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VARIABLE VISCOSITY EFFECTS ON CALENDERING VISCOPLASTIC FLUIDS

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Abstract. Non-isothermal flow of a variable viscosity non-Newtonian fluid between a pair of counter-rotating cylinders with equal speed and equal size rolls is analyzed to study theoretically the effect of viscous dissipation on the exiting sheet thickness for the power law plastic fluid model. A regular perturbation method based on the lubrication approximation theory is used to uncouple the momentum and the energy equations to provide numerical results of the effects of temperature profiles on the final sheet thickness. The heat transfer analysis of Calendering Non-Newtonian fluids is an important area on polymer processing. Here, we are interested in studying the heat transfer phenomena on calendering Non-Newtonian process using the power law model. This model, takes into account the effects of temperature on the consistency index, also the viscous dissipation. In this study the important parameters are the Nahme-Griffith number as a perturbation parameter, this one relates the temperature gradient generated by viscous dissipation to the temperature gradient necessary to modify the viscosity. In general form, the pressure distribution is determined from the momentum equation, and then we can calculate the velocity profile, finally the energy equation is solved to estimate the influence of variable viscosity and viscous dissipation on the final sheet thickness. The order of magnitude for the Graetz and Nahme-Griffith numbers were 10^1 and 10^{-3}, respectively. Finally the influence of power law index n and the flow rate on pressure and temperature are obtained.

1 INTRODUCTION

The heat transfer analysis on Calendering process is an important area for the design of calendar machines. The study of this class of heat transfer problems represents a fundamental branch that allows to obtain a better thermal design and control polymer processing performance. Here, we are interested in studying the calendaring process, in which the shear stress is represented by Herschel-Bulkley model; also the apparent viscosity of the fluid into this rheology model is variable on temperature. The process of calendering has been extensively studied by many researchers over the last 50 years. Starting with Ardichvili (1936), the work was extended to Newtonian and Bingham plastics by Gaskel (1950), to power law fluid by McKelvey (1962), Pearson (1966), to Herschel-Bulkley fluid by Souzanna Sofou and Evan Mitsoulis (2004) and recently a numerical simulation of calendering viscoplastic fluids was reported by Evan Mitsoulis (2008), and others. Non-Isothermal effects have been studied by Kiparissides and Vlachopoulos (1978), Ivan López Gómez, Omar Estrada and Tim Osswald (2006).

Although the foregoing works are essential contributions to the study of Calendering phenomena, they were only reserved for those situations where the momentum equation is not coupled with energy equation, so the velocity and pressure fields are not affected by temperature (isothermal case). The main contribution in this study is to investigate the influenced of temperature on velocity and pressure fields in the limit when the Bingham tends to zero. In this case, the zone of interest is a viscous fluid that obeys the power law model.

2 FORMULATION AND ORDER OF MAGNITUDE ANALYSIS

The physical model under study is shown in figure 1. We begin with the plausible argument that the most important dynamic events occur in the region of the minimal roll separation-the nip region. A non-Newtonian fluid flowing non-isothermally between two cylinders of radius R, rotating at the same speed U is considered. The cylinder surface is maintained at constant temperature T_0. The fluid can be represented by the power law model of consistency factor K_0, and power law index n. The clearance between the rolls, $2H_0$ is so small as to be negligible in comparison with length and radius of the cylinders. Then it is reasonable to assume that the flow is nearly parallel, so that the general movement of the fluid is mainly in the x direction, the velocity of the fluid in the y direction is small. The gradient in the x direction of the velocity in the same direction is negligible compared to its gradient in the y direction and the pressure gradient is a function of x only.

Due to the symmetry of the physical model, we consider only for convenience the upper side of this configuration. Therefore, we select as the origin of the coordinate system, whose y axis points up, i.e., in the opposite direction of the gravity vector, and the x axis points in the direction of the flow.

