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A HEURISTIC FRAMEWORK FOR RELIABILITY BASED OPTIMIZATION OF LAMINATED COMPOSITE STRUCTURES

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Abstract. The design of anisotropic laminated composite structures is very susceptible to changes in loading, angle of fiber orientation and ply thickness. Thus, optimization of such structures, using a reliability index as a constraint, is an important problem to be dealt. The problem of structural optimization of laminated composite materials with reliability constraint using a genetic algorithm and two types of neural networks is addressed in this paper. The reliability evaluation is performed using, alternatively, the following methods: First Order Reliability Method (FORM), FORM with Multiple Check Points (MCP), Standard Monte Carlo (MC) and Monte Carlo with Importance Sampling (MC-IS). The optimization process is performed using a genetic algorithm. To overcome high computational cost, Multilayer Perceptron or Radial Basis Artificial Neural Networks are used. This methodology can be used without loss of accuracy and large computational time savings, even when dealing with structures with non-linear behavior, as it is shown by some numerical examples.

1 INTRODUCTION

In the optimization of laminated composite structures, the design variables related to the optimal configurations may be the ply number, fiber orientation angles, thickness of each layer, number of materials and the sequence of lamination. The result of the optimization procedure consists of systems with anisotropic mechanical behavior that are also highly sensitive to the direction of applied loads. Any change in the applied load direction, fiber orientation or thickness of the layers may affect the stress state, leading to a reduction in the structural performance or in the reliability index. Thus, design of structures with anisotropic laminated composite materials should take into account such uncertainties in loads and material properties. Consequently, reliability of such optimized designs becomes especially important in the field of laminated composite structures (Mitsunori et al., 1987).

The main objective of this paper is to present a new methodology to determine the optimal configuration of laminated composite structures with reliability constraints.

The optimization process is performed using a genetic algorithm (GA). Genetic algorithms are optimization tools based on the concepts of natural selection and survival of the fittest individual with respect to some criterion. The design of the optimal sequence of layers in laminated composite materials (with their respective thickness and fiber orientation angles) is a minimization problem and due to its characteristics, genetic algorithms are more convenient than a gradient based optimization method, which often converge to solutions that represent local minima (Goldberg, 1989). Moreover, in commercial projects of this type of structure, fiber orientation angles, number and thickness of layers are discrete variables, a fact that encourages the use of genetic algorithms, because this tool is suitable for computational problems involving discrete variables and combinatorial optimization.

In this paper the reliability analysis is carried out using one of the following methods: First Order Reliability Method (FORM), modified FORM with multiple check points (FORM-MCP), Standard or Direct Monte Carlo (MC) and Monte Carlo with Importance Sampling (MCIS). These methods and concepts related to structural reliability are widely covered in texts such as Ang and Tang (1984), Haldar & Mahadevan (1999), Melchers (1999), among others, as well as a large number of articles published in several international journals. The Tsai-Wu criterion is adopted as the limit state function used to evaluate the reliability index (Daniel & Ishai, 1994; Jones, 1999; Gurdal et al., 1999).

The finite element analysis (FEA) was performed using the discrete Kirchhoff triangular element (DKT) for thin plate bending (Bathe & Batoz, 1980), coupled with the constant strain triangular element (CST) to take into account membrane effects. The element was adapted to analyze laminated composite structures, following the classical theory of laminates (Jones, 1999, Daniel & Ishai, 1994).

In order to reduce the computational cost in the reliability based optimization of laminated composite structures, two artificial neural networks were used: Multilayer

Perceptron Neural Network (MPNN) and Radial Basis Neural Network (RBNN) (Haykin, 1994 and Gomes, 2004).

2 COMPOSITE MATERIALS FAILURE CRITERION AND RELIABILITY ANALYSIS

There are several failure criteria for composite laminates reinforced by fibers, such as maximum strain, Tsai-Hill, Hoffman and Tsai-Wu (Kaw, 2006). Among these methods, Tsai-Wu criterion is the most widely used by several authors. This criterion takes into account the interactions between different stress components. The two coordinate systems used here are shown in Figure 1, where *1* and *2* represent the axes of reference, *x-y* the material principal axes and θ is the angle between the axes *x* and *1* and between axes *y* and *2*. Since the stress components in the direction of the reference axes (*1-2*) are rotated to the material principal axes (*x-y*), the Tsai-Wu criterion for plane stress state can be evaluated using the following expression:

$$F_x S_x + F_y S_y + F_{xx} S_x^2 + F_{yy} S_y^2 + F_{ss} S_{xy}^2 + 2F_{xy} S_x S_y = 1 \quad (1)$$

where $F_{xx} = 1/R_x R'_x$, $F_x = 1/R_x - 1/R'_x$, $F_{yy} = 1/R_y R'_y$, $F_y = 1/R_y - 1/R'_y$, $F_{ss} = 1/R_s^2$ and $F_{xy} = F_{xy}^* \sqrt{F_{xx} F_{yy}}$. The factor F_{xy}^* is taken as being equal to -1/2. The subscripts *x* and *y* indicates, fiber orientations, while *s* means shear. The symbols with apostrophe indicate compression strengths, whereas symbols without apostrophe indicate tensile strengths. R_x is the ultimate longitudinal tensile strength, R'_x is the longitudinal ultimate compressive strength, R_y is the ultimate transverse tensile strength, R'_y is the ultimate transverse compressive strength and R_s is the in-plane shear strength.

Assuming an elastic material behavior, a Tsai-Wu factor λ, which multiplies all stress tensor components, can be evaluated concerning the safety margin of the stress state. This is indicated by Daniel and Ishai (1994) by solving the following equation for λ:

$$\lambda^2 (F_{xx} S_x^2 + F_{yy} S_y^2 + F_{ss} S_{xy}^2 + 2F_{xy} S_x S_y) + \lambda (F_x S_x + F_y S_y) - 1 = 0 \quad (2)$$

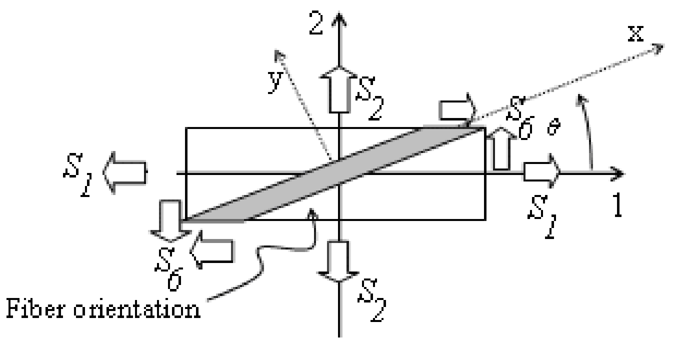

Figure 1: Coordinate systems for unidirectional composite materials.

