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SENSITIVITY OF STRUCTURAL RISK OPTIMIZATION WITH RESPECT TO EPISTEMIC UNCERTAINTIES

André T. Beck[a], Wellison J.S. Gomes[b], Felipe A.V. Bazán[c]

Department of Structural Engineering, University of São Paulo,
Av. Trabalhador São-carlense 400, 13566-590,São Carlos, SP, Brazil, http://www.set.eesc.usp.br

[a]*atbeck@sc.usp.br,* [b]*wellison63@hotmail.com,* [c]*favb@sc.usp.br*

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Abstract. In the context of structural design, risk optimization allows one to find a proper point of balance between the concurrent goals of economy and safety. Risk optimization allows the designer to find the optimum level of safety for a structure, in order to minimize total expected costs. Expected costs of failure are evaluated from nominal failure probabilities, which reflect the designers degree of belief in the structures performance. Such failure probabilities are said to be nominal because they are evaluated from imperfect and/or incomplete mechanical, mathematical and probabilistic models. Model uncertainty, together with other epistemic uncertainties, are likely to affect the solution of risk optimization problems. In this paper, the concept of robust optimization is used in order to study the sensitivity of structural risk optimization with respect to epistemic uncertainties. The investigation is based on a simple, illustrative problem, but should serve as a starting point for the construction of a robust version of the risk optimization problem. This formulation should lead to an optimum point of balance between economy and safety, which is insensitive to uncertainties in the problems solution models.

1 INTRODUCTION

Results of structural optimization should be robust with respect to the uncertainties inherently present in resistance of materials and structural loads. This notion has led to the development of different approaches to structural optimization: stochastic or robust optimization (Kall and Wallace, 1994; Birge and Louveaux, 1997, Beyer and Sendhoff, 2007; Schueller and Jensen, 2009), fuzzy optimization (Möller and Beer, 2004) and reliability-based structural optimization (Cheng et al., 2006, Silva et al., 2010). The robust formulation yields multi-objective optimization problems, where the mean performance of the system should be maximized whereas performance variance should be minimized. The balance between these objectives is a subjective choice of the analyst. In reliability-based design optimization, a probabilistic measure of system performance, also subjectively chosen by the analyst, is used as design constraint. Consequences of failure are not explicitly taken into account by these formulations. However, the main effect of uncertainties is the possibility of reaching a state of undesirable system performance. This possibility can be measured in terms of probability, and then multiplied by the cost (monetary measure) of failure. The resulting term, also known as expected cost of failure, can be incorporated in the objective function, leading to an unconstrained optimization problem (minimization of total expected costs). This formulation, also known as risk optimization (Beck and Verzenhassi, 2008), allows one to find the optimum point of compromise between different possible failure modes, as well as the optimum safety margin with respect to each failure mode.

2 FORMULATION OF RISK OPTIMIZATION PROBLEM

2.1 General formulation

Let **X** and **z** be vectors of structural system parameters. Vector **X** represents all random system parameters, and includes geometric characteristics, resistance properties of materials or structural members, and loads. Some of these parameters are random in nature; others cannot be defined deterministically due to uncertainty. Typically, resistances parameters can be represented as random variables and loads are modeled as random processes of time. Vector **z** contains all deterministic system parameters, like partial safety factors, design life, parameters of the inspection and maintenance programs, etc. Vector **z** may also include some parameters of random variables in **X**.

The existence of uncertainty implies risk, that is, the possibility of undesirable structural responses. The boundary between desirable and undesirable structural responses is given by limit state functions $g(\mathbf{z},\mathbf{x})=0$, such that:

$$D_f = \{\mathbf{z},\mathbf{x} \mid g(\mathbf{z},\mathbf{x}) \leq 0\} \text{ is the failure domain}$$
$$D_s = \{\mathbf{z},\mathbf{x} \mid g(\mathbf{z},\mathbf{x}) > 0\} \text{ is the safety domain} \tag{1}$$

Each limit state describes one possible failure mode of the structure, either in terms of performance (serviceability) or ultimate capacity of the structure. The probability of undesirable structural response, or probability of failure, is given by:

$$P_f(\mathbf{z},\mathbf{X}) = P[g(\mathbf{z},\mathbf{X}) \leq 0] \tag{2}$$

where P[.] stands for *probability*. The probabilities of failure for individual limit states and for system behavior are evaluated using traditional structural reliability methods such as FORM and SORM (Melchers, 1999; Ang and Tang, 2007).

The life-cycle cost of a structural system subject to risk can be decomposed in an initial or construction cost, cost of operation, cost of inspections and maintenance, cost of disposal and expected cost of failure. For a given failure mode, the expected cost of failure ($\mathrm{J}^{\text{expected}}$) - or failure risk - is given by the product of failure cost ($\mathrm{J}^{\text{failure}}$) by failure probability:

$$\mathrm{J}^{\text{expected}}(\mathbf{z},\ \mathbf{X}) = \mathrm{J}^{\text{failure}}(\mathbf{z})\, P_f(\mathbf{z},\ \mathbf{X}) \tag{3}$$

Failure costs include the costs of repairing or replacing damaged structural members, removing a collapsed structure, rebuilding it, cost of unavailability, cost of compensation for injury or death of employees or general users, penalties for environmental damage, etc. All failure consequences have to be expressed in terms of monetary units, which can be a problem when dealing with human injury, human death or environmental damage. Evaluation of such failure consequences in terms of the amount of compensation payoffs allows the problem to be formulated, without directly addressing the question.

For each structural component and for each failure mode, there is a corresponding failure cost term. The total (life-cycle) expected cost of a structural system becomes:

$$\begin{aligned}\mathrm{J}^{\text{total}}(\mathbf{z},\mathbf{X}) = {} & \mathrm{J}^{\text{initial or construction}}(\mathbf{z}) \\ & + \mathrm{J}^{\text{operation}}(\mathbf{z}) \\ & + \mathrm{J}^{\text{inspection and maintenance}}(\mathbf{z}) \\ & + \mathrm{J}^{\text{disposal}}(\mathbf{z}) \\ & + \sum_{\text{failure modes}} \mathrm{J}^{\text{failure}}(\mathbf{z}).P_f(\mathbf{z},\mathbf{X})\end{aligned} \tag{4}$$

The initial or construction cost increases with the safety coefficients used in design and with the practiced level of quality assurance. More safety in operation involves more safety equipment, more redundancy and more conservatism in structural operation. Inspection cost depends on intervals, quality of equipment and choice of inspection method. Maintenance costs depend on maintenance plan, frequency of preventive maintenance, etc. When the overall level of safety is increased, most cost terms increase, but the expected costs of failure are reduced.