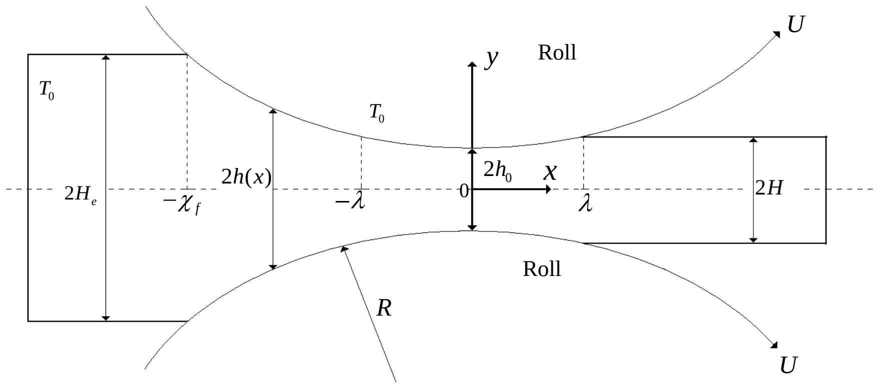

Figure.-1. Physical model sketch

The general governing equations in physical units are written in the following form,

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \tag{2}$$

$$\rho c_p \bar{u}\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial y^2} + \tau_{xy}\frac{\partial \bar{u}}{\partial y} \tag{3}$$

$$\chi = \lambda, \qquad p = dp/dx = 0 \tag{4}$$

$$\chi = -\chi_f, \qquad p = 0, \qquad T = T_0 \tag{5}$$

$$y = \pm h(x), \qquad \bar{u} = U, \qquad T = T_0$$

$$y = 0, \qquad \frac{\partial \bar{u}}{\partial y} = 0, \qquad \frac{\partial T}{\partial y} = 0 \tag{6}$$

From the momentum equation for the fluid in the longitudinal direction x, it can be shown that the pressure is of the order of,

$$p_c \sim K\left(U/h_0\right)^n \tag{7}$$

Besides the orders of magnitude of the important variables on calendaring process are presented in the following form,

$$x \sim \left(2Rh_0\right)^{1/2}, \qquad y \sim h_0, \qquad Q_c \sim 2Uh_0$$

$$\bar{u} \sim U, \qquad \bar{v} \sim \left(\frac{h_0}{2R}\right)^{1/2} U \tag{8}$$

3 MATHEMATICAL FORMULATION

In this section, we present the non-dimensional governing equations needed to solve the calendering heat transfer problem. Based on the above order of magnitude analysis, we introduce the following non-dimensional variables.

$$\chi = \frac{x}{\sqrt{2Rh_0}}\,,\ \eta = \frac{y}{h_0},\ h(x) = H_0\left(1+\frac{x^2}{2Rh_0}\right),\ \bar{P} = \frac{p}{K_0}\left(\frac{h_0}{U}\right)^n\sqrt{\frac{h_0}{2R}}\,,\ Q = \frac{\bar{Q}(x)}{2Uh_0} \tag{9}$$

$$u = \frac{\bar{u}}{U}\,,\ v = \frac{\bar{v}}{U}\,,\ \theta = \frac{(T-T_0)}{U^{n+1}h_0^{1-n}}\frac{\lambda_f}{K_0}\,,\tau = \frac{\bar{\tau}_{yx}}{K_0}\left(\frac{h_0}{U}\right)^n = (1-Na\theta)\left|\frac{\partial u}{\partial \eta}\right|^{n-1}\frac{\partial u}{\partial \eta} \tag{10}$$

Replacing the non-dimensional variables into the momentum equation eq.(2), we obtain the non-dimensional form of this,

$$\mathrm{Re}^*\left(\left(\frac{H_0}{2R}\right)^{\frac{1}{2}} u\frac{\partial u}{\partial \chi} + v\frac{\partial u}{\partial \eta}\right) = -\left(\frac{H_0}{2R}\right)^{\frac{1}{2}}\frac{\partial P}{\partial \chi} + \frac{\partial}{\partial \eta}\left[(1-Na\theta)\left|\frac{\partial u}{\partial \eta}\right|^{n-1}\frac{\partial u}{\partial \eta}\right] \tag{11}$$

