Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 6605-6618 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

BIOMECHANICAL MODEL AND CONTROL OF HUMAN POSTURAL SYSTEM AND SIMULATION BASED ON STATE-DEPENDENT RICCATI EQUATION

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Keywords: dynamic model, musculoskeletal system, state-dependent Riccati equation.

Abstract. The biomechanical model of a human postural system and the simulation of behaviour in movement can be applied in several areas, such as sports, engineering and medicine. The purpose of this work is obtain a dynamic and control model that represents a simplified postural system. The description of kinematic and dynamic links movements is based on Newton-Euler and Euler-Lagrange formulation. The resulting movements and forces are produced by sets of joint torque actuators. These dynamic models are non-linear with multiples input and output and many degrees of freedom. The model must be generic enough for accept several muscle models and control techniques. In this paper, it was used a control based on state dependent Riccati equation theory. A geometric model for simulations of postural control is obtained with Matlab/Simulink software.

1 INTRODUCTION

Researchers on robotic enthusiastically dreamed with smart machines that realize movements and tasks that a human can realize, with many expectation about modern control. Therefore, progresses in robotic control did not correspond to the expectation and the biggest difficulties were about the understanding of human motion in day-tasks. Human beings can manipulate objects and realize movements and complex tasks with facility and ability, through a biological evolution and trainee. A lot of researchers from several areas are involving in dynamic of human body. While a comprehensive theory of human movement is still far away, it is true that great progress has been made in the last few decades, with important contributions coming from researchers engaged in robotics. Mathematical models of biological systems are a field with the biggest increase in scientific development in the present-day. Although all techniques in simulation and mathematical models, the generalized application in musculoskeletal system is very complex (Thelen, 2006). The biomechanic model of musculoskeletal system and the simulation of the system behavior in motion can contribute to understanding the relationship among musculoskeletal properties and movement and articulation forces with application on medicine, diseases (Anderson et al. 2001, Pandy,1995) and sports (Thelen, 2005).

In the context of optimal control theory we consider the property of global stability of nonlinear systems with regulation control law based on the solution of a State Dependent Riccati Equation (SDRE) (Cloutier et al. 1996). Such method is known to yield a solution for the quadratic optimal control problems (Hammett et al. 1998). This control problem applied to the class of dynamical systems we are dealing with, the biomechanical model defines a topology where the solution is a set of control signals.). The SDRE nonlinear regulator produces a closed-loop solution which is locally asymptotically stable (Mracek and Cloutier, 1998; Banks et al. 2007). The procedure for drive the tip position to a desired point via SDRE technique considers successive optimal solutions for static equations and stabilized the system by feedback control. The advantage of this approach is the possibility of finding as muscle torque and kinematical trajectories, without previous measurement of the motion (Kuo, 1995; Menegaldo et al., 2003).

2. HUMAN DYNAMIC MODEL

In order to surpass the numerical difficulties associated with the optimal control solution, simpler biomechanical planar models with dynamic torque actuators of posture were adopted. The human system include four rigid segments representing the foot, the leg, the thigh and the upper part of the body, which are linked by three articulations, ankle, knee and hip joints modeled as frictionless hinges as shown in figure 1

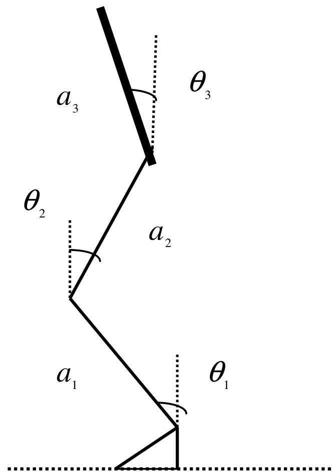

Figure 1. simpler biomechanical postural planar model.

In Figure 1, the vector of generalized variables is $q = [\theta_1, \theta_2, \theta_3]$ that represents articulations angle.