Any change in $\mathbf{z}$ that affects cost terms is likely to affect the expected cost of failure. Changes in $\mathbf{z}$ which reduce costs may result in increased failure probabilities, hence increased expected costs of failure. Reduction in expected failure costs can be achieved by targeted changes in $\mathbf{z}$, which generally increase costs. This compromise between safety and costs is typical of structural systems.

The structural risk optimization problem can be stated as finding:

$$\mathbf{z}^* = \arg\min\{\mathrm{J}^{\text{total}}(\mathbf{z},\mathbf{X}) : \mathbf{z} \in S\} \tag{5}$$

where $S = \left\{\mathbf{z}_{\text{inf}} \le \mathbf{z} \le \mathbf{z}_{\text{sup}}\right\}$ and $\mathbf{z}_{\text{inf}}$ and $\mathbf{z}_{\text{sup}}$ are the lower and upper bounds of the design variables.

2.2 Elementary risk optimization problem

The risk optimization problem formulation just presented is general but quite involving for a study of the effects of epistemic uncertainties. In the present paper, an elementary form of the risk optimization problem is studied. This elementary form involves a time-invariant reliability problem (only random variables), hence no operation, inspection or maintenance costs. Moreover, the elementary risk optimization problem is based on the fundamental (demand-capacity or stress-strength) reliability problem, which involves only two random variables, *R* for resistance and *S* for stress. For this elementary but fundamental problem, the limit state equation is:

$$g(R,S) = R - S \tag{6}$$

An analytical solution to this elementary problem is available when both random variables have normal (Gaussian) distributions (Ang and Tang, 2007). Hence, it is assumed herein that:

$$\begin{aligned} R &\sim N(\mu_R, \sigma_R) \\ S &\sim N(\mu_S, \sigma_S) \end{aligned} \tag{7}$$

where μ and σ are the mean value and standard deviation of the respective variables. The failure probability can hence be calculated in closed form:

$$P_f = \Phi(-\beta) = \Phi\left(-\frac{\mu_R - \mu_S}{\sqrt{\sigma_R{}^2 + \sigma_S{}^2}} \right) \tag{8}$$

where β is the reliability index and Φ(·) is the cumulative distribution function for a standard normal variable $Y \sim N(0,1)$. In order to formulate the design problem, the central safety factor λ is introduced:

$$\lambda = \frac{\mu_R}{\mu_S}, \quad \text{hence } \mu_R = \lambda \mu_S, \quad \lambda > 1 \tag{9}$$

To simplify the problem even further, it will be assumed that the coefficient of variation (ρ) for both variables *R* and *S* is the same:

$$\rho_S = \frac{\sigma_S}{\mu_S} = \rho_R = \rho, \qquad \text{hence } \sigma_R = \rho \lambda \mu_S \tag{10}$$

Introducing Eqs. (9) and (10) in Eq. (8), one obtains, after some elementary algebra:

$$P_f(\lambda, \rho) = \Phi\left(\frac{1-\lambda}{\rho\sqrt{1+\lambda^2}} \right) \tag{11}$$

In Eq. (11), the central safety factor λ is the (risk) optimization variable and (ρ) is a measure of the problem´s uncertainty. For this elementary risk optimization problem there is only one possible failure mode. The initial or construction cost is assumed proportional to λ, and the cost of failure is kλ, where k is an additional problem variable. Hence, the objective function becomes:

$$\mathrm{J}^{\text{total}}(\lambda,\mathrm{k},\rho) = \mathrm{J}(\lambda,\mathrm{k},\rho) = \mathrm{J}^{\text{initial}}(\lambda) + \mathrm{J}^{\text{expected}}(\lambda,\mathrm{k},\rho) = \lambda + \mathrm{k}\,\lambda\, P_f(\lambda,\rho) \tag{12}$$

The elementary but fundamental risk optimization problem is to find:

$$\lambda^* = \arg\min_{\lambda} \{J^{total}(\lambda, k, \rho): \quad \lambda > 1\} \tag{13}$$

The KKT necessary conditions for λ^* to be a solution to the above problem are:

$$\begin{aligned}\frac{\partial J(\lambda,k,\rho)}{\partial\lambda} &= 1 + k\,\lambda\,\frac{\partial P_f(\lambda,\rho)}{\partial\lambda} + k\,P_f(\lambda,\rho)\\ &= 1 + k\left[\Phi\left(\frac{1-\lambda}{\rho\sqrt{1+\lambda^2}}\right) - \frac{\lambda}{\rho\sqrt{1+\lambda^2}}\phi\left(\frac{1-\lambda}{\rho\sqrt{1+\lambda^2}}\right)\left(1-\lambda(\lambda-1)(1+\lambda^2)^{-3}\right)\right]\\ &= 0\end{aligned} \tag{14}$$

2.3 Variants of the elementary risk optimization problem

The elementary but fundamental risk optimization problem stated in Eqs. (12) and (13) has many variants, according to the values of the coefficient of variation (ρ) and the failure cost multiplier k. Typical values of ρ for civil engineering problems are within $\{0.05 \leq \rho \leq 0.5\}$. The cost of failure for civil engineering structures, following the Joint Committee on Structural Safety (JCSS, 2001), is classified as:

$$\begin{aligned}\text{minor consequences:}&\quad k < 2\\ \text{moderate consequences:}&\quad 2 \leq k < 5\\ \text{severe consequences:}&\quad 5 \leq k < 10\\ \text{extreme consequences:}&\quad 10 \leq k\end{aligned} \tag{15}$$

In the design of structures with extreme failure consequences ($k \geq 10$), a detailed risk-benefit analysis is suggested (JCSS, 2001). Since this is the objective of risk optimization, this range of values $\{10 \leq k \leq 20\}$ is investigated herein.

Figure 1 illustrates the objective (cost) function (Eq. 12) for different, typical values of ρ and k. Figure 2 shows the derivative with respect to λ of these functions, where the roots of Eq. (14) can be observed (λ^*). In Figure 1 it can be observed that for ρ=0.1 the cost function is sharper, having a more clearly defined minimum. In Figure 2 it is also observed that the minima do not change much with respect to the failure cost multiplier k. On the other hand, for ρ=0.3 cost functions are broader, and their minima are not clearly marked. The minimum points change significantly with failure cost k, but optimum costs do not change much within different λ^*. Hence, it can already be observed in these figures that results of risk optimization (λ^* and $J(\lambda^*)$) are less sensitive to perturbations when problem uncertainty is larger (larger ρ) and when failure consequences are around k=10.