Where the Non-dimensional parameters that appear in the above equation are the modified Reynolds, Nahme-Griffith, Graetz and Peclet numbers, defined respectively as:

$$\mathrm{Re}^* = \frac{\rho U^{2-n}H_0^n}{K_0}\,, \qquad Na = a\left(\frac{K_0U^{n+1}H_0^{1-n}}{\lambda_f}\right)$$

$$Gz = \left(\frac{H_0}{2R}\right)^{\frac{1}{2}} Pe\,, \qquad Pe = \frac{\rho c_p U H_0}{\lambda_f} \tag{12}$$

Herein, the inertial, pressure and viscous terms are of order unit, while the Reynolds number is of order 10^{-4}, in these processes, hence the left hand side of eq.(11) is negligible compared with the other ones. The relationship (11), can be reduced as,

$$\left(\frac{H_0}{2R}\right)^{\frac{1}{2}}\frac{\partial P}{\partial \chi} = \frac{\partial}{\partial \eta}\left[(1-Na\theta)\left|\frac{\partial u}{\partial \eta}\right|^{n-1}\frac{\partial u}{\partial \eta}\right] \tag{13}$$

$$\eta = 0, \qquad \frac{\partial u}{\partial \eta} = 0 \tag{14}$$

$$\eta = 1+\chi^2, \qquad u = 1 \tag{15}$$

$$\bar{P} = \frac{d\bar{P}}{d\chi} = 0, \qquad \chi = \lambda \tag{16}$$

$$\bar{P} = 0, \qquad \chi = -\chi_f \tag{17}$$

$$u = 1 + \left(\frac{n}{n+1}\right)\left(\frac{h_0}{2R}\right)^{1/2n}\left(\frac{d\bar{P}}{d\chi}\right)^{1/n}\left(\eta^{\frac{n+1}{n}} - (1+\chi^2)^{\frac{n+1}{n}}\right), \qquad \frac{d\bar{P}}{d\chi} > 0 \tag{18}$$

$$u = 1 - \left(\frac{n}{n+1}\right)\left(\frac{h_0}{2R}\right)^{1/2n}\left(-\frac{d\bar{P}}{d\chi}\right)^{1/n}\left(\eta^{\frac{n+1}{n}} - (1+\chi^2)^{\frac{n+1}{n}}\right), \qquad \frac{d\bar{P}}{d\chi} < 0 \tag{19}$$

$$\frac{\partial \bar{P}}{\partial \chi} = -\left(\frac{2n+1}{n}\right)^n \left(\frac{2R}{h_0}\right)^{1/2} \frac{(\lambda^2-\chi^2)\left|\lambda^2-\chi^2\right|^{n-1}}{\left(1+\chi^2\right)^{2n+1}} \tag{20}$$

$$\bar{P}(\chi) = -\left(\frac{2n+1}{n}\right)^n \left(\frac{2R}{h_0}\right)^{1/2} \int_{-\chi_f}^{\lambda} \frac{(\lambda^2-\chi^2)\left|\lambda^2-\chi^2\right|^{n-1}}{\left(1+\chi^2\right)^{2n+1}} d\chi \tag{21}$$

With regard to energy equation, and making a change of transversal variable to get a rectangular computational domain, the non-dimensional energy equation for the fluids is,

$$u(\chi,\eta)\frac{\partial \theta}{\partial \chi} = \frac{1}{(Gz)}\frac{\partial^2 \theta}{\partial \eta^2} + \frac{(1-Na\theta)}{(Gz)}\left|\frac{\partial u}{\partial \eta}\right|^{n-1}\left(\frac{\partial u}{\partial \eta}\right)^2 \tag{22a}$$