A mathematical expression for unidirectional composite failure may be written as follows:

$$g(\mathbf{X}) = g(x_1, x_2, ..., x_n) \leq 0 \tag{3}$$

where $g(\mathbf{X})$ represents the safety margin and $\mathbf{X}$ is the n-dimensional vector of random variables $(x_i, i = 1, 2, ..., n)$ that affects the material strength or structural behavior and $g(\mathbf{X}) \leq 0$ means failure while $g(\mathbf{X}) > 0$ means that the material is in the safety domain. Sometimes, function $g(\mathbf{X})$ is referred as the limit state function. Generally speaking, the failure probability can be evaluated using the joint probability density function $f_X(x_1, x_2, ..., x_n)$ given by the following expression:

$$P_f = \iint \cdots \int_D f_X(x_1, x_2, ..., x_n) dx_1 dx_2 ... dx_n \tag{4}$$

where D means the failure domain ($g(\mathbf{X}) \leq 0$). Let consider a thin plate of composite material subjected to a plane stress state, as indicated by Figure 1, where the random variables $\mathbf{X}$ are the stress components S_1, S_2 and S_6, the experimental material strengths along fiber and transversal directions $R_x, R'_x, R_y, R'_y, R_s$ and fiber orientation angles θ. Rotating these stress components to the fiber direction and distributing to the other layers of the composite accordingly to ply ratios, one may obtain the stress state acting on a lamina (S_x, S_y, S_{xy}). So, in this case, for instance, one may assume $\mathbf{X} = \{S_1, S_2, S_6, R_x, R'_x, R_y, R'_x, R_s, \theta\}^T$ as the vector of random variables. It must be noticed that the random stress components S_1, S_2, S_6 generate random stress components S_x, S_y, S_{xy} on the fiber and transversal directions, which requires a structural analysis to be evaluated.

Substituting equation (1) into (3), the limit state function $g(\mathbf{X})$, at a particular point in the composite material, becomes:

$$g(\mathbf{X}) = 1 - (F_x S_x + F_y S_y + F_{xx} S_x^2 + F_{yy} S_y^2 + F_{ss} S_{xy}^2 + 2F_{xy} S_x S_y) \tag{5}$$

It must be emphasized that this equation should be verified at the top, middle and bottom of each layer belonging to the composite material. The integration of equation (4) becomes hard if equation (5) is used as limit state function, since the problem deals with several random variables and the stress state is a function of geometrical dimensions and external loads. Besides, function $f_X(\mathbf{X})$ is not known *a priori* because usually there are not enough available statistical data. In this paper, using a finite element analysis, a limit state function is built based on the Tsai-Wu factors $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, ...\}^T$ evaluated at each element integration point and at the top, middle and bottom of each layer, as expressed by the following equation:

$$g(\mathbf{X}) = min(\lambda_i) - 1 \tag{6}$$

This equation holds provided a first ply failure for the composite is assumed. Therefore, if the minimal Tsai-Wu factor at any point is less than a unit value, this will mean failure ($g(\mathbf{X}) < 0$), otherwise all the stress states in the composite do not provoke failures.

In order to determine the failure probability, reliability analyses are preformed using standard methods such as the Direct Monte Carlo (MC) , Monte Carlo with importance Sampling (MCIS), First Order Reliability Method (FORM) and FORM with Multiple Check Points (FORM-MCP). Details about MC, MCIS and FORM can be found in Melchers (1999) and Ang *et al* (1984). FORM-MCP was presented by Miki (1986). In this case, sample points are searched close to the boundary of the failure and safety regions. FORM method is used instead of Monte Carlo Simulations with Importance Sampling in order to evaluate multiple design points.

The parameter used to distinguish among different design points is the same indicated by Miki (1986) and Shao *et al.* (1992): the angle between the vector of design variables for each new design point should be larger than a previously specified value θ (in this paper, θ must be larger than 10^{-3}). The search is performed in random directions; the number of searches is a multiple of the number of random variables. Figure 2 shows the multiple check point criterion in the standard non-correlated space for three random directions and three limit state functions. $H_i(\mathbf{U})$ are the limit state functions in the non correlated standard space obtained from $g(\mathbf{X})$, which is the limit state function in the real space. Then, the resulting values for failure probability and the reliability index are given by $P_f = \sum_i P_f^i$ and $\beta = \Phi^{-1}(P_f)$, respectively, where $\Phi^{-1}(.)$ is inverse of the cumulative standard probability function. Index i indicates the number of random directions. For the case of Figure 2, i=1, 2, 3. More details can be found in Miki (1986). It is important to point out that in this work the failure of one layer represents the failure of the whole system, criterion which is known as first ply failure.

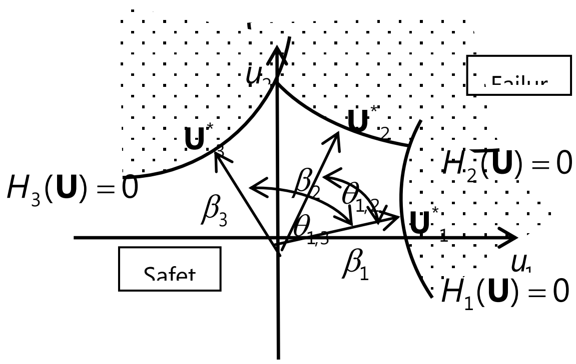

Figure 2: FORM with multiple check points in the standard non-correlated space.

3 GENETIC ALGORITHM (GA)

Genetic Algorithm (GA) is a computational search tool based on concepts of natural selection and survival of the fittest individual. One aspect of fundamental importance in the GA is the way the solutions are tracked. Instead of using derivatives or gradients, as in deterministic optimization algorithms, GA works with the objective function based on simple values of individuals. This feature makes the method suitable for problems involving discontinuous functions, and/or non-defined derivatives like in integer programming. Moreover, unlike deterministic optimization methods, which perform the search focusing on a single solution at a time, the GA works with a population of individuals in each generation. Thus, as several search points are considered, the convergence or stagnation to local minima, if the starting point is poorly chosen, is prevented. All these aspects result in increased chances of finding the optimal solution, even on problems that have hard search spaces with multiple local minimum (Goldberg, 1989).

The design of the optimal sequence of layers in laminated composite materials is a problem of global minimum. Due to the stochastic characteristics of Genetic Algorithms, they are more suitable than gradient based optimization methods, which often converge to solutions representing a local minimum. Moreover, in commercial designs fiber orientation angles and the amount and thickness of layers are discrete variables, a fact which confirms the suitability of Genetic Algorithms for these kinds of problems.

The design variables used in the optimization process will be the fiber orientation angles and thickness of the layers of laminated composite material.

4 ARTIFICIAL NEURAL NETWORKS(ANN)

Artificial Neural Networks (ANN) may be characterized as computational models based on parallel distributed processing with particular properties such as the ability to learn, to generalize, to classify and to organize data. There are several models that have been developed for different specific computational tasks. Multilayer Perceptron Neural Networks (MPNN) and Radial Basis Neural Networks (RBNN) are used. Both types of Networks have a supervised training, feed-forward architecture and they have been widely used as universal approximations for unknown functions of several variables with several outputs. More details can be found in Gomes *et al* (2004).

4.1 Generation of sample data for Artificial Neural Network training

To generate the sample data for artificial neural network training it is first carried out a search on random directions (in the non-correlated standard space) for points in the vicinity of the limit state function $H(\mathbf{U})=0$ in the standard non-correlated space. Once such points are found, the mean values of the distribution functions of the design variables are shifted in order to obtain samples (using Monte Carlo

Method) near the neighborhood of the safety/failure domain. Another set of random samples centered on mean values of the random variables are added to the original sample set in order to give a better behavior to the fitted limit state function that are located far from the failure domain. This is especially important if a gradient based method like FORM is used, but not so important when Monte Carlo method is used. Figure 3 shows schematically how this sample data are generated in the non-correlated standard Gaussian space for a limit state function of two random variables. In this paper, the number of random directions is three times the number of random variables.