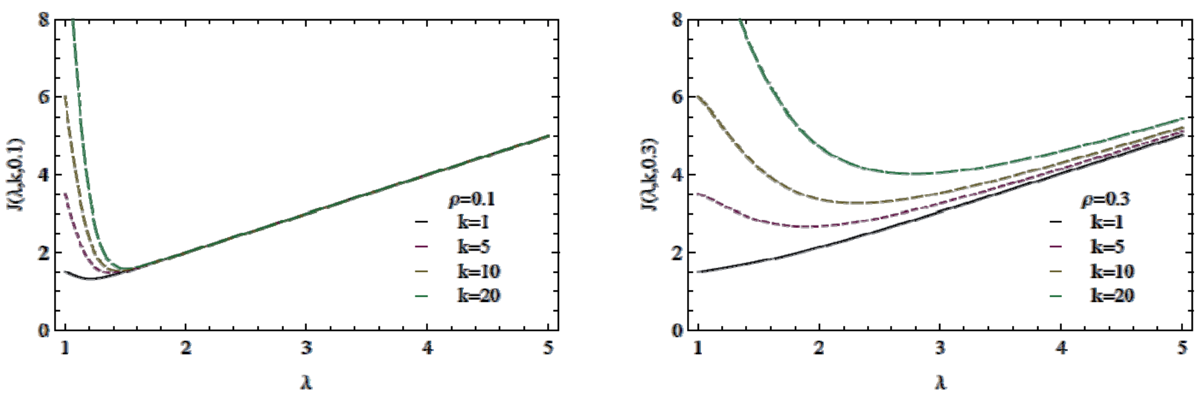

Figure 1: Cost functions J() in terms of c.o.v. (ρ), failure cost (k) and opt. parameter (λ).

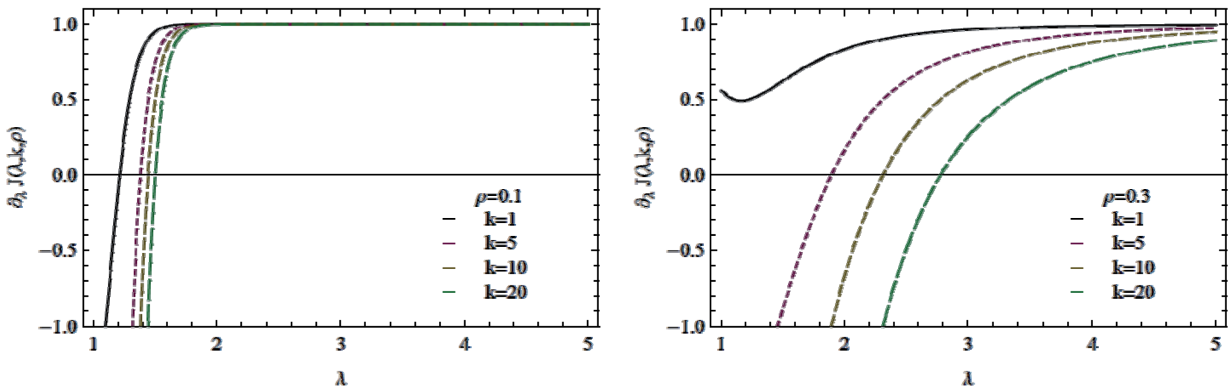

Figure 2: Derivative of cost functions J() in terms of opt. parameter (λ).

3 UNCERTAINTIES AFFECTING RISK OPTIMIZATION PROBLEM

Different types of uncertainties can affect the performance of engineering systems. One important distinction is between intrinsic and epistemic uncertainties. Intrinsic or aleatoric uncertainty is due to the inherent variability of physical processes, and includes uncertainty in resistance and other parameters of structural materials, load actions (environmental actions, in particular), structural dimensions, etc. Epistemic uncertainties are related to the level of knowledge about the problem, and include statistical uncertainties, model errors and phenomenological uncertainty. Intrinsic uncertainty is unavoidable and largely irreducible, whereas epistemic uncertainty can, in principle, be reduced by improving the level of knowledge about the problem. Broadly speaking, intrinsic uncertainties can be quantified and represented as random variables or stochastic processes, as do some types of epistemic uncertainties. This is called the probabilistic quantification of uncertainties. However, due to lack of knowledge, most forms of epistemic uncertainties cannot be quantified or represented probabilistically. Epistemic uncertainties admit a possibilistic representation using, for example, interval analysis or fuzzy number approaches. The interested reader is referred to Kiureghian and Ditlevsen (2009) for a general discussion on the aleatoric/epistemic classification of uncertainties.

In general robust optimization applications, no distinction needs to be made between intrinsic and epistemic uncertainties (Beyer and Sendhoff, 2007). In the present application to risk optimization, it is assumed that all uncertainties (either intrinsic or epistemic) that can be quantified probabilistically have been included in the formulation of the underlying reliability problem (Eq. 2). Hence, robustness of the risk optimization problem is sought with respect to epistemic uncertainties that cannot be described probabilistically. In the present paper, sensitivity of the risk optimization problem with respect to these epistemic uncertainties is investigated by using a possibilistic (interval) representation. In an future extension of the present work, a robust version of the risk optimization problem will be constructed by describing the same uncertainties using fuzzy variables (following, for example, Hanss and Turrin, 2010).

In the present paper, only uncertainties that can be represented within the simplified but fundamental risk optimization problem are discussed. With this objective, a perturbed version of the problem is introduced below:

$$\begin{aligned} \mathrm{J}_p &= \mathrm{J}_p^{\text{total}}(\lambda, \mathrm{k}, \rho, \delta\rho, \delta\mathrm{m}, \delta\mathrm{k}) = \lambda + \lambda\ \mathrm{k}\ (1{+}\delta\mathrm{k})\ 10^{\delta\mathrm{m}}\ P_f\left(\lambda, \rho(1{+}\delta\rho)\right) \\ \lambda_p^{\ *} &= \arg\min_{\lambda} \{\mathrm{J}_p^{\text{total}}(\lambda, \mathrm{k}, \rho, \delta\rho, \delta\mathrm{m}, \delta\mathrm{k}) : \ \ \lambda > 1\} \end{aligned} \tag{16}$$

The meaning of the perturbations δρ and δk is presented in the sequence. Variable δm is technically not a perturbation but an "order of magnitude" multiplication factor. For simplicity, however, we refer to Eq. (16) as a perturbed objective function.

3.1 Statistical uncertainties in random variable parameters

One type of epistemic uncertainty is the so-called statistical uncertainty, which is related to the limited number of samples that is used to characterize the probability distribution and the parameters of the random variables/processes describing intrinsic uncertainties. Statistical uncertainty can be reduced by increasing the number of samples and it could, theoretically, be eliminated by using infinite samples. Neither is the case for most real world applications, where number of samples is limited by budgetary constraints. Hence, statistical uncertainty is always present, in smaller or greater amounts.