Replacing the velocity and the viscous dissipation on the above equation we get,

$$\left[1+\lambda^2\left(\frac{2n+1}{n+1}\right)\left[1-Y^{\frac{n+1}{n}}\right]-\chi^2\left(\frac{n}{n+1}\right)\left[1-\left(\frac{2n+1}{n}\right)Y^{\frac{n+1}{n}}\right]\right]\frac{\partial \theta}{\partial \chi} = \frac{1}{Gz\left(1+\chi^2\right)}\frac{\partial^2 \theta}{\partial Y^2} + \frac{1}{Gz}\left(\frac{2n+1}{n}\right)^{n+1}\frac{\left(\lambda^2-\chi^2\right)^{n+1}}{\left(1+\chi^2\right)^{2n+1}}Y^{\frac{n+1}{n}} \tag{22b}$$

$$u(\chi,\eta) = \left[1+\lambda^2\left(\frac{2n+1}{n+1}\right)\left[1-Y^{\frac{n+1}{n}}\right]-\chi^2\left(\frac{n}{n+1}\right)\left[1-\left(\frac{2n+1}{n}\right)Y^{\frac{n+1}{n}}\right]\right]$$

The boundary conditions for the energy equation (22b) are,

$$\begin{aligned} &\chi = -\lambda, \quad && \theta = 0 \\ &Y = 1, && \theta = 0 \\ &\frac{\partial \theta}{\partial Y} = 0, && \theta = 0 \end{aligned} \tag{22c}$$

The equation of energy was solved by a marching finite difference method of implicit type for the upper half of the flow field. The following finite difference approximations were used for the temperature derivatives.

$$\frac{\partial \theta_0}{\partial \chi} = \frac{\theta_{j,i+1}-\theta_{j,i}}{\Delta \chi} \tag{23}$$

$$\frac{\partial^2 \theta_0}{\partial Y^2} = \frac{\theta_{j+1,i+1}-2\theta_{j,i+1}+\theta_{j-1,i+1}+\theta_{j+1,i}-2\theta_{j,i}+\theta_{j-1,i}}{2\Delta Y^2} \tag{24}$$

Replacing the finite difference approximations on eq.(22), and applying the Crank-Nicolson method, we obtain the Crank-Nicolson finite difference equation for the internal nodes, defined as,

$$-M_{j,i}d\theta_{j-1,i+1}+2\left(1+M_{j,i}d\right)\theta_{j,i+1}-M_{j,i}d\theta_{j+1,i+1}=M_{j,i}d\theta_{j-1,i}+ 2\left(1-M_{j,i}d\right)\theta_{j,i}+M_{j,i}d\theta_{j+1,i}+N_{j,i}\Delta\chi \quad (25)$$

In the next section, we present the numerical results, on pressure and temperature fields to calendering non-newtonian fluids.

4 RESULTS

The characteristic values of the parameters involved on calendering process are presented in Table 1

Fluid	Re^*	Na	K_0 $\left(Pa.s^n\right)$	λ_f (W/mK)	c_p (J/kgK)	H_0 m	U (m/s)	R m	n
Polystyrene (PS) $T_0=463K$	$2X10^{-5}$	0.0001	$4.47X10^4$	0.17	2100	0.00025	0.166	0.15	0.22
PVC $T_0=458K$	$1.25X10^{-4}$	0.0024	$5X10^4$	0.16	2200	0.0006	0.05	0.30	0.50

Table 1.- Characteristics values on calendering

In the figures 2-4, the non-dimensional axial pressure field on calendering process is shown for a viscoplastic fluid obeying the power law. In these figures we can see the power law index influence on pressure field for various dimensionless flow rate.

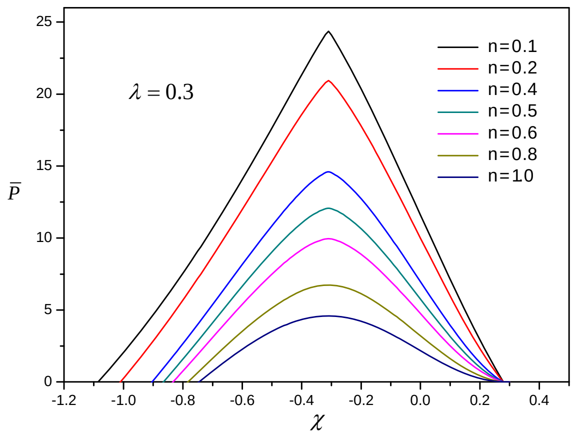

Figure 2. Non-dimensional axial pressure field for different power law index with $\lambda=0.3$

