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STABILIZED FINITE ELEMENT METHODS FOR THREE-PHASE POROUS MEDIA FLOWS

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Abstract. This work presents a stabilized finite element method for the simplified black-oil model, which is of great importance in reservoir simulation. This model considers that the three fluids, water, oil and gas are incompressible and assumes no mass transfer among phases. We apply a stabilized finite element method, using the CAU shock-capturing technique, to discretize the equations in space. A segregated solution technique is used to solve the coupled differential equation system. We simulate a one-dimensional gas-water injection case in a petroleum reservoir. The results obtained in this work are compared with the analytical solution and also with other numerical solution computed with a coupled multiscale technique. We can conclude that the computational strategy utilized in this work yields results comparable to those obtained with the coupled multiscale method.

1. INTRODUCTION

Simulation of three-phase porous media flows is of great interest in the petroleum industry since it permits to perform predictions of well productivity. These simulations are also indeed important due to the fact that the governing equations that represent flows in petroleum reservoirs are much similar to those used for basin modeling (Hantschel and Kauerauf, 2009).

The most common model used to describe three-phase porous media flows is the black-oil model (Peaceman, 1977). This model is composed by large, coupled, strong nonlinear and time dependent partial differential equations. The complexity of these equations makes very important to start studying a simpler model, the simplified black-oil (Juanes and Patzek, 2005, Abreu, 2003, Sesini, 2006). This model is smaller in the sense that fewer partial differential equations are involved and the nonlinearities are not as strong as in the full black-oil model.

To advance in time the simplified black-oil model coupled differential equation system two methods are used (Juanes, 2005): the simultaneous or fully implicit and the sequential solution. The first method solves all of the coupled nonlinear equations simultaneously and implicitly. It is a stable method and can be used with very large time steps. Even so, this method demands lots of memory and has comparatively high computational cost. The sequential solution method advances in time each equation separately. Each equation may be solved explicitly or implicitly. This method is less stable, consequently uses smaller time steps, but requires less memory space and has low computational complexity compared to the simultaneous method.

Different methods are usually applied to discretize the governing equations when flexible meshes are used. Meshes can be structured or unstructured: control volume methods as CVFA (Control Volume Function Approximation) (Li et al., 2003), the hybrid finite element method as described in Abreu (2003) and stabilized finite element methods as SUPG (Streamline-Upwind / Petrov-Galerkin) (Coutinho and Alves, 1999; Sesini *et al.*, 2010) and ASGS (Algebraic Subgrid Scale) (Juanes and Patzek, 2005). SUPG consists in adding a stabilized term to the classical Galerkin formulation to magnify the stability in the direction of the streamlines. ASGS consists in a multiple-scale decomposition into resolved grid scales and unresolved subgrid scales, yielding a stable formulation for advection-dominated flows.

Stabilized finite element methods are often supplemented by a shock-capturing technique to control the instabilities occurring at high gradient regions and from sharp moving fronts. Coutinho and Alves (1999) and Sesini et al. (2010) employ the SUPG stabilized finite element method plus the CAU shock-capturing technique, introduced earlier by Galeão e Dutra do Carmo (1988), to simulate viscous-fingering in miscible flows in porous media. They reported very good results. At last, it is important to note that there is a necessity of developing stable, robust, efficient and accurate methods to simulate three-phase porous media flows.

In this work we introduce a sequential finite element method for the simplified black-oil model, using the Galerkin method to discretize the pressure equation and the SUPG stabilized finite element method with the CAU shock-capturing technique to discretize the water and gas saturation equations. In the following section we describe briefly the governing equations and in the subsequent section we present the discrete equations. In the section 4 we describe the numerical example studied and show the numerical results obtained. Finally in the last section we present the conclusions of this work.

2. GOVERNING EQUATIONS

We present in this section the governing equations of the simplified black-oil model to simulate three-phase flows in porous media (Juanes and Patzek, 2005). The full black-oil

model assumes that no mass transfer occurs between the water phase and the other two phases (oil and gas) and also that oil does not volatilize. The component gas can be found in the gas phase or also dissolved in the oil phase. The mathematical problem can be described by a pressure equation and the water and gas saturation equations (Peaceman, 1977).