There are many statistical model fitting approaches which allow statistical uncertainties in model parameters (mean, standard deviation) to be quantified probabilistically (Montgomery and Runger, 2009). Since this paper addresses a generic problem, it is assumed that no probabilistic description of model parameters is available. The simplified risk optimization problem considered herein is insensitive to a translation of the means (μ_R and μ_S). It is not insensitive to a perturbation of the means. However, since uncertainty in mean values is generally smaller than uncertainty in higher statistical moments, perturbation of the means is not considered in Eq. (16). However, a perturbation of the coefficient of variation (ρ) is assumed. This perturbation reflects the uncertainty in the standard deviation of both *R* and *S* random variables.

Figure 3 illustrates the isolated effect of perturbations δρ=0±0.2 in the objective function (Eq. 16) of the simplified risk optimization problem (for δm=δk=0), for ρ=0.1 and ρ=0.3 and for different cost multipliers k. It can be observed in Figure 3 that the effect of such perturbation is different for the optimum value λ_p^* and for the cost function $J_p(\lambda_p^*)$. For ρ=0.1, for example, the change in λ_p^* is relevant, but the change in $J_p(\lambda_p^*)$ is much smaller. The change in objective function $J_p(\lambda^*)$ is actually more relevant for the present investigation. The dotted vertical lines in Figure 3 indicate the change in perturbed cost functions for the optimum solution λ^* of the original, unperturbed problem. The optimum solution λ^* is found for the original problem, but the actual cost could be any of the perturbed cost functions $J_p(\lambda^*)$. Hence, a measure of the error introduced by the epistemic uncertainties is given as:

$$\varepsilon_{\mathrm{p}} = \frac{\mathrm{J}_p(\lambda^*) - \mathrm{J}(\lambda^*)}{\mathrm{J}(\lambda^*)} \tag{17}$$

In a robust risk optimization problem, this error should be minimized. In Figure 3 it can already be observed that the largest effects of perturbations δρ=0±0.2 occur for ρ=0.3 and k=20 (bottom right). Two types of errors can be derived from Eq. (17) and identified in Figure 3. The upper curves are obtained for $J_p(\lambda^*, \delta\rho{=}{+}0.2)$ and lead to positive errors, where actual (perturbed?) design costs are likely to be larger than the predicted, unperturbed cost $J(\lambda^*)$. The lower curves are obtained for $J_p(\lambda^*, \delta\rho{=}{-}0.2)$ and lead to negative errors, where actual design costs are likely to be smaller than the predicted, unperturbed costs, but where the possibility for even further improvements is lost.

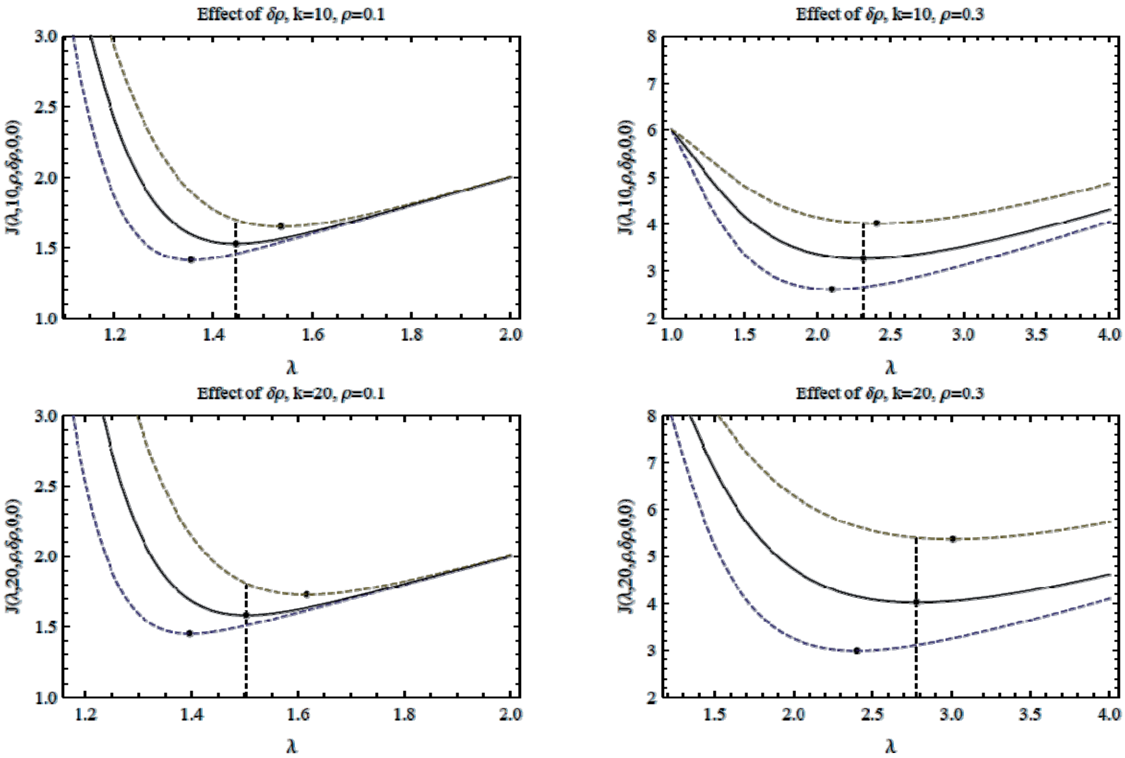

Figure 3: Effect of isolated perturbations $\delta\rho=0\pm0.2$ in cost functions and λ^*.

One kind of statistical uncertainty relates to the choice of probability distribution function (PDF) used to describe a given set of experimental observations. Hypothesis testing and distinct fitting tests (chi-square, KS, Anderson-Darling) can be used, but no definite conclusion about the correct PDF can be made. Usually, a number of different PDF´s will pass the fitting test and could be deemed acceptable to represent the same set of data. The failure probability (Eq. 2), however, is known to be highly sensitive to the tails of the assumed distributions. Hence, one significant source of epistemic uncertainty is the arbitrary choice between different acceptable PDFs. The simplified risk optimization problem presented herein is not appropriate for an investigation of this uncertainty, because a closed form solution is only available for Gaussian distributions. Nevertheless, it is assumed that uncertainty in probability distribution models induces an uncertainty in the calculated failure probabilities (Eq. 2), and this can be incorporated in the uncertainties described in section 3.3. In a future extension of this investigation, the principle of maximum entropy will be used to more accurately investigate the effect of uncertainty in probability distribution models.