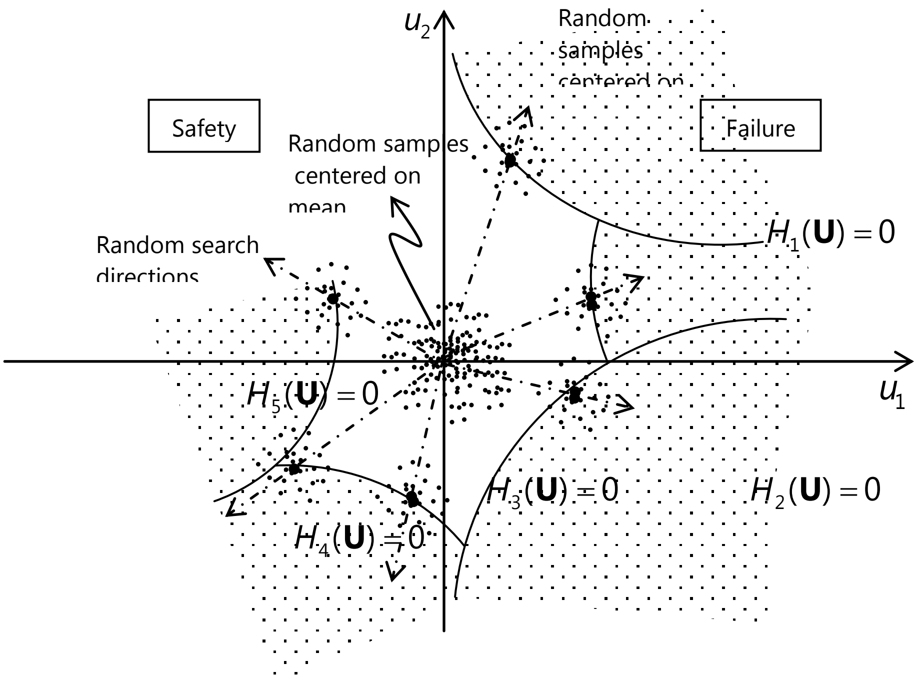

Figure 3: Generation of sample data set for neural network training.

5 NUMERICAL RESULTS

5.1 Example 1 – Optimization of a laminated composite plate with reliability constraint

This example deals with the minimization of the total thickness of a laminated composite plate with linear behavior. The total number of layers is N and the thickness of layer i is h_i *(i=1,2,...,N)*. In all cases studied here, the cost function was the total thickness of the plate (h_t) and the constraint was the minimum reliability index required by the system (β_{req}), which is a value defined by the user. The optimization problem takes the following form:

$$\begin{array}{lll} \text{Find} & h_i & (i = 1, 2, \ldots, N) \\ \text{such that} & h_t = \sum_{i=1}^{N} h_i & \text{is a minimum} \\ \text{subjected to} & \beta \geq \beta_{req} & \end{array} \qquad (7)$$

The fiber orientation angle of each ply of the laminated composite plate with four layers remains constant and according to the following distribution [0^0,45^0,45^0,0^0], as shown in Figure 4.

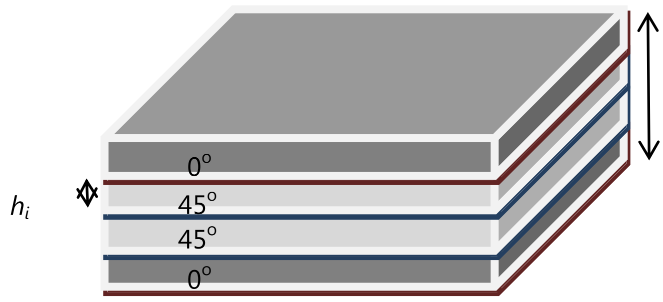

Figure 4: Laminated composite plate with four layers.

The material used here was Graphite / Epoxy (T300/5208). In Table 1 the deterministic mechanical properties are presented. In this example nine random variables were considered, where four variables are the applied loads N_1, N_2, N_{12} and M_1, arranged as shown in Figure 5, and five variables are strengths $R_x^T, R_x^C, R_y^T, R_y^C$ e R_{xy}, where indexes T and C mean, respectively, tension and compression whereas R_{xy} is the shear strength. In Figure 5, (*x, y*) is the fiber orientation system and (*1, 2*) is the global system. The statistical properties of the random variables are listed in Table 2, where COV means the coefficient of variation.

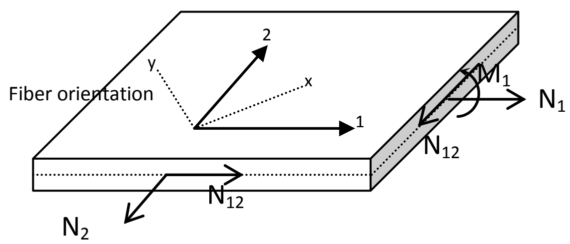

Figure 5: Loads acting on the laminated composite plate.

Material	E_1	E_2	E_{12}	ν_{12}
T300/5208 Grafite/Epóxi	181 GPa	10.30 GPa	40 GPa	0.28

Table 1: Deterministic mechanical properties.

No.	Symbol	Unit	Mean Value	Coeff. of Variation	Distribution type
1	N_1	KN/m	100.0	0.20	Lognormal
2	N_2	KN/m	200.0	0.20	Lognormal
3	N_{12}	KN/m	40.0	0.20	Lognormal
4	M_1	N.m/m	0.1	0.20	Lognormal
5	R_x^T	MPa	1500.0	0.20	Lognormal
6	R_x^C	MPa	1500.0	0.20	Lognormal
7	R_y^T	MPa	40.0	0.20	Lognormal
8	R_y^C	MPa	246.0	0.20	Lognormal
9	R_{xy}	MPa	68.0	0.20	Lognormal

Table 2: Statistical properties of random variables.

In all simulations, it was adopted a target reliability index constraint of $\beta_{req} = 3.0$. The optimization was performed using a GA, which input data are listed in Table 3. The reliability index was calculated using Monte Carlo, Monte Carlo with Importance Sampling, FORM and FORM with Multiple Check Points (FORM-MCP). The limit state function considered here was the Tsai-Wu failure criterion and the stress state at the local axes of the laminated composite plate was determined employing the classical theory of composite plates using a closed form solution (Jones, 1999, Daniel & Ishai, 1994) and after a finite element program, which uses the discrete Kirchhoff triangular element (DKT) (Bathe & Batoz, 1980), coupled with a constant stress triangular element (CST), following the classical theory of laminates (CTL) (Daniel & Ishai, 1994).

Number of individuals of each population	30
Maximum number of generations	30
Crossover probability	100%
Probability of mutation	1%
Stopping criterion (standard deviation of individuals of the population)	$1.0\text{x}10^{-5}$
Number of design variables (thickness)	4
Lower limit of the design variables (m) - L	$0.5\text{x}10^{-3}$
Upper limit of design variables (m) - U	$3.0\text{x}10^{-3}$
Number of bits of each design variable - n	16

Table 3: Data Input for the genetic algorithm program.