In the simplified black-oil model is considered that the three fluids, water, oil and gas are immiscible, incompressible and there are no internal sources or sinks. Mass transfer between phases and gravitational and thermal effects are neglected. The governing equations in this case, in a domain with boundary Γ in a time interval [0, T] can be written as

- Water saturation equation:

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \mathbf{v}_w = 0, \text{em } \Omega \times \left[0,\ \mathrm{T}\right] \tag{1}$$

- Gas saturation equation:

$$\phi \frac{\partial s_g}{\partial t} + \nabla \cdot \mathbf{v}_g = 0, \text{em } \Omega \times \left[0,\ \mathrm{T}\right] \tag{2}$$

- Oil saturation equation:

$$\phi \frac{\partial s_o}{\partial t} + \nabla \cdot \mathbf{v}_o = 0, \text{em } \Omega \times \left[0,\ \mathrm{T}\right] \tag{3}$$

where the subscripts *w*, *o* and *g* denote the phases water, oil and gas. In these equations ϕ is the porosity, $\mathbf{v}_i$ and s_i are the velocity and the saturation of the *i* phase, respectively.

The velocity of each phase, $\mathbf{v}_i$, is represented by Darcy's law:

$$\mathbf{v}_i = -\mathbf{K}\lambda_i \nabla p_i, i = w, o, g \tag{4}$$

where **K** is the absolute permeability tensor depending only on the position, λ_i and p_i are the mobility and the pressure of the *i* phase, respectively. The mobility is given by

$$\lambda_i = \frac{k_i}{\mu_i} \tag{5}$$

where k_i and μ_i are the relative permeability and the viscosity of phase *i*, respectively.

Each relative permeability k_i depends on the saturations of the phases. These curves are obtained experimentally and typical cases for two phase flows are presented in Peaceman (1977). The porosity ϕ is considered constant and as thermal effects and compressibility are neglected the viscosity μ_i is assumed constant too.

The saturations of the water, oil and gas phases satisfy the constraint:

$$s_w + s_o + s_g = 1 \tag{6}$$

The phase pressures are related by the capillary pressures p_{cow} and p_{cgo} as:

$$p_{cow} = p_o - p_w \tag{7}$$

$$p_{cgo} = p_g - p_o \tag{8}$$

These capillary pressures p_{cow} and p_{cgo} are functions of the saturations of the phases and also measured experimentally just as described above for each relative permeability k_i.

The fractional flow functions f_i are defined as,

$$f_i = \frac{\lambda_i}{\lambda_w + \lambda_o + \lambda_g} \tag{9}$$

Substituting the expression (4) for the velocities of the phases and the constraint condition (6) into the mass conservation equations (1) to (3), and with a little bit of algebraic manipulation we get the following three partial differential equations (10) to (12):

$$\nabla \cdot \mathbf{v}_{\mathrm{T}} = 0 \tag{10}$$

$$\phi \frac{\partial s_w}{\partial t} + \mathbf{v}_{\mathrm{T}} \left(\frac{\partial f_w}{\partial s_w} \nabla s_w + \frac{\partial f_w}{\partial s_g} \nabla s_g \right) + \nabla \cdot \left[\mathbf{D}_{ww} \nabla s_w + \mathbf{D}_{wg} \nabla s_g) \right] = 0 \tag{11}$$

$$\phi \frac{\partial s_g}{\partial t} + \mathbf{v}_{\mathrm{T}} \left(\frac{\partial f_g}{\partial s_g} \nabla s_g + \frac{\partial f_g}{\partial s_w} \nabla s_w \right) + \nabla \cdot \left[\mathbf{D}_{gw} \nabla s_w + \mathbf{D}_{gg} \nabla s_g) \right] = 0 \tag{12}$$

where $\boldsymbol{D}_{ww}$, $\boldsymbol{D}_{wg}$, $\boldsymbol{D}_{gw}$ and $\boldsymbol{D}_{gg}$ are diffusion-like tensors defined by