3.2 Uncertainty in failure costs

This uncertainty relates to the deterministic unpredictability of (future) failure costs. In real engineering structures, such uncertainties arise, for example, from fluctuations in the price of commodities, since failure costs are always paid in the future. For failures involving human injury or death, or environmental damage, failure costs can always be quantified (from the practical managers point of view, at least) from the amount of past compensation payoffs for similar accidents. Clearly, such values cannot be defined with certainty. For the general risk optimization problem considered herein, these uncertainties cannot be quantified. However, an "order of magnitude" interval can be determined based on intuition. Figure 4 illustrates the effects of isolated perturbations $\delta k=0\pm0.2$ in objective function (Eq. 16), for $\delta\rho=\delta m=0$. It is observed that a perturbation of the same order (as δρ=0±0.2) produces much smaller effects for optimum points (λ_p^*) and for perturbed cost functions, $J_p(\lambda^*)$. Moreover, it can also be observed that the largest effects of perturbations δk=0±0.2 occur for ρ=0.3 (right).

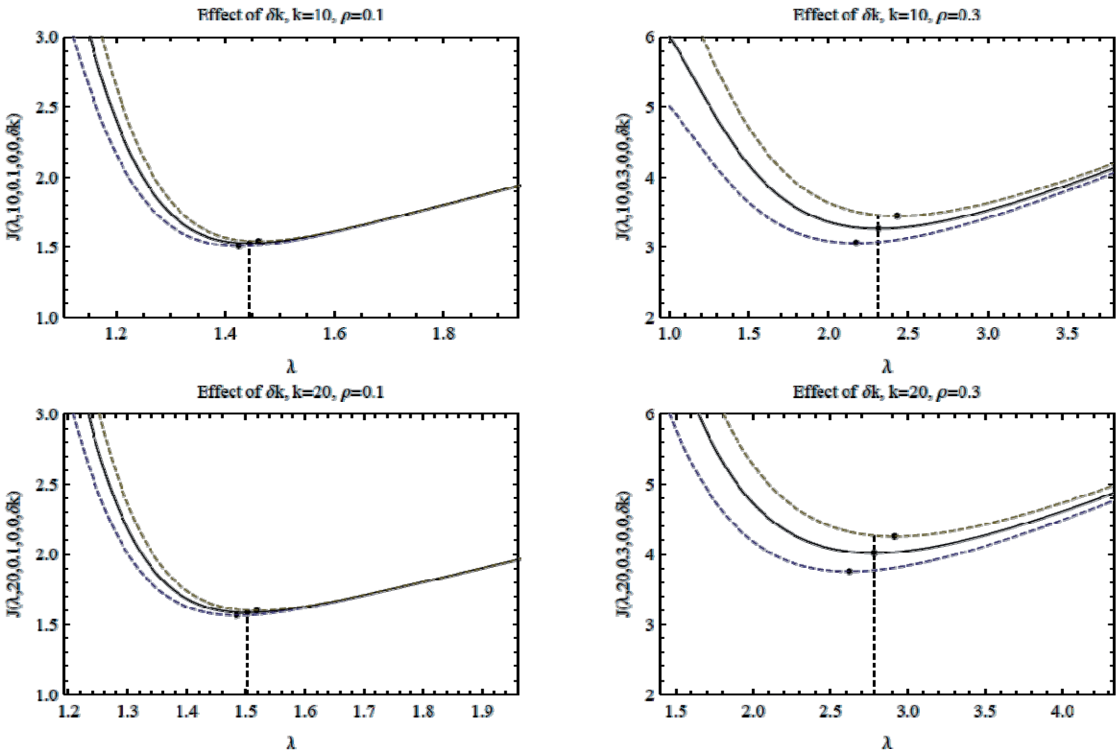

Figure 4: Effect of isolated perturbations δk=0±0.2 in cost functions and λ^*.

3.3 Model errors and phenomenological uncertainty

Model errors arise from the inability of structural load and resistance models to exactly predict loads, load effects and resistance of structural members or systems. In civil engineering design, it is customary to formulate limit state functions in terms of load effects and resistance of structural members. A resistance model gives the resistance of a structural member in terms of member dimensions and resistance of structural materials. The resistance of a reinforced concrete element, for example, is a function of yield stress (σ_s) and bar areas (A_s) of reinforcement steel, concrete strength and area (σ_c, A_c), cross-section dimensions (B, H) and other relevant variables:

$$R^{\text{model}} = R(\sigma_c, \sigma_s, A_c, A_s, B, H, \ldots) \tag{18}$$

Some model errors can be described probabilistically, by comparing model predictions with experimental results. In the example above, model error samples can be obtained from experimental results where the model parameters are varied:

$$M_e = \frac{R^{\text{experimental}}}{R^{\text{model}}} \tag{19}$$

When experimental results are available, model error can be represented by a proper PDF and incorporated into the formulation, becoming a new but quantifiable source of uncertainty.

The new resistance estimate, incorporating model uncertainties, becomes:

$$R = M_e R^{\text{model}} \tag{20}$$

It is assumed herein that such quantifiable model error uncertainties have already been incorporated in the load (*S*) and resistance (*R*) variables in the limit state equation (Eq. 6). Model error uncertainties, however, are not always quantifiable. Additional uncertainties exist. The actual resistance of a structural element, as constructed and in service condition, cannot be captured by laboratory experiments, for example. Manufacturing and actual service conditions (boundary conditions, loading positioning and orientation, etc) of the element cannot be predicted exactly. Hence, there are always additional sources of model uncertainties that cannot be quantified probabilistically. In this paper, interest lies in the effect of these epistemic uncertainties on the nominal, calculated failure probabilities (Eqs. 2 and 8) and on the results of the risk optimization problems. To take into account epistemic uncertainities that directly affect the nominal, calculated failure probabilities, the perturbation δm was included in Eq. (16). One such epistemic uncertainty is the probability distribution model for random variables *R* and *S*, as mentioned before.

One large source of uncertainty in most engineering problems is of phenomenological origin. Phenomenological uncertainty describes uncertainty related to the analyst´s understanding of the phenomena that actually control the behavior of the systems he designs. One typical example are structural failures that occur under failure modes that, for the original designer of the structure, were unpredictable or unimaginable. The history of structural engineering design has many examples of such failures. Dynamic wind excitation, that led to the collapse of the Tacoma Narrows strait bridge, was not known or not regarded as a relevant failure mode for which bridges had to be designed in those years. The World Trade Center, in New York, had been designed to withstand the collapse of a small aircraft, and a small fire fuelled by office material like paper, carpets, timber boards and so on. The enormous intensity of the fire fuelled by large amounts of jet fuel was not anticipated by the designers, but ultimately led to the collapse of the towers.