2.1. Position and Orientation of a Rigid Body

An human link, in this case, a leg, can be seen, in a mechanic point of view, like a kinematic open chain, formed by rigid bodies connected by rotation joints (Yang, 1990),. An end is connected to the base and the other one to the terminal element (foot). The structure movement is realized by a composition of elementary movements for each link, with respect to the preceding. In order to simulate a movement like walking or pedaling it is necessary a description of position and orientation of joints and links. It is also necessary a derivation of kinematic equations of leg, describing the position and orientation of the terminal element, as function of joint variables with respect to a reference coordinates system. These equations can be obtained through Denavit-Hartenberg convention (Siciliano and Valavanis, 1998; Bottega 2005). We express the transformation of coordinates that relate the system O_i with the system O_{i-1}, through the following steps:

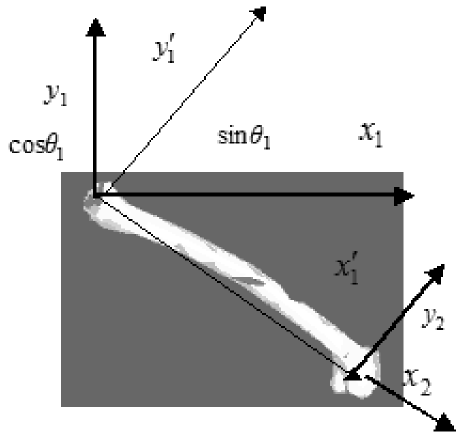

Figure 2. Different coordinated systems.

1. It starts from coordinated system O_{i-1}.

2. It does the rotation θ_i around of z_i axis. This operation take to the system O'_{i-1}, described by homogeneous rotation transformation matrix.

$$A_{\mathrm{i-1}}^{\mathrm{i}} = \begin{bmatrix} \cos\theta_{\mathrm{i}} & -\sin\theta_{\mathrm{i}} & 0 & 0 \\ \sin\theta_{\mathrm{i}} & \cos\theta_{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

3. It dislocates the coordinated system O'_{i-1} in a_i, through x'_{i-1} axis. This operation take to the system O_i, described by homogeneous dislocation transformation matrix.

$$D_{\mathrm{i-1}}^{\mathrm{i}} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Finally, it is obtained the transformation of coordinates

$$Q_{i-1}^{i} = A_{i-1}^{i} D_{i-1}^{i} \tag{3}$$

For successions of several transformations, as a musculoskeletal system, the position and the orientation of terminal element is

$$T_0^3 = Q_0^1 Q_1^2 Q_2^3 \tag{4}$$

4.2. Geometric Jacobian

Once knew the direct kinematic equations, we obtain the relationship among velocity of joints and linear and angular velocities of links, through the geometric jacobian (Bottega, 2005). These relations are necessaries for the derivation of movement equation of the musculoskeletal model as a whole.

The linear and angular velocity of a point p of the terminal element are expressed, like free vector in function of velocity of the joints $\dot{q} = \dot{\theta}$, with relations as

$$\begin{aligned} \dot{p} &= J_p(q)\dot{q}, \\ w &= J_0(q)\dot{q} \end{aligned} \tag{5}$$

which can be written in the following form

$$v = \begin{bmatrix} J_p \\ J_0 \end{bmatrix} \dot{q} = J(q)\dot{q}\,, \tag{6}$$

where the transformation matrix $J_{6\times n}$ is called geometric jacobian. The Equation (4) can be written in vectors

$$J = \begin{bmatrix} J_{p_1} & \cdots & J_{p_n} \\ J_{O_1} & \cdots & J_{O_n} \end{bmatrix}, \tag{7}$$

where $J_{pi}(q_i)\dot{q}_i$ represents the contribution of joint *i* to the linear velocity of the terminal link, while $J_{0i}(q_i)\dot{q}_i$ represents the contribution of this joint to the angular velocity of the terminal link.

4.3. Lagrange's Formulation

In order to obtain a set of differential equations of motion to adequately describe the dynamics of the musculoskeletal system, the Lagrange's approach can be used (Cannon, 1982). A system with n generalized coordinates $q_i = \theta_i$ must satisfy n differential equations of the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \xi_{f_i}, \quad \text{i=1,...,n}\,, \tag{8}$$

where ξ_{f_i} are the generalized forces with respect to the generalized coordinates q_i. L is the so called Lagrangian which is given by

$$L = \mathrm{T} - U\,, \tag{9}$$

where T represents the kinetic energy of the system and U the potential energy.

The Equation 8 define the relations among the generalized forces applied on the system and the joints velocity and acceleration.

4.4. Kinetic Energy

The kinetic energy of link *i* can be expressed as

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(q)\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{q}^T B(q)\dot{q} \tag{10}$$

where

$$B(q) = \sum_{i=1}^{n}\left(m_{l_i} J_p^{l_i T} J_p^{l_i} + J_0^{(l_i)T} I_{l_i} J_0^{(l_i)}\right) \tag{11}$$

is the generalized inertia matrix.

