 3-member planar frame, Displacements X LoadStep for the continuous case using one member group.

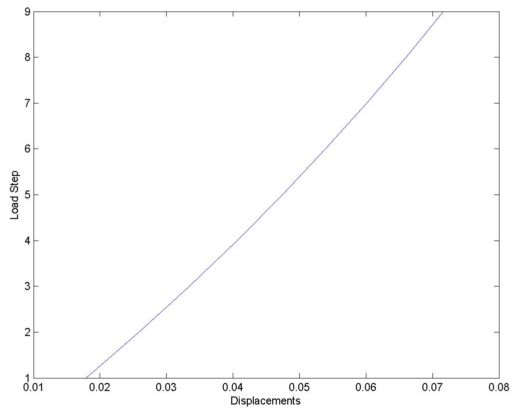


Figure 12: 3-member planar frame, Displacements X LoadStep for the discrete case using one member group.

## 6 CONCLUSIONS

In this paper, a GA is used to minimize the weight of planar frame structures considering linear static analysis as well as nonlinear static analysis. the design variables are the dimensions of the cross-sectional areas of the members ant they can be discrete or continuous. To solve the nonlinear equilibrium equation of the structure the iterative Newton-Raphson's Method was adopted. The algorithm used in this paper are previously presented in Lemonge et al. (2010) applied to optimization of geometrically nonlinear dome structures.

One can observe from the results of the analysis the importance of carry out the linear and nonlinear procedures since these analysis can lead to different final weights. It is possible to

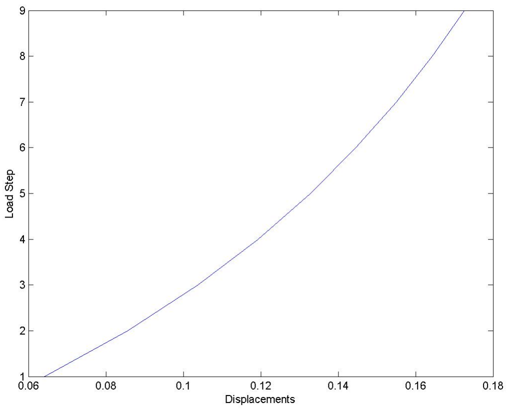


Figure 13: 3-member planar frame, Displacements X LoadStep for the discrete case using one member group

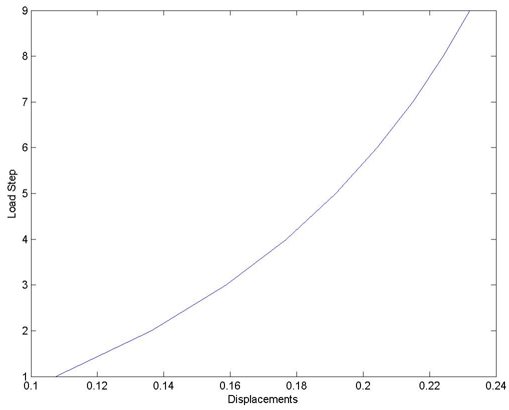


Figure 14: 3-member planar frame, Displacements X LoadStep for the discrete case using one member group

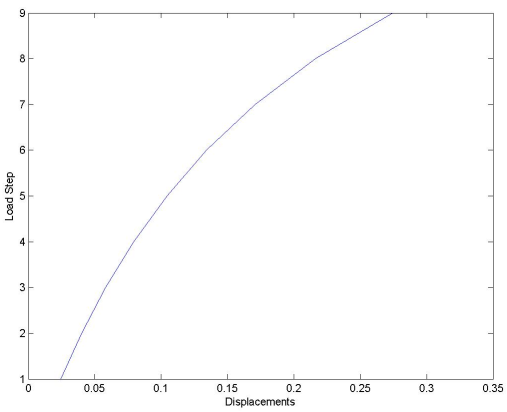


Figure 15: 5-member planar frame, Displacements X LoadStep for the continuous case using one member group.

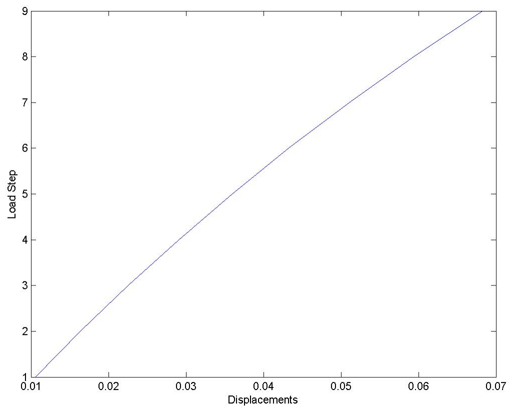


Figure 16: 5-member planar frame, Displacements X LoadStep for the discrete case using one member group

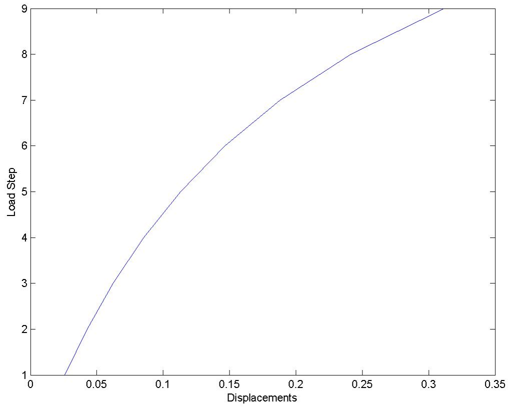


Figure 17: 5-member planar frame, Displacements X LoadStep for the continuous case using two member groups.