Phenomenological uncertainty can be very large, and can affect the nominal, calculated, failure probabilities by orders of magnitude. Model error and distribution model uncertainties will add to the uncertainty affecting calculated failure probabilities. In order to study the effects of epistemic uncertainties in calculated failure probabilities, "order of magnitude" perturbations δm were introduced in Eq. (16).

Figure 5 illustrates the effects of isolated perturbations δm=0±1 in objective function (Eq. 16), for δρ=δk=0. This "perturbation" of ±one order of magnitude (10^{-1}, 10^{0} and 10^{+1}) is considered representative of the combined effects of phenomenological, model error and distribution model uncertainties. It is observed in Figure 5 that such perturbations change the risk optimization objective function quite dramatically. For ρ=0.1 the change in λ_p^* is not too large, but the change in objective function $J_p(\lambda^*)$ is already significant. For ρ=0.3 both λ_p^* and $J_p(\lambda^*)$ vary significantly. The change in $J_p(\lambda^*)$ is more relevant for the perturbation δm=+1, since this increases the nominal, calculated failure probability by a factor of ten, largely increasing expected costs of failure. For perturbation δm=−1, the cost $J_p(\lambda^*)$ is actually smaller than $J(\lambda^*)$, and also not much different than $J(\lambda^*)$.

It is clear from Figures 3, 4 and 5 that perturbations δm=0±1 have greater impact than δρ=0±0.2 or δk=0±0.2. However, the combined effects of these perturbations can have an even greater impact on results of risk optimization.

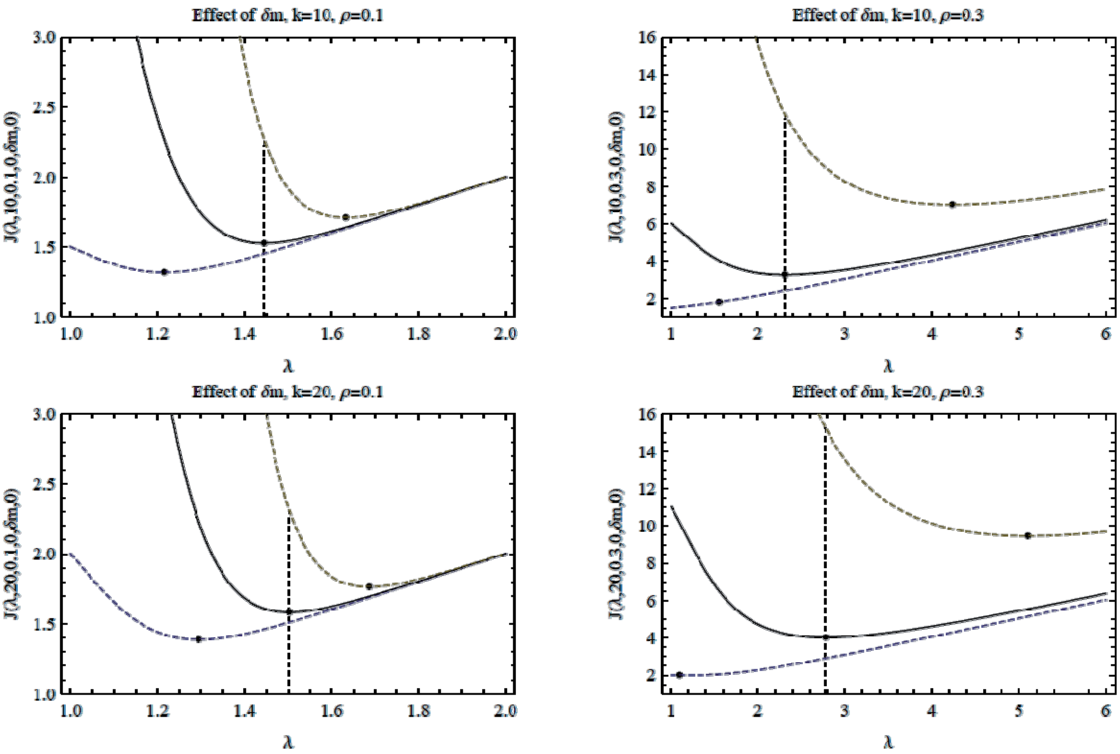

Figure 5: Effect of isolated perturbations δm=0±1 in cost functions and λ^*.

4 EFFECT OF COMBINED PERTURBATIONS IN TOTAL EXPECTED COST

The combined effect of perturbations δρ=0±0.2, δk=0±0.2 and δm=0±1 is studied for the different configurations of the fundamental risk optimization problem (parameters ρ and k). The coefficient of variation is varied from ρ=0.05 to ρ=0.4 with intervals of 0.05. The failure cost multiplier is varied from k=5 to k=20 with intervals of 2.5. Each of the resulting problems is solved for λ^* and the perturbed cost functions $J_p(\lambda^*)$ are evaluated, for all combinations of the three three-level perturbations. The maximum, mean and minimum values of the perturbed cost functions are evaluated. Corresponding maximum, mean and minimum measures of the error are obtained following Eq. (17). In fact, it turns out that not all perturbation combinations need to be evaluated. Following the results presented in Figures 3 to 4, the maximum error is obtained for $J_p(\lambda^*$, k, ρ, δρ=+0.2, δm=+1, δk=+0.2), and the minimum error is obtained for $J_p(\lambda^*$, k, ρ, δρ=−0.2, δm=−1, δk=−0.2).

The mean value of perturbed cost functions $J_p(\lambda^*)$ is evaluated from an integral over all perturbations of a function interpolated over the evaluated points, and divided by the size of the perturbation intervals. The mean value yields a measure of the "mean" error, and is less extreme than the maximum and minimum measures.

Figure 6 illustrates the results obtained for ρ=0.1 and ρ=0.3. The minimum error is nearly zero for ρ=0.1 and less than -0.5 (-50%) for ρ=0.3, which is very acceptable. The mean error is nearly zero for ρ=0.1 and less than around 50% for ρ=0.3.

The maximum error is much larger, and varies around 2 (200%) for ρ=0.1 and between 5 and 7.5 for ρ=0.3. This is quite large, but still smaller than the order of magnitude of the perturbations themselves (±1000%). Also, one should recall that the maximum error is a worst-case scenario: it corresponds to the worst combination of the three perturbations. Figure 7 shows maximum and minimum errors as a function of problem parameters ρ and k. It is observed that the maximum error increases almost linearly with parameters ρ and k. The minimum error varies very little with k, but increases almost quadratically with ρ.

It is unclear whether such magnitude of maximum errors renders the solution of risk optimization problems unrealistic or meaningless. On the positive side, however, is the observation that perturbations are not amplified, that is: the maximum error is large, but it is still smaller than the actual perturbations causing it. This, together with the fact that maximum errors are worst-case scenarios, should be sufficient to suggest that realistic results can still be obtained even in the presence of large epistemic uncertainties.