Since the GA is based on a binary codification, each design variable will present discrete values that depend on the number of bits used for the codification. The resolution for each design variable can be calculated using the following expression (Goldberg, 1989):

$$R = \frac{U - L}{2^n - 1} \tag{8}$$

where n is the number of bits given to each design variable, while U and L are, respectively, upper and lower limits of the design variables. In this example the resolution for the minimization of the thickness is R=3.815x10^{-5} mm. The search space, which corresponds to the number of thickness combinations, is $(2^{16})^4$=1.84x10^{19} which is unworthy for exhaustive search. Equation (9) shows how the cost function, which depends on the sum of the thickness of each ply, a penalty factor (which was adopted as being 10^5) and the reliability index of each individual, is evaluated.

$$\text{Minimize} \quad (\sum_{i=1}^{4} h_i)[1+(\beta_{req}-\beta)^2 10^5] \tag{9}$$

where h_i , $i=1,4$ represents the thickness of each layer. Figures 6 to 9 show results for the optimal solution (where evolution of the thickness of the different layers and the total thickness of the plate along different generations are presented) for some methods to evaluate the reliability index. The local stress state was obtained using a closed solution for laminated composite rectangular plates, given by Kaw (2006).

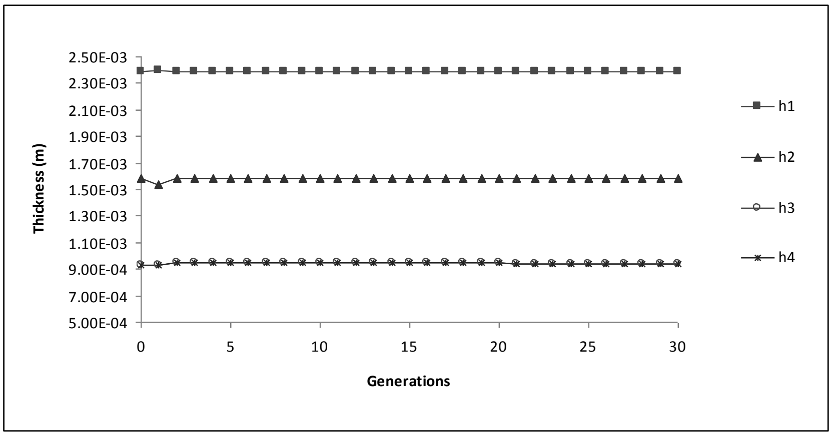

Figure 6: Layer thickness for the optimal solution of the laminated composite plate using Direct Monte Carlo Method for reliability index evaluation (the limit state function is determined analytically).

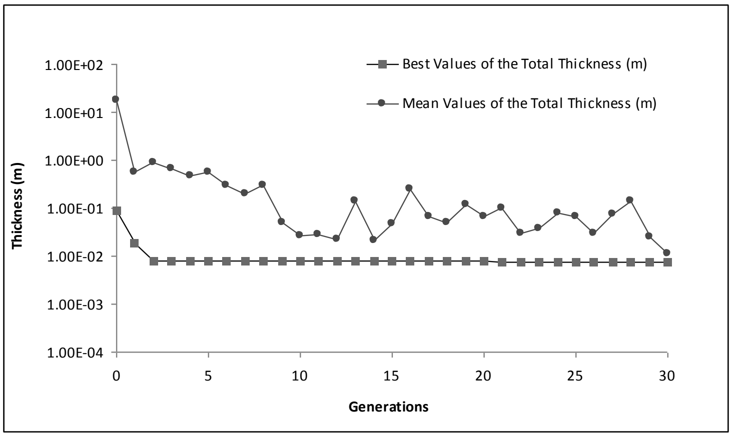

Figure 7: Total thickness of the best individual and mean values of the population's total thickness of the laminated composite plate using Monte Carlo Method for reliability index evaluation (the limit state function is determined analytically).

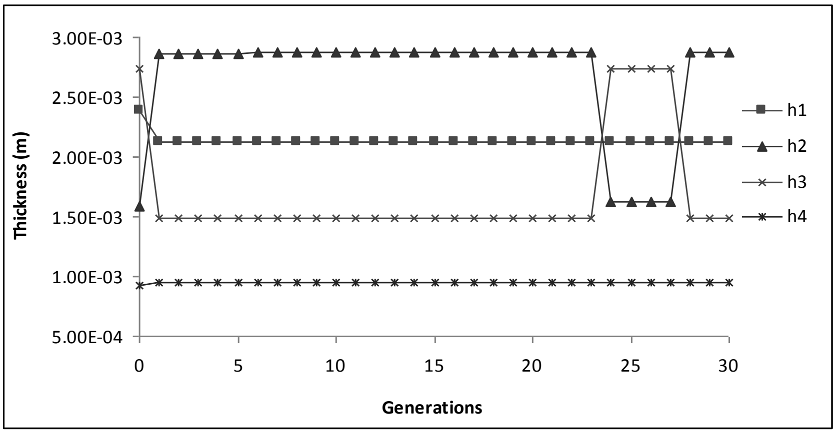

Figure 8: Layer thickness for the optimal solution of the laminated composite plate using FORM to calculate the reliability index (the limit state function is determined analytically).

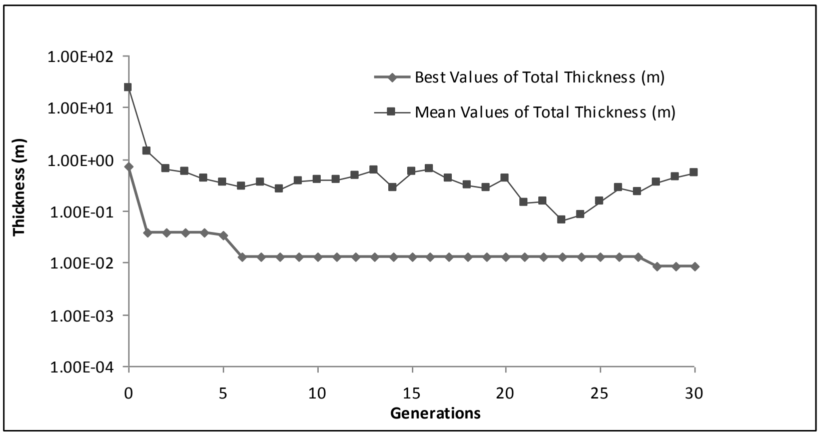

Figure 9: Total thickness of the best individual and mean values of the population's total thickness of the laminated composite plate using FORM to calculate the reliability index (the limit state function is determined analytically).

It can be noticed in the previous figures that the optimization process with GA converged, in most cases, due to the criterion based on the maximum number of generations instead of diversity criterion, even when optimum value was early found. The diversity parameter (which is the coefficient of variation of the cost function, i.e., the standard deviation divided by the mean value) used as convergence criterion does not seem to be a suitable parameter to indicate convergence. This shows that a more suitable convergence criterion reducing the number of generations and excessive number of simulations must be used. Perhaps the reduction of the heuristic parameter indicating probability of mutation may reduce the diversity parameter. This is an issue to be investigated in future papers.

As observed in results presented in Figures 7, 9 and 12, it should be clear that the total thickness for initial generations is weighted by the constraint violation (in this case the reliability constraint), which is the way as Penalization Techniques account for constraints (see Eq. 9) justifying values of the total thickness that are higher than the maximum physical total thickness.