$$\boldsymbol{D}_{ww} = -\boldsymbol{K} f_w (\lambda_o + \lambda_g) \frac{dp_{cwo}}{ds_w} \tag{13}$$

$$\boldsymbol{D}_{wg} = \boldsymbol{K} f_w \lambda_g \frac{dp_{cgo}}{ds_g} \tag{14}$$

$$\boldsymbol{D}_{gw} = \boldsymbol{K} f_g \lambda_w \frac{dp_{cwo}}{ds_w} \tag{15}$$

$$\boldsymbol{D}_{gg} = -\boldsymbol{K} f_g (\lambda_w + \lambda_o) \frac{dp_{cgo}}{ds_g} \tag{16}$$

We consider no-flow boundary conditions,

$$\mathbf{v}_i . \mathbf{n} = 0, i = w,\ o,\ g, \mathbf{x} \in \Gamma \tag{17}$$

where $\mathbf{n}$ is the unit outward normal to the boundary Γ. In this work we use p_o, s_w and s_o as the unknowns. The corresponding initial conditions are

$$p_o(\boldsymbol{x}, 0) = p_o^0(\boldsymbol{x}), \boldsymbol{x} \in \Gamma \tag{18}$$

$$s_w(\boldsymbol{x}, 0) = s_w^0(\boldsymbol{x}), \boldsymbol{x} \in \Gamma \tag{19}$$

$$s_o(\boldsymbol{x}, 0) = s_o^0(\boldsymbol{x}), \boldsymbol{x} \in \Gamma \tag{20}$$

In the next section we will see the finite element formulations applied to the simplified black-oil model.

3. SEMI-DISCRETE FINITE ELEMENT FORMULATION

In this section we present the SUPG stabilized finite element method applied to discretize the equations in space of the simplified black-oil model, using a sequential method to solve the coupled system. We also describe the CAU shock-capturing technique added to the formulation to stabilize the solution in the shock regions.

3.1 Sequential formulation of saturation equations

We describe in this section only the finite element stabilized formulation applied to the water and gas saturation equations. The solution of the pressure equation is trivial when there are no sources or sinks.

Considering a domain Ω subdivided in *nel* elements, Ω_e, e=1, 2,…,nel, where $\Omega=\bigcup_{e=1}^{nel} \Omega e$ and $\Omega i \cap \Omega j=\emptyset$. The interpolation functions spaces for water and gas saturations (s_w^h, s_g^h) and the weight functions space w^h are defined as

$$S_w^h = \{ s_w^h / s_w^h \in [H^{1h}(\Omega)], s_w^h / \Omega_e \in [P^1(\Omega^e)], s_w^h(t) = \overline{s}_{wi}^h \text{ em } \Gamma_i\} \tag{21}$$

$$S_g^h = \{ s_g^h / s_g^h \in [H^{1h}(\Omega)], s_g^h / \Omega_e \in [P^1(\Omega^e)], s_g^h(t) = \overline{s}_{gi}^h \text{ em } \Gamma_i\} \tag{22}$$

$$W^h = \{ w^h / w^h \in [H^{1h}(\Omega)], w^h / \Omega_e \in [P^1(\Omega^e)], w^h = 0 \text{ em } \Gamma\} \tag{23}$$

where $\boldsymbol{H}^{1h}(\Omega)$ is the finite-dimensional space defined on Ω and $P^1(\Omega_e)$ indicates first order polynomials in Ω_e.

Considering a standard discretization of Ω in finite elements, the Streamline-Upwind/Petrov-Galerkin stabilized formulation for the saturation equations of each phase (water and gas) is written as

$$\int_\Omega w^h (L_w^h(s_w^h, \mathbf{v}_{aw}^h))d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \tau_w L_w^* w^h L_w^h(s_w^h, \mathbf{v}_{aw}^h) d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \delta(s_w^h) \nabla w^h \nabla s_w^h d\Omega = 0 \tag{24}$$

$$\int_\Omega w^h (L_g^h(s_g^h, \mathbf{v}_{ag}^h))d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \tau_g L_g^* w^h L_g^h(s_g^h, \mathbf{v}_{ag}^h) d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \delta(s_g^h) \nabla w^h \nabla s_g^h d\Omega = 0 \tag{25}$$