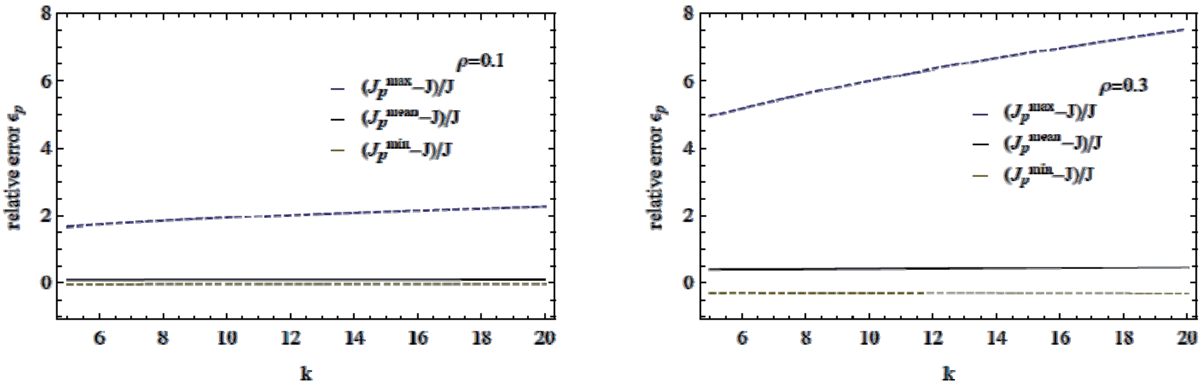

Figure 6: Maximum, mean and minimum errors in cost functions $J_p(\lambda^*$, k, ρ, δρ, δm, δk) for combined effect of perturbations, ρ=0.1 and ρ=0.3.

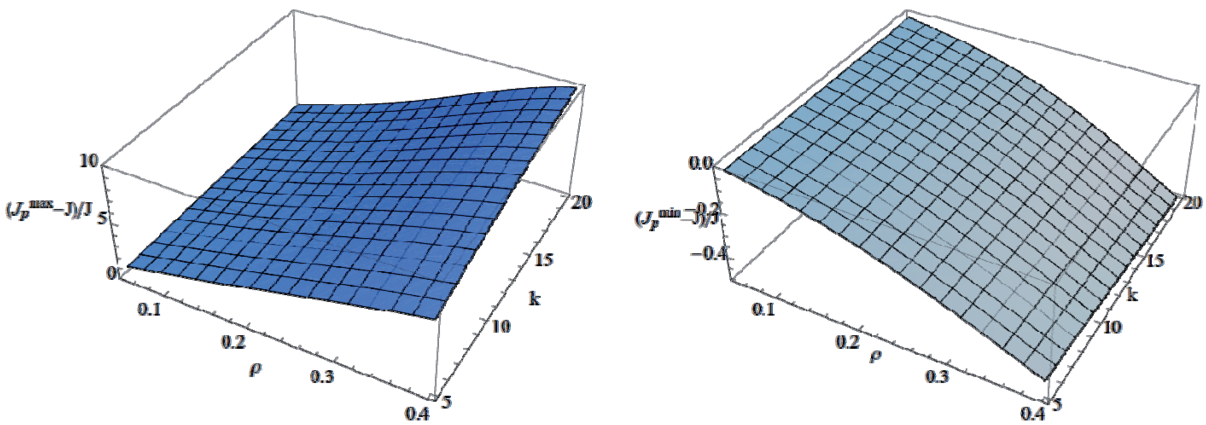

Figure 7: Maximum (left) and minimum (right) relative errors in cost functions $J_p(\lambda^*$, k, ρ, δρ, δm, δk) for combined effect of perturbations.

5 FUZZY ANALYSIS OF THE EFFECT OF EPISTEMIC UNCERTAINTIES

The maximum errors computed in last section are worst-case scenarios, corresponding to the worst combination of the three assumed perturbations. A less dramatic scenario is obtained by representing the same epistemic uncertainties using fuzzy variables (Möller and Beer, 2004). A fuzzy variable with triangular (linear) shape can be represented as:

$$x = \text{tri}(x_i, \bar{x}, x_f) \tag{21}$$

where $\bar{x}$ is the so-called center value or nominal value, and $[x_i, x_f]$ is the interval of definition. For a triangular fuzzy variable, the following membership function $m_x(x) \in [0,1]$ is defined:

$$m_x(x) = \min\left[\max\left(0, 1 - \frac{\bar{x} - x}{\bar{x} - x_i}\right), \max\left(0, 1 - \frac{x - \bar{x}}{x_f - \bar{x}}\right)\right] \quad \forall x \in \mathbb{R} \tag{22}$$

Other types of fuzzy variables are available, but the triangular shape is sufficient for the present analysis. Figure 8 illustrates the membership function for a variable $x = \text{tri}(0.8, 1.0, 1.2)$, which is used to represent the uncertainties in ρ and k. A similar representation is used for the uncertainty in the calculated failure probabilities:

$$\begin{aligned} \rho &= \bar{\rho} \cdot \text{tri}(0.8, 1.0, 1.2) \\ k &= \bar{k} \cdot \text{tri}(0.8, 1.0, 1.2) \\ dm &= \text{tri}(-1.0, 0.0, 1.0) \end{aligned} \tag{23}$$

The membership function is equal to one for the center value, $m_x(\bar{x}) = 1$. For decreasing values of m_x, nested intervals are obtained. Figure 8 (right) illustrates the computation of intervals for variable $x = \text{tri}(0.8, 1.0, 1.2)$ and for m_x=0.2.

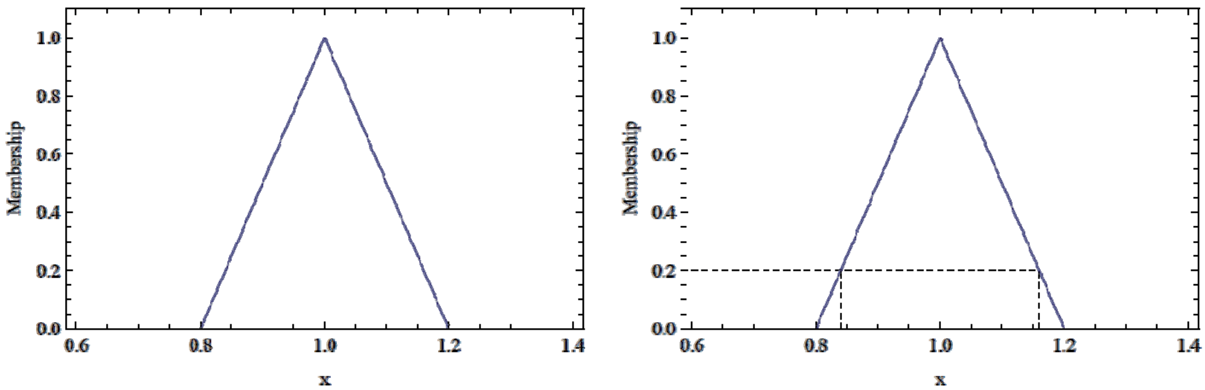

Figure 8: Fuzzy representation of membership function (left) for parameter x=tri(0.8, 1.0, 1.2) and determination of interval corresponding to membership level m_x=0.2 (right).