Fig. 8 and Fig. 10 suggest that there are symmetrical layer configurations having similar values for the reliability index. This may hinder the algorithm in order to reach the minimum cost function.

The other tests (Monte Carlo with Importance Sampling, FORM and FORM-MCP using finite element analysis) show similar results and behavior regarding evolution of design variables and cost function. In the examples where the stress state was obtained by a finite element analysis, the simulations were performed using only FORM and modified FORM (FORM-MCP) to obtain the reliability index, since Monte Carlo methods would give a very expensive processing time.

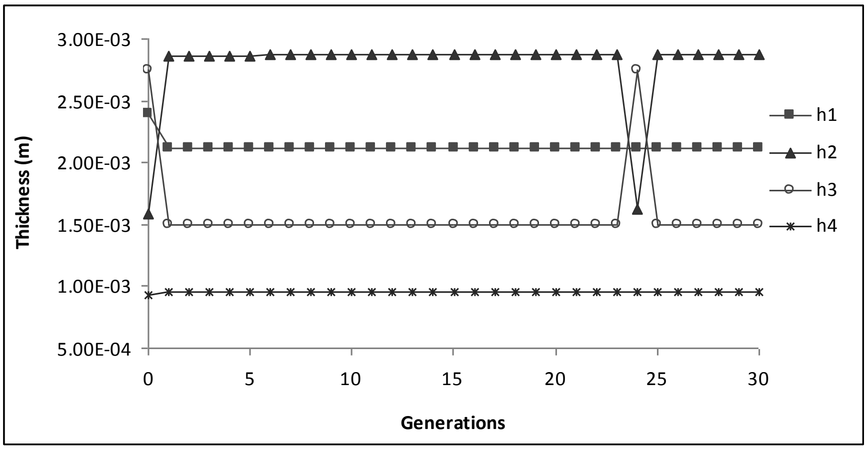

Figure 10: Total and layers thickness for the optimal solution of the laminated composite plate using modified FORM (FORM-MCP) to calculate the reliability index (the limit state function is determined analytically).

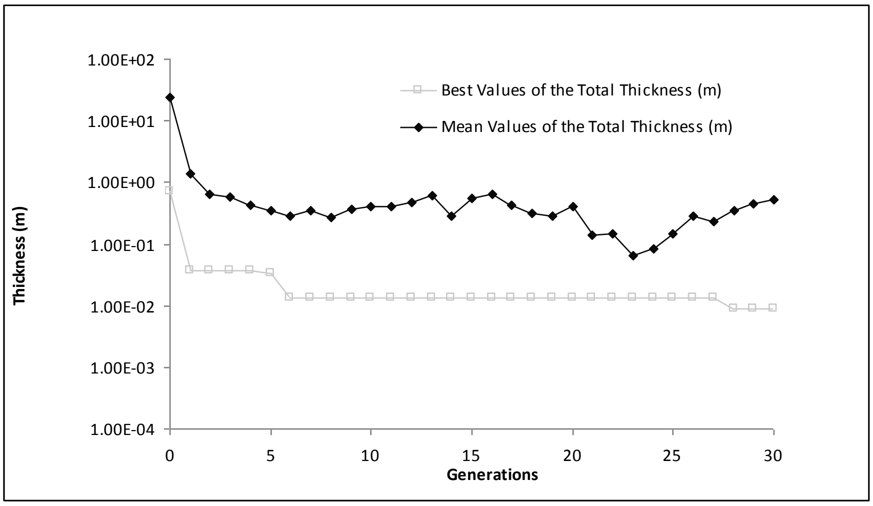

Figure 11: Total thickness of the best individual and mean values of the population's total thickness of the laminated composite plate using modified FORM (FORM-MCP) to calculate the reliability index (the limit state function is determined analytically).

In this example, RBNN and MPNN were also used in order to reduce the processing time spent in the optimization process. The training procedure used here was to train the networks, so that they could provide directly the reliability index, from a particular configuration of the laminate (one specific individual of the GA population). The reliability index used for training the neural networks was calculated using FORM (while the value of the limit state function was obtained using a finite element program). A total number of 300 samples, collected according to section 4.1 were used. The network architectures were (4:300:1) for RBNN and (4:10:10:10:1) for MPNN.

Table 4 summarizes all the tests performed in this work and a comparison of the computational cost using Artificial Neural Networks and finite elements is presented. The processing time for FORM with Multiple Check Points (FORM-MCP) using finite element to evaluate the limit state function was considered as the reference for processing time comparisons.

Table 5 presents the relative errors using neural networks with reference to the solution with FORM and finite elements for the design variables.

The results show a drastic reduction in processing time when the optimization is performed using neural networks to simulate the calculation of reliability index. The relative errors are small and does not exceed 3.51% (thickness h_4 using RBNN).

Method	Relative processing Time (s)	h_1 (m)	h_2 (m)	h_3 (m)	h_4 (m)	h_t (m)	β
Monte Carlo +closed form solution	2.677	$2.39x10^{-3}$	$1.58x10^{-3}$	$2.74x10^{-3}$	$9.37x10^{-4}$	$7.66x10^{-3}$	3.000
Monte Carlo with Importance Sampling +closed form solution	$8.026x10^{-1}$	$2.12x10^{-3}$	$2.87x10^{-3}$	$1.50x10^{-3}$	$9.50x10^{-4}$	$7.30x10^{-3}$	2.981
Modified FORM+closed form solution	$1.466x10^{-1}$	$2.12x10^{-3}$	$2.87x10^{-3}$	$1.49x10^{-3}$	$9.50x10^{-4}$	$7.42x10^{-3}$	2.978
FORM+closed form solution	$1.796x10^{-3}$	$2.12x10^{-3}$	$2.87x10^{-3}$	$1.49x10^{-3}$	$9.50x10^{-4}$	$7.42x10^{-3}$	2.978
Modified FORM+FEM	1.000	$2.390x10^{-3}$	$1.580x10^{-3}$	$2.740x10^{-3}$	$9.40x10^{-4}$	$7.650x10^{-3}$	2.999
FORM+FEM	$9.475x10^{-2}$	$2.390x10^{-3}$	$1.580x10^{-3}$	$2.740x10^{-3}$	$9.40x10^{-4}$	$7.65x10^{-3}$	2.995
RBNN-training	$2.500x10^{-2}$						
RBNN-simulation	$3.381x10^{-5}$	$2.377x10^{-3}$	$1.576x10^{-3}$	$2.724x10^{-3}$	$9.075x10^{-4}$	$7.585x10^{-3}$	3.000
MPNN-training	$2.260x10^{-2}$						
MPNN-simulation	$2.305x10^{-5}$	$2.390x10^{-3}$	$1.541x10^{-3}$	$2.745x10^{-3}$	$9.319x10^{-4}$	$7.608x10^{-3}$	3.000

Table 4: Comparison of processing time using neural networks and finite element for the optimization of the laminated composite plate thickness.

Method	Error h_1 (%)	Error in h_2(%)	Error in h_3(%)	Error in h_4(%)	Error in h_t(%)
RBNN	0.544	0.253	0.584	3.510	0.850
MPNN	0.000	2.468	0.182	0.862	0.549

Table 5: Relative errors of design variables (%) using neural networks and finite elements for the thickness optimization of a laminated composite plate.