The differential operators $L(s_w^h, \mathbf{v}_T \frac{\partial f_w}{\partial s_w})$ and $L(s_g^h, \mathbf{v}_T \frac{\partial f_g}{\partial s_g})$ are defined for the phases water and gas respectively by

$$L_w(s_w^h, \mathbf{v}_T \frac{\partial f_w}{\partial s_w}) = \phi \frac{\partial s_w^h}{\partial t} + \mathbf{v}_T \frac{\partial f_w}{\partial s_w} \nabla s_w^h + \mathbf{v}_T \frac{\partial f_w}{\partial s_g} \nabla s_g^h + \nabla \cdot (\mathbf{D}_{ww} \nabla s_w^h + \mathbf{D}_{wg} \nabla s_g^h) \tag{26}$$

$$L_g(s_g^h, \mathbf{v}_T \frac{\partial f_g}{\partial s_g}) = \phi \frac{\partial s_g^h}{\partial t} + \mathbf{v}_T \frac{\partial f_g}{\partial s_g} \nabla s_g^h + \mathbf{v}_T \frac{\partial f_g}{\partial s_w} \nabla s_w^h + \nabla \cdot (\mathbf{D}_{gg} \nabla s_g^h + \mathbf{D}_{gw} \nabla s_w^h) \tag{27}$$

The differential operator for each phase $L_i^*(w^h)$ is defined as it follows

$$L_i^* w^h = \mathbf{v}_T \frac{\partial f_i}{\partial s_i} \cdot \nabla w^h \tag{28}$$

In equations (26) and (27) the first term is the Galerkin formulation, the first summation of element level integrals is the SUPG stabilization term and the second summation represents

the CAU shock- capturing term added to the formulation to avoid spurious oscillations around shock regions.

The stabilized formulation SUPG involves a parameter τ that is dependent on the characteristic size of the element. Different forms to obtain τ and the element characteristic size for predominantly advective flows have been proposed. These forms are initially entranced by the works of Brooks and Hughes (1982), Hughes and Brooks (1979), Hughes and Tezduyar (1984), Tezduyar and Hughes (1982) and Tezduyar and Hughes (1983). Later Tezduyar and Park (1986) and Tezduyar and Ozawa (2000) introduce other expressions. The parameters they propose are computed from the element-level matrices and vectors, and these automatically take into account the local length scales as well as the advection field.

The stabilization parameter using here for each phase τ_i, introduced in the works of Coutinho and Alves (1996) and Codina (1993) is defined by

$$\tau_i = \frac{1}{2}\frac{h^e}{\left\| \mathbf{v}^e_{\ T} \dfrac{\partial f_i}{\partial s_i}^{\,e} \right\|} min\left(\frac{Pe^e}{3}, 1.0 \right) \tag{29}$$

where the denominator is the apparent velocity in the element and Pe^e is the local Peclet number defined as follows

$$Pe^e = \frac{1}{2} h^e \frac{\left\| \mathbf{v}^e_{\ T} \dfrac{\partial f_i}{\partial s_i}^{\,e} \right\|^3}{\left(\mathbf{v}^e_{\ T} \dfrac{\partial f_i}{\partial s_i}^{\,e} \right)^T \mathbf{D}_{ii} \mathbf{v}^e_{\ T} \dfrac{\partial f_i}{\partial s_i}^{\,e}} \tag{30}$$

The shock-capturing term added to the variational formulation also involves a parameter δ. This parameter also can be obtained using different expressions and in most cases depends nonlinearly on the element level residual. For a recent review see John and Knobloch (2007).