4.5. Potential Energy

The potential energy is given by

$$U = -\sum\left(m_{l_i} g_0^T p_{l_i}\right) \tag{12}$$

where g_0 is the gravity vector expressed in the base frame.

5. Equations of motion

By taking Equation 10 and Equation 12 , into account, the Lagrangian of Equation 8 can be written as

$$\sum_{j=1}^{n} b_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}(q)\dot{q}_k\dot{q}_j + g_i(q) = \xi_{f_i} \text{ ,i=1,...,n ,} \tag{13}$$

where $\mathbf{h}_l(\mathbf{q})$ represents a vector with centripetal, Coriolis and gravitational forces given by

$$h_{ijk} = \frac{\partial b_{ij}}{\partial q_k} - \frac{\partial b_{jk}}{\partial q_i}\,. \tag{14}$$

Finally, the equations of motion Eq.(13) for our system, which are modeled as a set of coupled rigid bodies are of the form

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u} \text{ ,} \tag{15}$$

where $\mathbf{M(q)}$ is the mass matrix, $\mathbf{g(q)}$ is the gravity vector, $\mathbf{u}$ is the $n \times 1$ vector of applied joint torques and $\mathbf{C(q,\dot{q})\dot{q}}$ is the Coriolis matrix and $q_i = \theta_i$ are the joint angle generalized coordinates.

With an appropriate coordinated system, the Jacobian of linear velocity Eq.(7) is given by

$$J_p^{(l_1)} = \begin{bmatrix} -l_1 s_1 & 0 & 0 \\ l_1 c_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_p^{(l_2)} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{16}$$

$$J_p^{(l_3)} = \begin{bmatrix} -l_3 s_{123} - l_2 s_{12} - l_1 s_1 & -l_3 s_{123} - l_2 s_{12} & -l_3 s_{123} \\ l_3 c_{123} + l_2 c_{12} + l_1 c_1 & l_3 c_{123} + l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix} \quad , \tag{17}$$

where $c_{(12...n)}$ and $s_{(12...n)}$ indicate, respectively, cossino and sino of ($\theta_1 + \theta_2 + ... + \theta_n$).

The Jacobians of angular velocity of baricenters are

$$J_0^{(l_1)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \; J_0^{(l_2)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \; J_0^{(l_3)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}. \tag{18}$$

3. CONTROL MODEL

3.1 Non-linear feedback controller: state-dependent Riccati equation (SDRE) technique

In nonlinear systems the matrices of the state are dependent on the variables of the problem. An optimal control of a dynamic model requires a formulation that seeks to minimize a cost functional and comply with restrictions on the model, which can be dynamic equilibrium equations, boundary conditions and / or others.

A problem for a system with the coefficients of the matrices of state, state dependent, can be formulated as follows (Mraček and Cloutier, 1998): minimize the cost functional,

$$\mathbf{J}_u = \frac{1}{2}\int_{t_0}^{\infty}\left[\mathbf{x}^{\mathrm{T}}\mathbf{Q}(x)\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}(x)\mathbf{u}\right]dt\,, \tag{19}$$

from the state **x** and control **u** subject to the system of nonlinear constraints

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(x) + \mathbf{B}(x)\mathbf{u} \\ \mathbf{y} &= \mathbf{S}(x)\mathbf{x} \\ \mathbf{x}(\mathbf{0}) &= \mathbf{x}_0 \\ \mathbf{x}(\infty) &= \mathbf{0} \end{aligned} \tag{20}$$

where $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$ and $\mathbf{Q} \in \Re^{n \times n}$ is symmetric positive semidefinite, and $\mathbf{R} \in \Re^{m \times m}$ is symmetric positive definite. The state-dependent Riccati equation (SDRE) approach for obtaining a suboptimal solution of this problem is

1 – Use direct parameterization to bring the nonlinear dynamics to the state-dependent coefficient form.