5.2 Example 2 –reliability based optimization of the ply angles of the layers on a laminated composite shell with non-linear behavior subjected to an external pressure load.

Problem description

In this example, the reliability index is calculated using a finite element model of a semi-cylindrical shell with geometric nonlinear behavior. An external pressure load P = 250000 Pa acting along the outer surface of the structure is considered. The dimensions and boundary conditions, taken from Almeida and Awruch(2009) are shown in Figure 12.

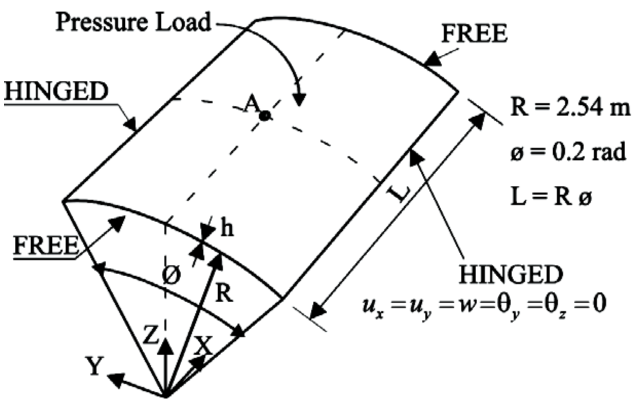

Figure 12: Composite shell under pressure load (Almeida and Awruch, 2009).

The total thickness of the laminated composite shell is 12.6 mm , with 28 plies and fiber orientation given by $\left[90^0_4, \mp \quad \mp \quad \right]_S$ measured with respect to the longitudinal direction of the shell. The material considered is glass-epoxy, which mechanical properties and strengths are E_1 = 39GPa, E_2 = 8.6 GPa, E_{12} = 3.8 GPa and ν_{12} = 0.28 R_x^t = 1080 MPa, R_x^c = 620 MPa, R_y^t = 39 MPa R_y^c = 128 MPa, R_{xy} = 89MPa. The limit state function is the Tsai-Wu criterion. The load-displacement curve for point A, using deterministic values, is shown in Figure 13.

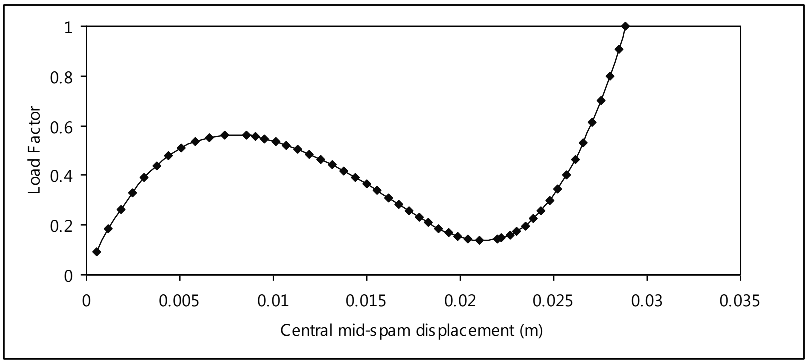

Figure 13: Mid-span non-linear load-deflection curve for the hinged semi-cylindrical shell

Five design variables are considered for this example, as indicated in Table 6.

Random Variable	Unit	Mean Value	Coeff. of Variation	Distribution Function
R_x^t	Pa	$1.08\text{x}10^9$	0.2	Log-Normal
R_x^c	Pa	$6.2\text{x}10^8$	0.2	Log-Normal
R_y^t	Pa	$3.9\text{x}10^7$	0.2	Log-Normal
R_y^c	Pa	$1.28\text{x}10^8$	0.2	Log-Normal
R_{xy}	Pa	$8.9\text{x}10^7$	0.2	Log-Normal

Table 6: Statistical parameters for random variables.

In the optimization of the fiber orientation angles, deterministic parameters, such as symmetry of the laminate arrangement and the possibility of a maximum of four contiguous layers with the same fiber orientation angle, were also used. The same random variables mentioned above, namely, the five parameters of the Tsai-Wu failure surface were assumed. This case is an optimization problem with seven parameters (because of the five random parameters, the symmetry of the laminate arrangement and the maximum number of contiguous layers with the same fiber orientation angle). These design variables could assume, for constructive practical reasons, discrete values, and the following values were adopted: 45^o, 0^0, 45^o e 90^o. Thus, using a genetic algorithm, a number of bits per design variable equal to 2 was defined, so that for the fiber orientation angles there are a binary encoding 00, 01, 10, 11, giving 4^7 = 16384 fiber orientation angles combinations.

The cylindrical shell of twenty eight layers was previously analyzed and the failure probability using Tsai-Wu criterion as ultimate limit state was evaluated. In this case, the reliability index of the structure against ultimate failure is β = 2.063 using Monte Carlo method with Importance Sampling and the Finite Element Method. This index was confirmed by several other methods of reliability assessment. The configuration of the laminate in this case is $\left[90^0_2, 90^0_2, \pm 45^0, 90^0_2, 90^0_2, \pm 45^0, 90^0_2\right]_s$.

Optimization of ply angles using the finite element method

Due to processing time required for the analysis, only the optimization based in ply angles was performed, imposing as a constraint a constant value of the reliability index equal to β = 5.0 employing FORM for reliability assessment. The finite element mesh as well as the parameters for the nonlinear analysis is the same used previously. The parameters used by the genetic algorithm are shown in table 7.

The obtained results using finite element analysis to evaluate the limit state function are given in Figure 14 and Figure 15.

No. of design variables (n)	7
Discrete values Fo design variables	-45^o, 0^0, 45^o and 90^o
Design variable's No. of bits (b)	2
Probability of Mutation (p_M)	1%
Probability of **C**rossover (p_c)	90%
Population size (*npop*)	300
Maximum number of grnerations (*ngen*)	100
Cost Function to be minimized (f)	$f = c\,\lvert\beta - 5\rvert$
Stopping criterion by diversity of individual's cost function ($COV = \sigma_f / \mu_f$)	5%
Penalty coefficient c	100

Table 7: Genetic Algorithm (GA) parameters.

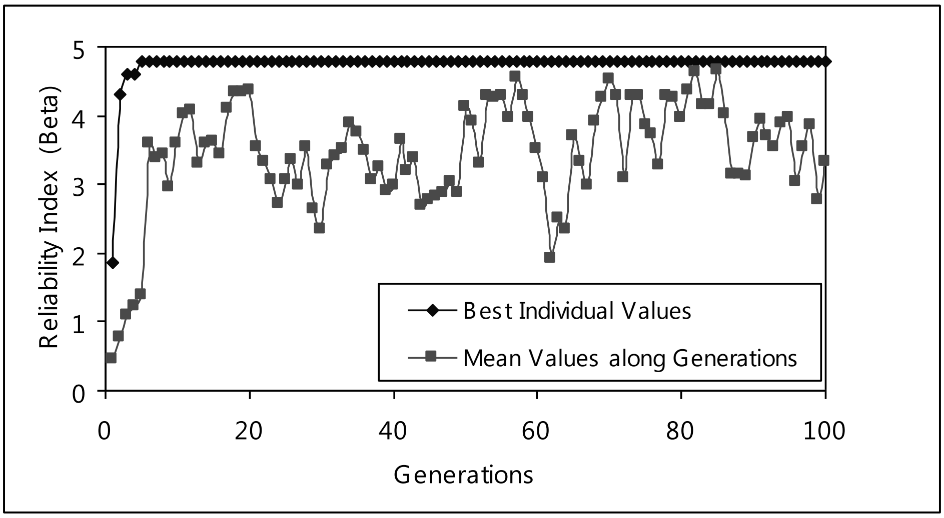

Figure 14: Reliability index for the best individual and generation mean values along generations using finite elements for limit state function evaluation.