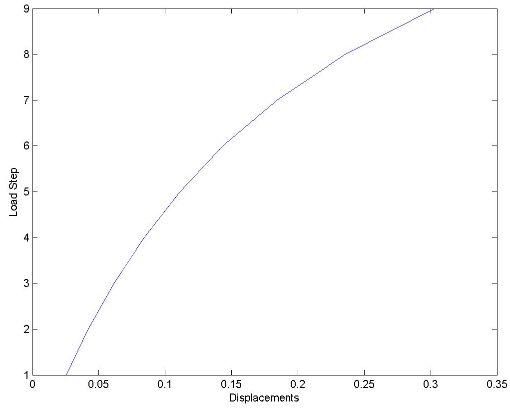


Figure 18: 5-member planar frame, Displacements X LoadStep for the discrete case using two member groups.

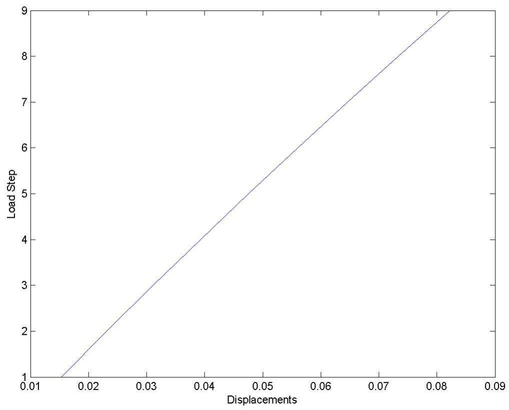


Figure 19: 6-member planar frame, Displacements X LoadStep for the continuous case using one member group.

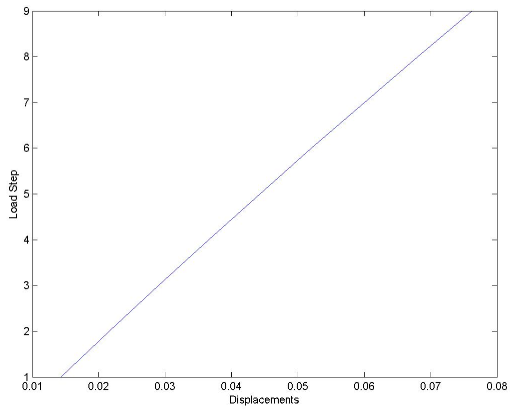


Figure 20: 6-member planar frame, Displacements X LoadStep for the discrete case using one member group.

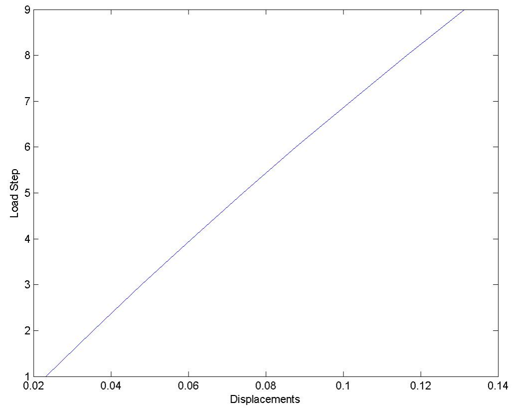


Figure 21: 6-member planar frame, Displacements X LoadStep for the continuous case using two member groups.

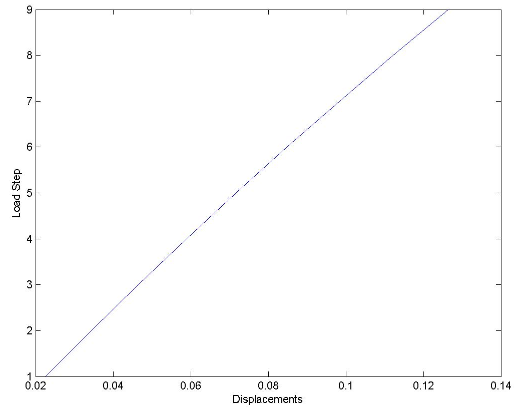


Figure 22: 6-member planar frame, Displacements X LoadStep for the discrete case using two member groups.

reach lighter weights considering nonlinear analysis in the structural optimization process when, for example, the structure present a gain of stiffness leading to advantages with respect to the economic aspects. Also it is possible to reach similar final weights but distinct cross-sectional characteristics of the members and, in this way, the designer have to be attempt in order to choice the adequate analysis to be conducted in the structural optimization.

The GA used in this paper has been improved in other aspects to solve more complex optimization problems considering nonlinear and snap-through analysis as well as problems of maximization of the critical load or limit point of instability.

Also, several aspects of the iterative methods have to be considered in order to accelerate the search of the optimum solutions. Aiming to provide a more appropriate statistical analysis, the number of runs of the GA in the numerical experiments should be increased in the future works.

## 6.1 Acknowledgments

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