The extreme of the pressure distribution occur at $\chi=-\chi_f$, and $\chi=\pm\lambda$. For a given dimensionless leave of distance $\chi=\lambda$, corresponding to a finite sheets, integration of the momentum equation is carried out until the pressure passes through zero . At this point is the entry to the domain and thus the value of $\chi = -\chi_f$, is found since here $\overline{P}\left(\chi = -\chi_f\right) = 0.0$

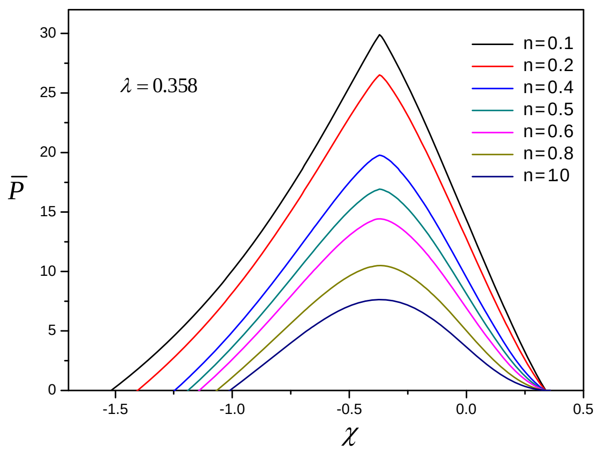

Fig. 3. Axial non-dimensional pressure distribution for different values of the power law index for a viscoplastic fluid with a dimensionless flow rate $\lambda = 0.444$ **.**

In figure 1, for $\lambda = 0.3$ and $n = 1$ the maximum pressure is located at $\chi = -0.3$ where P=4.58, while for the viscoplastic fluid P=24.11 with a power law index of $n = 0.2$, since the pressure in the last case is five times higher than the Newtonian case, we can find the influence of the parameter n, on the pressure field.

In figures 2 and 3, we show the influence of the non-dimensionless flow rate on the non-dimensionless pressure field, for n=0.1 the maximum pressure is shown on figure 2, where P=24.1115 while the maximum pressure on figure 3, is 1.56 times.

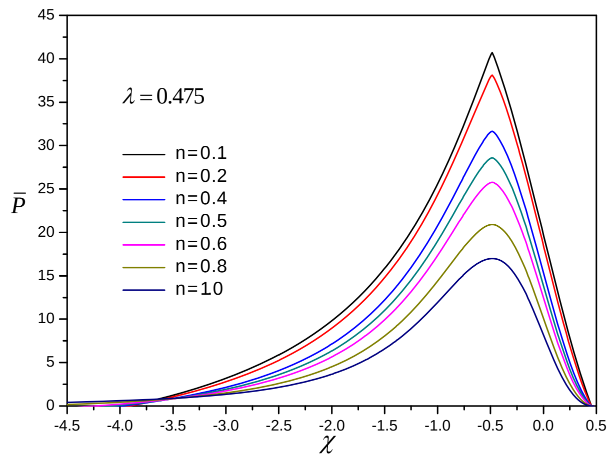

Fig. 4. Axial non-dimensional pressure distribution for different values of the power law index for a viscoplastic fluid with a dimensionless flow rate $\lambda = 0.475$ **.**

On the other hand, for $\chi \to -\infty$ the pressure distribution $P \to 0$ this is shown on figure 4. Under this assumption we get the maximum thickness value of the calendaring sheet, i.e. when $\lambda_\infty = 0.475$. From figures 1-6, we can see that the pressure into the nip region, exactly on $\chi = 0$ is the half value of the maximum pressure gets during the calendering process on $\chi = -\lambda$.