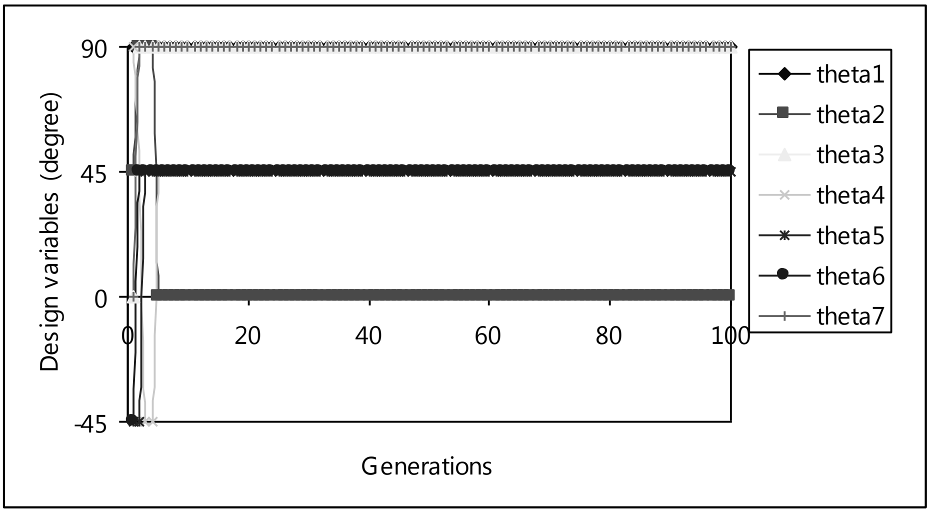

Figure 15: Reliability index of the best individual and generation mean values along generations using finite elements for limit state function evaluation.

The best combination of fiber orientation angles that provides reliability index closer to the desired value is $\left[90^0_2, 0^0_2, 90^0_2, 45^0_2, 45^0_2, 45^0_2, 90^0_2\right]_s$ and the reliability index value was β = 4.792. Obviously, in this case where design variables are discrete and cannot take any arbitrary value, the corresponding reliability indices may not reach exactly the required value, being the result of the optimization process the combination of angles that most closely approximates the required value for the reliability index. It should be noticed, therefore, that it is possible, keeping the same number of layers of the laminated composite structure, and changing only their fiber orientation angles, to increase the design reliability index from β=2.063 to β=4.792, which is an highly desirable situation, since there are no additional production cost of the new laminated composite material.

Optimization of the ply angles using Artificial Neural Networks

In this section neural networks are trained to substitute the application of a complete finite element analysis only in the part where the reliability analysis is performed. Thus, for a given combination of fiber orientation angles, the neural network is trained to help in the evaluation of the corresponding reliability index. The architecture of the neural network used here has seven inputs (ply angles) and one output (reliability index). In the cases of Multilayer Perceptron Neural Network and Radial Basis Neural Network, architectures of (7:10:10:10:1) and (7:120:1), respectively, were enough for training. The parameters used in neural networks, such as learning rate, tolerance for convergence, types of activation function, momentum, etc. are the same used in the previous example, changing only the architecture of the network.

The chosen training process consisted in the generation of 150 samples uniformly distributed over the search space (composed by 16384 combinations). For each of the 150 samples (combinations of fiber orientation angles) the reliability index using FORM was evaluated. This stage is the most time consuming in the analysis by neural networks, since finite element analysis are necessary for training sample generation. Thus, the generated samples were used for training and then the genetic algorithm was used to optimize the fiber orientation angles using the trained neural network. Optimization results by genetic algorithms using trained neural networks are presented in Figures 16 to 19.

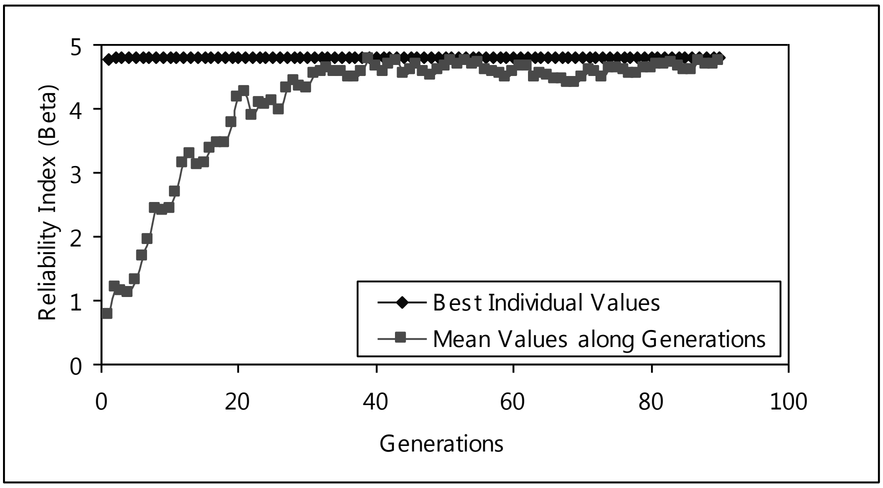

Figure 16: Reliability index history for the best individual and mean values along generations (Radial Basis Neural Network).

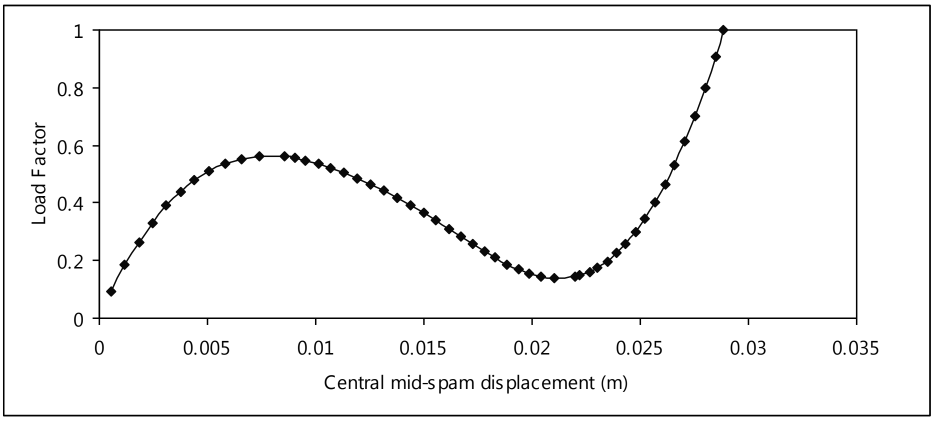

Figure 17: Mid-span non-linear load-deflection curve for a 12.6 mm thick hinged semi-cylindrical shell