Rewriting the nonlinear dynamics, Eq.(20) in the state-dependent coefficient form $\mathbf{f(x)} = \mathbf{A(x)x}$, (Banks et al. 2007) we have

$$\dot{\mathbf{x}} = \mathbf{A}(x)\mathbf{x} + \mathbf{B}(x)\mathbf{u} \tag{21}$$

For the multivariable case we consider an illustrative example, $\mathbf{f}(\mathbf{x}) = [x_2, x_1^3]^T$. The obvious parameterization is

$$\mathbf{A}_1(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ x_1^2 & 0 \end{bmatrix}. \tag{22}$$

However, we can find another parameterization

$$\mathbf{A}_2(\mathbf{x}) = \begin{bmatrix} x_2 / x_1 & 0 \\ x_1^2 & 0 \end{bmatrix} \tag{23}$$

by dividing and multiplying each component of $\mathbf{f(x)}$ by x_1. Since there are at least two parameterizations, there are an infinite number. This is true since for all $0 \le \alpha \le 1$,

$$\alpha \mathbf{A}_1(\mathbf{x})\mathbf{x} + (1 - \alpha)\mathbf{A}_2(\mathbf{x})\mathbf{x} = \alpha \mathbf{f}(\mathbf{x}) + (1 - \alpha)\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}). \tag{24}$$

The choice of the parameterizations to be done must be appropriate, in accordance with the system and control of interest. An important factor for this choice is not to violate the controllability of the system, that is, the matrix of dependent controllability of the state

$$[\mathbf{B}(\mathbf{x}) \quad \mathbf{A}(\mathbf{x})\mathbf{B}(\mathbf{x}) \quad \ldots \quad \mathbf{A}^{n-1}(\mathbf{x})\mathbf{B}(\mathbf{x})] \tag{25}$$

to have full rank.

2- Solve the state-dependent Riccati equation (SDRE) (Banks et al. 2007).

The Hamiltonian for the optimal control problem, Eq.(20) is given by

$$\mathbf{H}(\mathbf{x},\mathbf{u},\boldsymbol{\lambda}) = \frac{1}{2}\left(\mathbf{x}^T\mathbf{Q}(\mathbf{x})\mathbf{x} + \mathbf{u}^T\mathbf{R}(\mathbf{x})\mathbf{u}\right) + \boldsymbol{\lambda}^T\mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u}\,. \tag{26}$$

The co-state is assumed to be of the form $\boldsymbol{\lambda}=\mathbf{P}(x)\mathbf{x}$, that it has dependence of the state. Using this form of the co-state and differentiating the Hamiltonian of the problem in relation the **u**, gets the feedback control

$$\mathbf{u} = -\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}^T(\mathbf{x})\mathbf{P}(\mathbf{x})\mathbf{x}\,, \tag{27}$$

where $\mathbf{P}(x)$ is gotten of the solution of the state-dependent Riccati equation

$$\mathbf{A}^T(\mathbf{x})\mathbf{P}(\mathbf{x}) + \mathbf{P}(\mathbf{x})\mathbf{A}(\mathbf{x}) - \mathbf{P}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}^T(\mathbf{x})\mathbf{P}(\mathbf{x}) + \mathbf{Q}(\mathbf{x}) = \mathbf{0}\,. \tag{28}$$

Substituting the control Eq.(27) into Eq.(21), we find $\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{R}^{-1}\mathbf{B}(\mathbf{x})^T\mathbf{P}(\mathbf{x})\mathbf{x}$, thus getting the system in the closed form.

The optimality condition satisfies the solution of the local suboptimal control. In the infinite time, in the standard case of the linear quadratic Regulator (LQR) (with matrices of weight of the functional with constant coefficients) it is verified that this equation is local satisfied. For some special cases, as systems with little dependence of the state or few variable of state, the equation can be solved of analytical form (Shawky et al. 2007). On the other hand, a numerical solution can be gotten with a tax of enough great sampling. An approach, with local stability of the system of closed mesh is resulted of the use of the technique of the state-dependent nonlinear Riccati equations.

By Mracek and Cloutier (1998), an important factor of method SDRE is that it does not cancel the benefits that can come from the nonlinearities of the dynamic system. The reason for this can be that it does not demand dynamic inversion and nor linearizations in the feedback of the nonlinear system.