In this work we adopt the CAU shock-capturing technique introduced by Galeão and do Carmo (1988). In this technique the residual dependent parameter has the following form,

$$\delta_i = \alpha_i h^e \frac{\left| \mathbf{L}^e(s_i^{he}, \mathbf{v}^e_{\ T} \dfrac{\partial f_i}{\partial s_i}^{\,e}) \right|}{\left\| \nabla s_i^{he} \right\|} if \left\| \nabla s_i^{he} \right\| \neq 0 \tag{31}$$

where i=w,g and α_i is a parameter defined by

$$\alpha_i = \frac{1}{2} min\left(\frac{Pe^e_{\ //}}{4}, 0.7 \right) \tag{32}$$

The apparent velocity in the element projected in the direction parallel to the solution gradient is defined respectively by

$$\mathbf{v}^e_{\ T} \frac{\partial f_i}{\partial s_i}^{\,e}_{\ //} = \frac{\mathbf{v}^e_{\ T} \dfrac{\partial f_i}{\partial s_i}^{\,e} \cdot \nabla s_i^{he}}{\left\| \nabla s_i^e \right\|^2} \nabla s_i^{he} \tag{33}$$

and the local Peclet number correspondent to this apparent velocity is

$$Pe^{e}_{//} = \frac{1}{2} h^{e} \frac{\left\| \mathbf{v}^{e}_{T} \frac{\partial f_i}{\partial s_i}^{e}_{//} \right\|^{3}}{\left(\mathbf{v}^{e}_{T} \frac{\partial f_i}{\partial s_i}^{e}_{//} \right)^{T} \mathbf{D} \mathbf{v}^{e}_{T} \frac{\partial f_i}{\partial s_i}^{e}_{//}} \tag{34}$$

To discretize the governing equations in time is applied the finite difference method using the generalized trapezoidal rule (Hughes, 1987). To solve the nonlinear and nonsymmetric equations systems emanating from the discretization of the water and gas saturation equation an iterative predictor multi-corrector algorithm is employed where the linearized systems of equations are solved by GMRES(m) method with an element-by-element Gauss Seidel preconditioner. In all simulations we adopt the implicit, second-order member of the generalized trapezoidal rule as the basic time integrator and each nonlinear saturation equation is solved sequentially until a convergence criteria is met or for a fixed number of nonlinear iterations. The resulting time marching scheme can be viewed as a block-iterative predictor-multicorrector strategy, where each block corresponds to a saturation equation.

4. NUMERICAL RESULTS

In this section we present numerical results obtained applying the computational strategy described above to a one-dimensional and incompressible three-phase flow in porous media problem. We compare the numerical results with the analytical solution (Juanes, 2003) and a numerical solution presented in Juanes and Patzek (2005) adopted here as reference numerical solution.

4.1 Water-gas injection in a petroleum reservoir

This case consists in the simultaneous injection of water and gas into a porous medium filled with oil, gas and a little quantity of water as described in Juanes (2003).

The capillary diffusion is considered constant. Then the diffusive terms in equations (11) and (12) are given respectively as it follows

$$\nabla \cdot (\varepsilon_w \nabla s_w) \tag{35}$$

$$\nabla \cdot (\varepsilon_g \nabla s_g) \tag{36}$$

where the capillary diffusion coefficients are set with small values, that is,

$$\varepsilon_w = 0.001, \quad \varepsilon_g = 0.002. \tag{37}$$

The relative permeability functions are

$$k_{rw} = s_w^2 \tag{38}$$

$$k_{ro} = (1 - s_w)(1 - s_g)(1 - s_w - s_g) \tag{39}$$

$$k_{rg} = \beta_g s_g + (1 - \beta_g) s_g^2 \tag{40}$$

where β_g=0.1, and the values for fluid viscosities of water, gas and oil respectively are:

$$\mu_w = 0.875, \ \mu_g = 0.03, \ \mu_o = 2. \tag{41}$$

It is imposed a constant initial condition

$$u(x,0) = u_d \tag{42}$$

and a Dirichlet boundary condition on the left side of the domain,

$$u(0,t) = u_e \tag{43}$$

The medium has the following initial saturations s_w= 0.05, s_g= 0.4, and s_o= 0.55. Water and gas are injected in such proportion that the saturations at the inlet are s_w = 0.85 and s_g = 0.15, respectively. It is assumed that the saturations injected are constant during the experiment. These values of initial and injected saturations are representative of a linear water-alternate-gas (WAG) injection process in a petroleum reservoir after primary production.