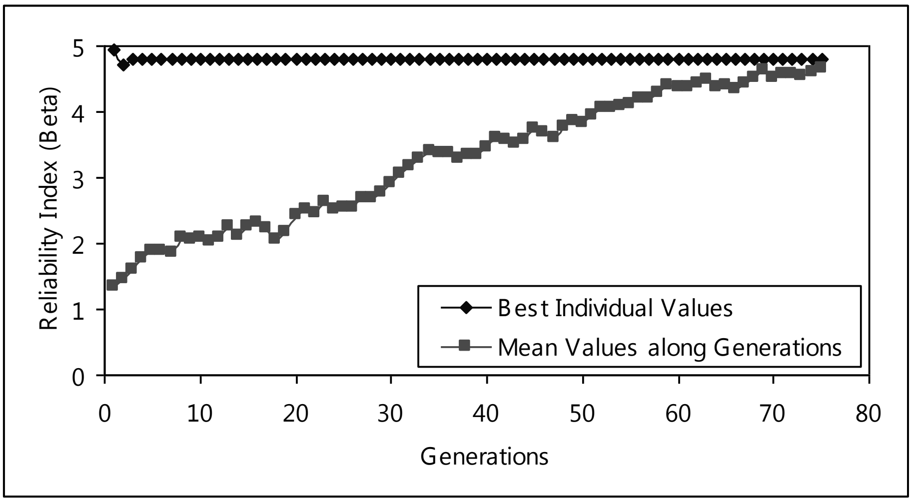

Figure 18: Reliability index of the best individual and mean values along generations (Multilayer Perceptron Neural Network).

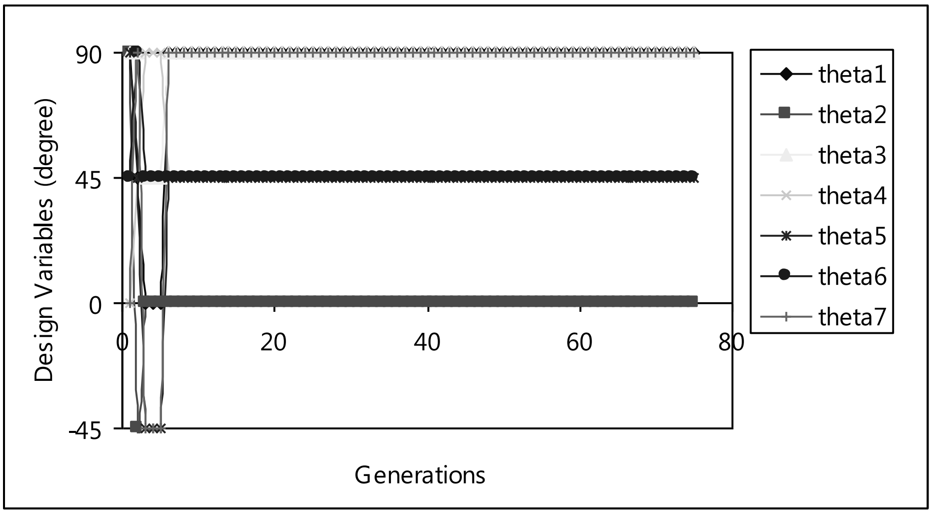

Figure 19: Fiber orientation of the best individual along generations (Multilayer Perceptron Neural Network).

In both cases, the optimum value of the combination of the fiber orientation angles was exactly the same which was found with the optimization using the finite element method, i.e. $\left[90_2^0, 0_2^0, 90_2^0, 45_2^0, 45_2^0, 45_2^0, 90_2^0\right]_s$ where the evaluated value of reliability index was β=4.791.

Comparisons regarding processing time

Table 8 shows the processing times, design variable values and reliability indexes obtained in the optimization using finite element analysis and artificial neural networks. It can be noticed that both ANN give large time savings on computer processing time. For both ANN, most of the processing time is spent in the training process, since the processing time spent by the ANN to calculate results is very small when compared with a complete Finite Element Analysis.

A small difference in the reliability index values using trained neural networks with respect to those obtained using complete finite element analysis may be explained by a lack of fit of the neural network with the training data or the number of samples used for the training process.

Method	Relative Processing Time	θ_1 (º)	θ_2 (º)	θ_3 (º)	θ_4 (º)	θ_5 (º)	θ_6 (º)	θ_7 (º)	β
GA+FORM+FEM	1.00	90	0	90	45	45	45	90	4.792
RBNN –training (150 samples)	1.05×10^{-1}								
GA+FORM+RNBR	3.25×10^{-4}	90	0	90	45	45	45	90	4.790
MPNN –training (150 samples)	$1.19 X 10^{-1}$								
GA+FORM+MPNN	2.33×10^{-4}	90	0	90	45	45	45	90	4.791

Table 8: Comparison of processing times for ply orientation optimization using the finite element method and artificial neural networks.

6 FINAL REMARKS

Some initial results were presented in this work dealing with reliability based optimization for structural problems involving laminated composite materials. A review of the issues addressed in this paper (genetic algorithm, artificial neural networks, reliability analysis and laminated composite material failure model) was briefly presented, and a methodology to reduce the processing time using trained ANN, when dealing with reliability based design optimizations, was proposed.

For the structural optimization, a Genetic Algorithm (GA) was used. GA are very suitable tools to obtain global optimal solution in problems where laminated composite materials are employed, because these materials handle with discrete variables (such as fiber orientation angles and number of layers) and multiple local optima are probable when dealing with reliability constraints. In some examples it was noticed that the convergence criterion used by the algorithm to stop the optimization process needs to be investigated since maximum number of iterations prevailed with respect to the diversity criterion (based on the coefficient of variation) leading to excessive number of iterations.

To evaluate the reliability index four classical methods were used: Standard or Direct Monte Carlo Method (MC), Monte Carlo Method with Importance Sampling (MCIS), First Order Reliability Method (FORM) and FORM with Multiple Check Points (FORM-MCP). In order to assess accuracy in the analysis, the first example uses both the Finite Element Method (FEM) and closed solutions for laminated composite plates to evaluate the limit state function (the Tsai-Wu failure criterion was adopted) regarding the reliability index. In optimization problems, where the reliability index is used as a constraint, a complete finite element analysis (FEA) is very expensive in terms of computer processing time (especially if MC or even MCIS are employed).

As an alternative to save computer processing time, trained Artificial Neural Networks (ANN) were used to evaluate the reliability index for the examples presented. Two types of ANN were used: Multilayer Perceptron Neural Network (MPNN) and Radial Basis Neural Network (RBNN). Their efficiency depends mainly of the chosen architecture and training process. In this work, both ANN reduced de computer processing time and the corresponding errors with respect to a complete FEA were very small.

In the reliability based optimization of the ply angles of the layers on a laminated composite shell with non-linear behavior, it can be noticed that the reliability index 5.0 was not attained since ply orientation has discrete values. The reliability based optimization resulted in a composite shell with reliability index about 4.792. Nevertheless it should be noticed that it was possible, keeping the same number of layers of the laminated composite structure and changing only ply orientation angles, to increase the reliability index of original design from β=2.063 to β=4.792. This is a highly desirable situation, since there are no additional production cost of the new laminated composite material.

In the last example, a small difference in the obtained reliability index value using trained neural networks with respect to those obtained using complete finite element analysis may be explained by a lack of fit of the neural network with the training data, indicating that the training process was not completed.

Future works would involve more complex problems with other limit state functions, such as delamination and hygrothermal effects. Improvements of the GA and the training process, as well as a parallel algorithm to solve large real problems, could also be implemented.

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