4. RESULTS

4.1 Dynamic System

The dynamic system defined by Eq. (15) can be parameterized in first order equations and written in the state-dependent coefficient form

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u}\\ \mathbf{y} &= \mathbf{S}(\mathbf{x})\mathbf{x}\end{aligned} \tag{29}$$

where $\mathbf{x} = \left[\theta_1 - \theta_{d1},\ \theta_2 - \theta_{d2},\ \theta_3 - \theta_{d3},\ \dot{\theta}_1 - \dot{\theta}_{d1},\ \dot{\theta}_2 - \dot{\theta}_{d2},\ \dot{\theta}_3 - \dot{\theta}_{d3}\right]^T$ is a time state-dependent, $\dot{\mathbf{x}} \in \Re^6$ is the vector of the first order time derivates of the states, defined as the difference between the regulated θ_i output and the value of the set-point θ_{di}, $\mathbf{u} \in \Re^3$ is the control vector, U is the control constraint set and $\mathbf{S}(\mathbf{x})$ is the output matrix. This system represents the constrains from the nonlinear regulator problem, together with $\mathbf{x}(t_0) = \mathbf{x}_0$, $\mathbf{x}(\infty) = 0$, respectively the initial and final conditions.

The coefficient dependent matrices are given by

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1} & -\mathbf{M}^{-1}(\mathbf{C}+\mathbf{g}) \end{bmatrix}, \quad \mathbf{B}(\mathbf{x}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix},$$
$$\mathbf{Q}(\mathbf{x}) = \mathbf{S}^T(\mathbf{x})\mathbf{S}(\mathbf{x}), \quad \mathbf{S}(\mathbf{x}) = diag\left\{\sqrt{q_i}\right\}_{i=1,\dots,6} \quad , \tag{30}$$

where $\mathbf{A} \in \Re^{6x6}$, $\mathbf{B} \in \Re^{6x3}$ and $\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$. It is assumed that **f(0)=0**, which imply that the origin is an equilibrium point.

A state feedback rather that output feedback is adopted to enhance the control performance. The non-quadratic cost function for the regulator problem is given by Eq. (19) where **Q(x)** is semi-positive-definite matrix and **R(x)** positive definite, chosen as **Q** = *diag*(50, ..., 50) and **R** = *diag*(1, ..., 1) and the control law is given by Eq. (27).

There are weighting matrices on the outputs and control inputs, respectively. For a pointwise linear fashion there matrices are assumed with constant coefficients.

It is shown in Mracek and Cloutier (1998) that

1) In the neighborhood Ω about the origin the SDRE method guarantees a closed-loop solution, local asymptotic stability.
2) In the scalar case, the SDRE method reaches the optimal solution of the feedback regulator problem performance index Eq. (30), even when **Q** and **R** are functions of *x*.
3) In general multivariable case, the SDRE nonlinear feedback controller satisfy the first necessary condition for optimality, $\mathbf{H_u} = \mathbf{0}$, (**H** is the Hamiltonian from the problem Eq. (29), while the second necessary condition for optimality, $\dot{\boldsymbol{\lambda}} = -\mathbf{H_x}$, is asymptotically satisfied at a quadratic rate as **x** goes to zero.
4) The system Eq. (29) is pointwise controllable and observable, for a region in neighborhood Ω about the origin. For controllability this mean $[\mathbf{B} \vdots \mathbf{A^n B}]_{n=1,\dots,5}$ from the static problem $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, in this neighborhood. SDRE method considers a solution for this static pointwise problem, for small time interval.

The SDRE technique to obtain a suboptimal solution for this problem has the following procedure (Mracek and Cloutier, 1998).

Step 1. Define the space-state model of the manipulator with the state-dependent coefficient form as in Eq. (30).
Step 2. Measure the state of the system **x**(*t*)**,** i.e., define $\mathbf{x(0)} = \mathbf{x_0}$, and choose the coefficients of weight matrices **Q** and **R**.
Step 3. Solve the Riccati equation, Eq. (28) for the state **x**(*t*), considering pointwise static solutions, i.e., solve $\mathbf{A^T P} + \mathbf{PA} - \mathbf{PBR^{-1}B^T P} + \mathbf{Q} = \mathbf{0}$ for each step.
Step 4. Calculate the input signal from Eq. (27)
Step 5. Integer the system Eq.(29) and update the state of the system **x**(*t*) with this results. Go to step 3.

4.2 Stability Analysis

Using the SDRE method implies that the controllability of the static problem depends on the size from the time step. In this work this chosen size was 0.001 sec. Controllability is lost for large time steps. In our case, with high frequencies, the time step is also important for characterizing correct frequency period.

The choice of the best values for the state weighting matrix **Q** is very important. A good choice can improve the efficiency of the controllers. In this work we have tested some weighting matrices and concluded that, for our control design, the good results are obtained with values around **Q**=*diag{50,…,50}*. Smaller or greater values affects the control efficiency.