4.2 Reference numerical solution

To compare the results obtained we use the numerical solution described in Juanes and Patzek (2005). They used a mesh with 40 elements, $h = 2.5e-2$, and a time step $\Delta t = 0.005$. Consequently the non-dimensional Peclet, *Pe*, and Courant, *cfl*, numbers for this case are respectively,

$$Pe = \frac{v_r h}{\varepsilon_r} \approx 30 \tag{44}$$

$$cfl = \frac{v_r \Delta t}{h} \approx 0.25 \tag{45}$$

where v_r and ε_r are the reference velocity and the capillary diffusion coefficient respectively. In this case the reference coefficients assume the following values:

$$v_r = 1,\ \varepsilon_r = 0.001 \tag{46}$$

Water and gas saturation equations are solved simultaneously in Juanes and Patzek (2005). The ASGS multiscale formulation is employed to discretize the equations in space. Stabilization parameters given by Hughes and Mallet (1986) are used. Also a multiscale shock-capturing technique to control spurious oscillations in the shock regions is added, where the nonlinear shock-capturing parameter has the following gradient-global form

$$\delta = C_{sc} h \frac{\left| \boldsymbol{R}^*(\boldsymbol{u}^h) \right|}{\left| \frac{\boldsymbol{U}_{sc}}{h} \right|} \tag{47}$$

where C_{sc} is a constant coefficient and $\boldsymbol{U}_{sc}$ is a vector of constant solution characteristic values, taken as,

$$\boldsymbol{U}_{sc} = (0.5, 0.5) \quad , \quad C_{sc} = 2 \tag{48}$$

and $\boldsymbol{R}^*(u^h)$ is the element residue.

4.3 Numerical solution

To simulate this one-dimensional problem we use a rectangular mesh of 50 cells along the longitudinal direction. Each cell is composed by 4 triangles totalizing 200 finite elements. Different time step sizes and multi-correction strategies, for the block-iterative predictor-multicorrector algorithm described in section 3, are employed to adjust these numerical parameters.

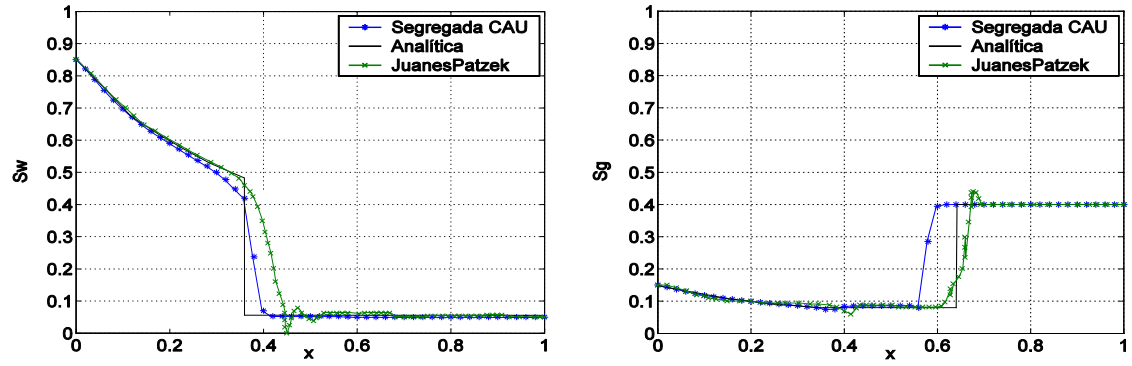
We use the following time step values:

$$\Delta t_1 = 0.005, \Delta t_2 = 0.01, \Delta t_3 = 0.02 \tag{49}$$

Therefore the respective Courant numbers, with $h = 0.01$ and $v_r = 1$, are

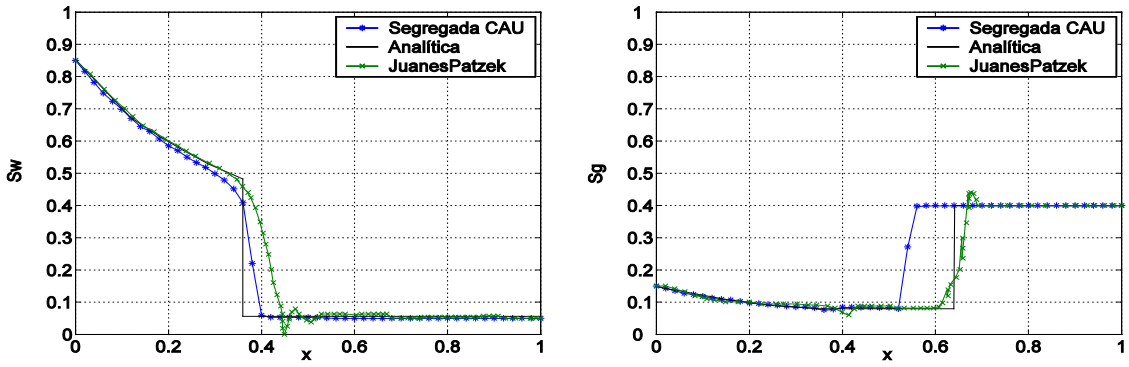
$$cfl_1 = \frac{\Delta t_1}{h} = 0.5, cfl_2 = 1, cfl_3 = 2 \tag{50}$$

In Figs. 1, 2 and 3 are shown the numerical results respectively obtained for each *cfl* and using 3 multi-corrections. We can note in these Figures that the more accurate result is the case using *cfl* = 0.5 (Fig. 1). The analytical and the reference numerical solutions are also plotted in the figures.



(a) (b)

Figure 1: Water Saturation (a) and Gas Saturation (b) with CAU Shock-capture Operator in the Time t = 0.5 with *cfl* = 0.5 and Maximum Number of Corrections Fixed in 3.



(a) (b)

Figure 2: Water Saturation (a) and Gas Saturation (b) with CAU Shock-capture Operator in the Time t = 0.5 with *cfl* = 1 and Maximum Number of Corrections Fixed in 3.

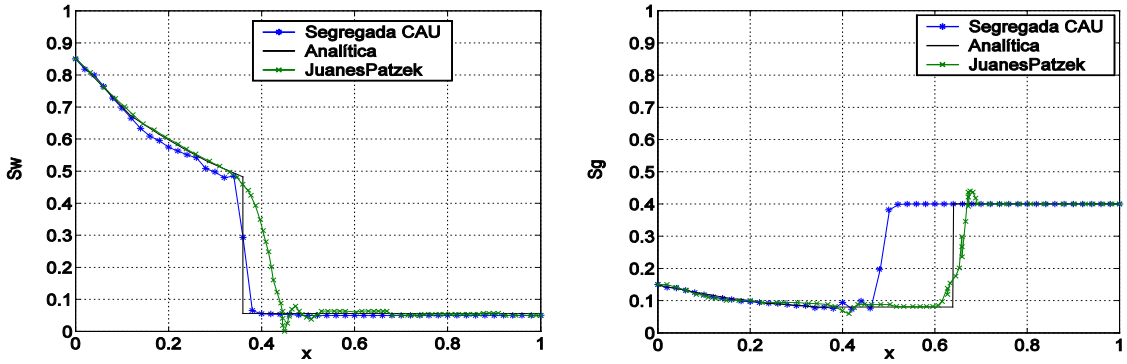


Figure 3: Water Saturation (a) and Gas Saturation (b) with CAU Shock-capture Operator in the Time t = 0.5 with cfl = 2 and Maximum Number of Corrections Fixed in 3.

The accuracy of the results obtained with the sequential method although using bigger step times, is comparable to the precision of the reference solution which uses a totally implicit technique. In Fig. 4 we show the results computed for the case cfl = 0.5 and for the maximum number of multi-corrections equal to 20. We can observe that this result is very close to the case with 3 multi-corrections without meaningful enhancements in the exactness of the solution. The time marching scheme as described in section 3 can be viewed as a block-iterative predictor-multicorrector strategy, where each block corresponds to a saturation equation. In Fig. 5 is presented the time history of the number of non-linear iterations of this time marching scheme, when the quantity of multi-corrections is set to 20.