In order to compute the relative fuzzy error (Eq. 17) from the fuzzy representation of epistemic uncertainties(Eq. 23), intervals are computed for each of the fuzzy variables and for m_x=0.9 to m_x=0.0, with intervals of 0.1. For each set of intervals, maximum and minimum errors are computed in the same way as for the perturbations in Section 4. For each membership level m_x, the corresponding interval of the fuzzy error is given by the minimum and maximum values of the relative error. Results are presented in Figure 9 for ρ=0.1 and ρ=0.3 and for k=10 and k=20. In Figure 9 it becomes clear that, although the maximum error can be very large, the fuzzy error is still very much concentrated around the nominal value of zero. It can also be observed that the fuzzy errors are skewed towards the larger, positive values, as could have been anticipated from the results shown in Figures 6 and 7.

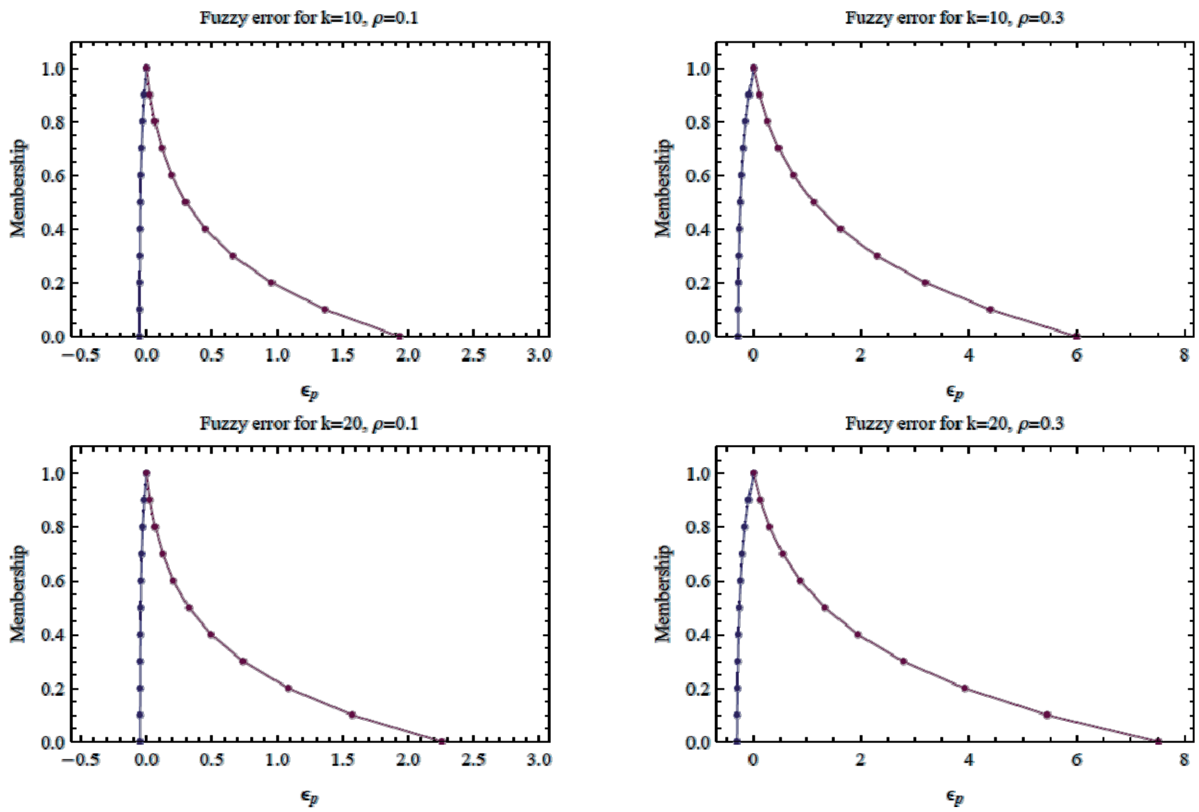

Figure 9: Relative fuzzy error in terms of parameters k and ρ for combined effect of fuzzy uncertainties.

6 CONCLUSIONS

In this article, the sensitivity of risk optimization problems with respect to epistemic uncertainties was investigated. In the structural risk optimization formulation it is assumed that all probabilistically quantifiable uncertainties (of intrinsic and epistemic types) have been incorporated in the structural reliability problem. This formulation, however, should be robust to uncertainties of the epistemic type, which cannot be quantified probabilistically. This is in contrast to general robust optimization formulations, where no distinction needs to be made between the different types or sources of uncertainty.

In the present paper, an elementary but fundamental risk optimization problem is investigated. Uncertainties in random variable parameters (coefficient of variation), failure cost and of model and phenomenological nature are considered by assuming possible intervals, based on expert (the authors) intuition. Perturbations corresponding to possibility intervals are investigated (±20% in coefficient of variation and cost terms, ±1000% in the nominal, calculated failure probabilities). Minimum, mean and maximum perturbation errors are computed.

Mean errors were found to be very close to zero. Minimum errors were found to be less than 50%, and as such are considered acceptable, given the broad range of investigated perturbations. Maximum errors of the order of 600 to 800% were found. It is unclear whether such magnitude of maximum errors renders the solution of risk optimization problems unrealistic or meaningless. On the positive side, however, is the observation that perturbations are not amplified, that is: the maximum error is large, but it is of the same order of magnitude of the actual perturbations causing it. Maximum errors are worst-case scenarios, which correspond to an unlikely combination of the studied perturbations.

Epistemic uncertainties affecting the fundamental risk optimization problem were also modeled using fuzzy variables of triangular (linear) shape. The resulting relative fuzzy error was shown to be highly skewed towards large, positive values. Still, it was observed that fuzzy errors are concentrated around the nominal values of zero, and that very large errors are obtained only for very unfortunate, unlikely combinations of epistemic uncertainties. This observation should be sufficient to suggest that realistic results can still be obtained even in the presence of large epistemic uncertainties.

The present paper addressed a very simple but fundamental form of the structural risk optimization problem. This investigation is already been extended to practical applications.

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