The stability analysis for this system may be examined around the origin (Shawky *et al.*, 2007). The linearization technique was used for

$$\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x},\ \mathbf{w}(\mathbf{x},\mathbf{u}) = \mathbf{B}(\mathbf{x})\mathbf{u}$$
$$\mathbf{J_f} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]_{\mathbf{x=0}},\quad \mathbf{J_h} = \left[\frac{\partial \mathbf{w}}{\partial \mathbf{u}}\right]_{\mathbf{x=0}}, \tag{31}$$

where $\mathbf{J_f}$ and $\mathbf{J_h}$ are the Jacobian matrices of **f(x)** and **w(x,u)** at **x = 0**, respectively. If the eigenvalues of the Jacobian have negative real part, the point **x = 0** is a locally stable equilibrium point. If one of the real part are positive, then the point **x = 0** is an unstable equilibrium point. In our case, $\mathbf{J_f} = \mathbf{A(0)},\ \mathbf{J_h} = \mathbf{B(0)}$. Then, a necessary condition for a local stability is that the pair $\{\mathbf{A(0)},\mathbf{B(0)}\}$ has to be stabilizable. It was obtained one positive eigenvalue, so that we have an unstable equilibrium point at the origin. Even so, the linearized system is pointweise controllable and observable for a region of interest Ω. This fact is shown in Shawky *et al.* (2007), and we have also verified the controllability for our system. The stability is obtained by full state feedback gain $\mathbf{u} = -\mathbf{kx}$.

5. SIMULATIONS

We consider a simplified model presented in Figure 1 that uses torque as variable of input control, where the mathematical model is given by Eq. (29). Let m_1, m_2 and m_3 be the masses, l_1, l_2 and l_3 the baricenter length, I_1, I_2 and I_3, the inertial moment and g, the gravity acceleration.

This model uses the following anthropometric and geometric parameters of musculoskeletal system (Menegaldo, 2003),

$l_1 = 0.24$, $l_2 = 0.27$, $l_3 = 0.1$,
$a_1 = 0.4$, $a_2 = 0.4$ $a_3 = 0.2$,
$m_1 = 14.0$, $m_2 = 6.7$, $m_3 = 0.5$,
$I_1 = 0.2640$, $I_2 = 0.1295$, $I_3 = 0.063$,
$g = 9.8$.

The movement of musculoskeletal system was simulated on PC, using MatLab/Simulink with time period $\Delta t = 1\ ms$, numerical method for differential equation solution by Runge Kutta, with 1 second period. In this case, we obtained the following results.

In order to check the performance of the controllers presented, using a path that represents an initial condition of the human body crouching down to erect condition to simulate a control of human posture (Pandy, 2001), where the initial conditions represent angles of approximately 57 degrees to the hip joint θ_3, approximately -57 degrees to the knee joint θ_2 and about 40 degrees to the ankle joint θ_1. In Figure 3 is shown by the trajectory angle of the joints, there is a convergence to zero of the angle of joints, representing the upright position of the model.

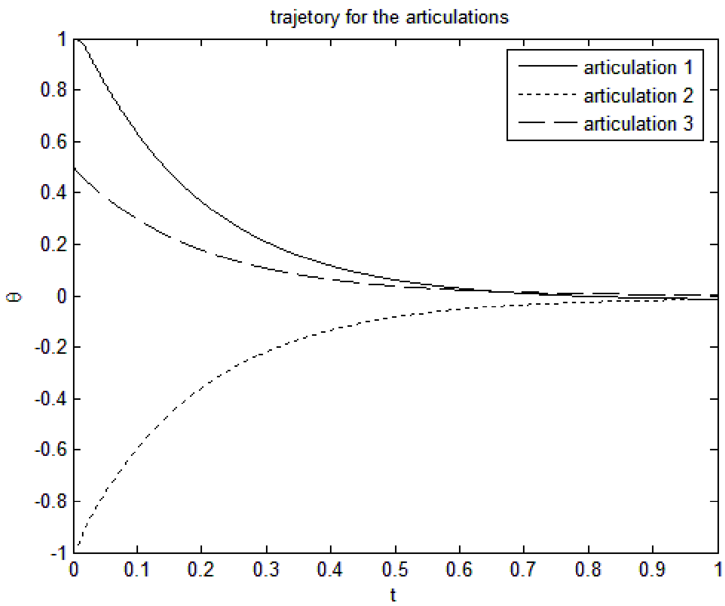

Figure 3. Tracking trajectory for angle of articulations to musculoskeletal system.