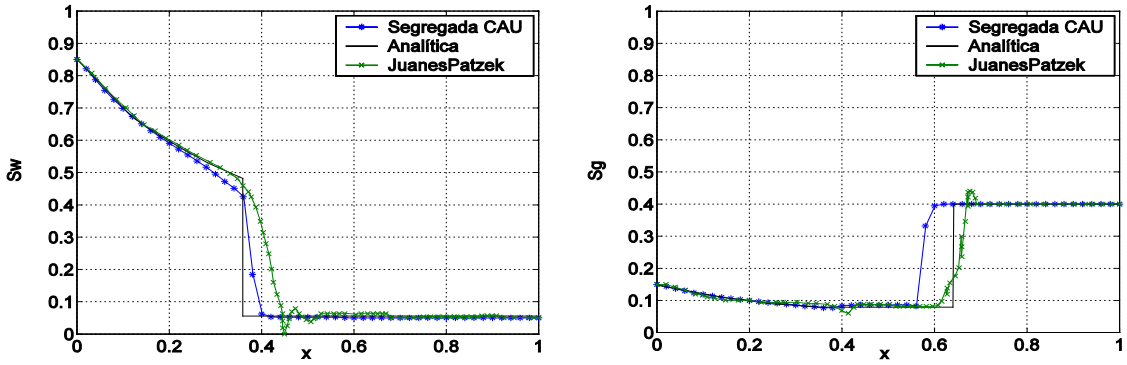


Figure 4: Water Saturation (a) and Gas saturation (b) CAU Shock-capture Operator in the Time t=0.5 with cfl = 2 and Maximum Number of Non-linear Iterations Fixed in 20

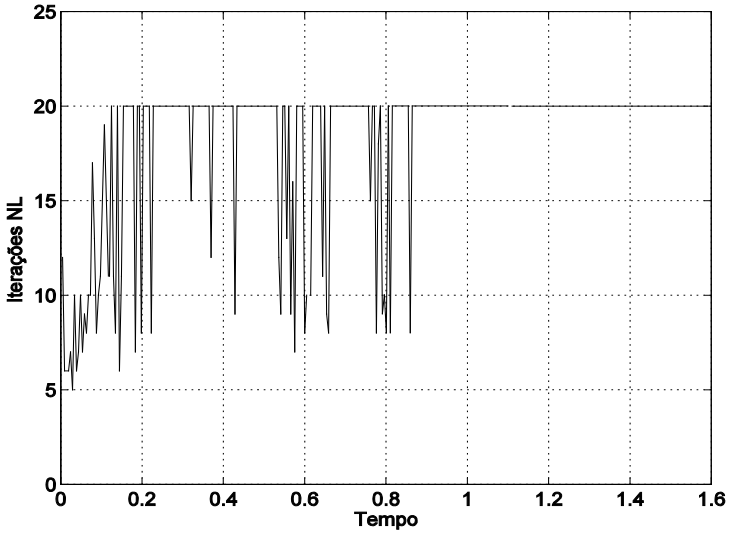


Figure 5: Non-linear Iterations for each Time Step.

5. CONCLUSIONS

We applied a stabilized numerical formulation to simulate a simplified black-oil model. The numerical method applied is essentially composed by the SUPG stabilized formulation to discretize the equations in space supplemented by the CAU shock-capturing technique to stabilize the solution in the shock regions. A sequential technique was used to solve the coupled equations system and the generalized trapezoidal method was applied to discretize the problem in time.

We simulated a one-dimensional three-phase flow case of simultaneous injection of water and gas in a petroleum reservoir (Juanes and Patzek, 2005). From the results obtained in this study, we can conclude that the numerical parameters that best represent the physical behavior of the problem for a mesh with 50 cells are: Courant number cfl = 0.5 and number of non-linear iterations for the multi-corrections fixed to 3.

We observed comparing the results with the analytical and numerical solutions that the sequential method combined with the formulation presented in this work is a competitive technique for the one-dimensional case studied here.

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