In Figure 4 there is a convergence to zero of the speed trajectories. The torques applied to joints, limited by the gain matrix of control, are shown in Figure 5.

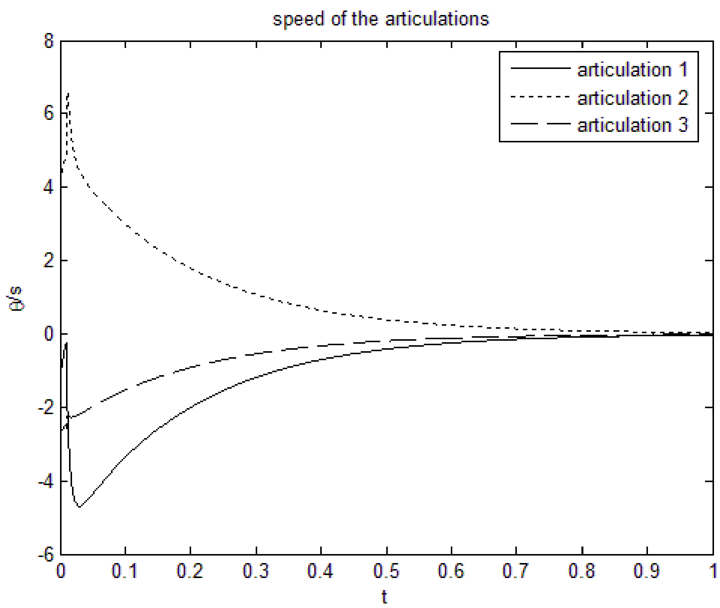

Figure 4. Tracking trajectory for the speed angle of articulations to musculoskeletal system.

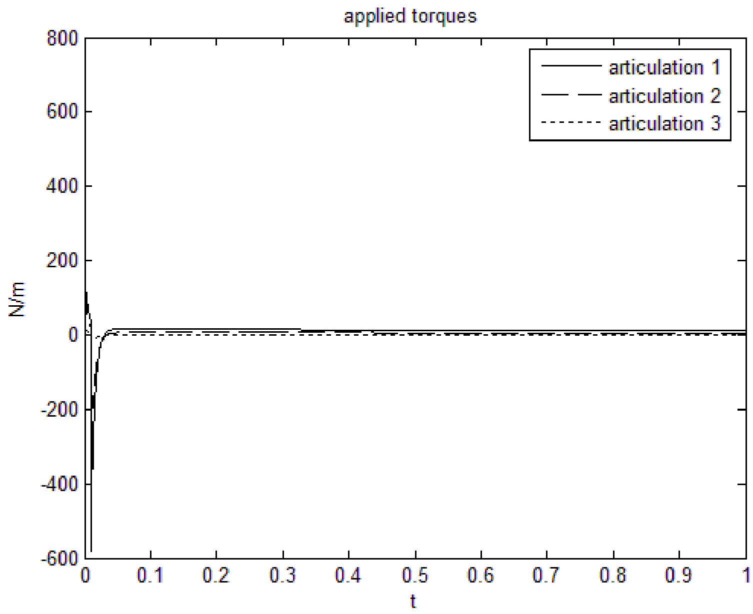

Figure 5. The torques applied to joints.

This simulation shows the good performance of the adaptive control system presented in both the stationary and the transient state.

6. CONCLUSIONS

The biomechanical model of a human musculoskeletal system and the simulation of his behavior in movement can be applied in several areas, such as sports, engineering and medicine. The objective of this work was obtain a dynamic model and control that represents the postural musculoskeletal system. The description of kinematic and dynamic links movements was based on Newton-Euler and Euler-Lagrange formulation. In this paper, it was used a control based on state-dependent Riccati equation (SDRE) technique. A geometric model for simulations was obtained with Matlab/Simulink software. A path that represents an initial condition of the human body crouching down to erect condition was used to simulate a control of human posture. We presented the tracking trajectories for angle of articulations to postural musculoskeletal system. The trajectory error remained next to zero with similar results to those published in literature. The simulation showed the good performance of state-dependent Riccati equation (SDRE) control technique to postural control presented here.

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