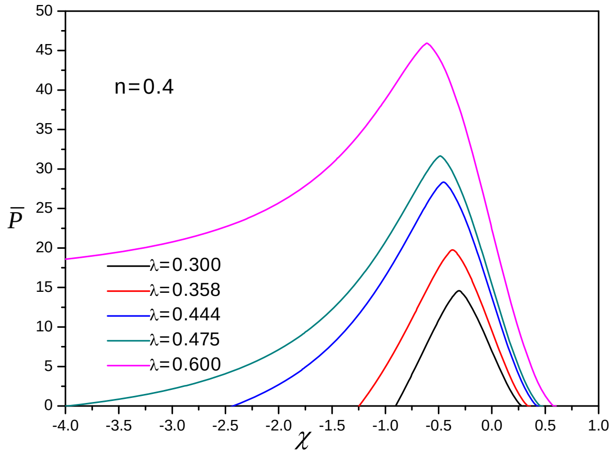

Fig. 5. Non-dimensional Axial pressure distribution for different values of nondimensionless flow rate for a viscoplastic fluid with a power law index $n = 0.4$

In figures 5 and 6 we show the hard influence of the flow rate on axial dimensionless pressure distribution for a value of the power law index. Two limits for λ are shown in these graphics, for the first case $\lambda \le 0.475$, it is possible to determine exactly the point where the rolls bite for the first time the finite sheet, this point is at, $-\chi_f$.

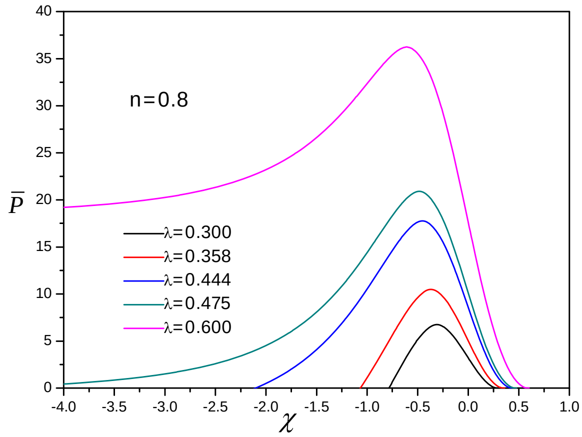

Fig. 6. Non-dimensional Axial pressure distribution for different values of non-dimensionless flow rate for a viscoplastic fluid with a power law index $n = 0.8$

The second limit is when $\lambda > 0.475$, because of the pressure distribution there is a backflow component which is superimposed onto the drag flow component. In this case there is a negative flow along the axis , and a circulation pattern develops in this case it was assumed that the calender was fed with a mass of fluid so large that an infinite reservoir of fluid existed upstream from the nip, these can be seen on figures 5 and 6. The development of the temperature profile is schematically shown in figures 7-8. The amount of shear is greater near the roll surfaces, hence the two maximum in the vicinity of these surfaces.

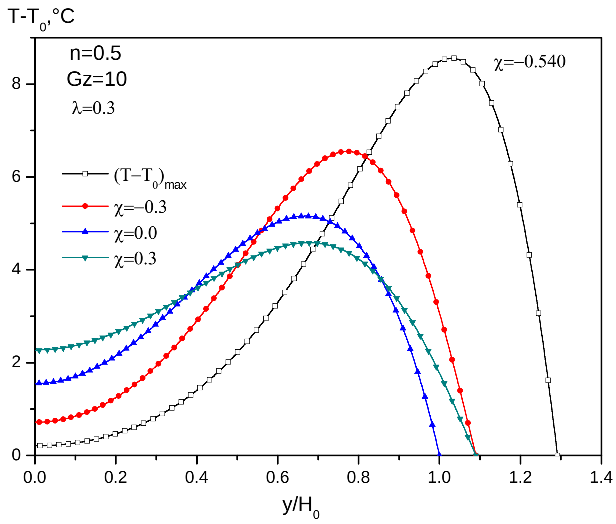

Figure 7. Transversal Temperature profile for the calendaring process with a power law index $n = 0.5$ and Graetzt number $Gz = 10$ with a non-dimensionless flow rate $\lambda = 0.3$ in the case of Newtonian fluid

In figures 7 and 8, we can see the hard influence of the power law index on the temperature profiles for the calendaring process, where in the case of Non-Newtonian fluid the temperature profiles is smaller than in the Newtonian case. The increase of temperature due to the viscous dissipation is more important on the vicinity of the rolls because of the movement of the rolls, and the temperature decreases as we approach the center of the nip, because of the influence of the drag in this zone is negligible.

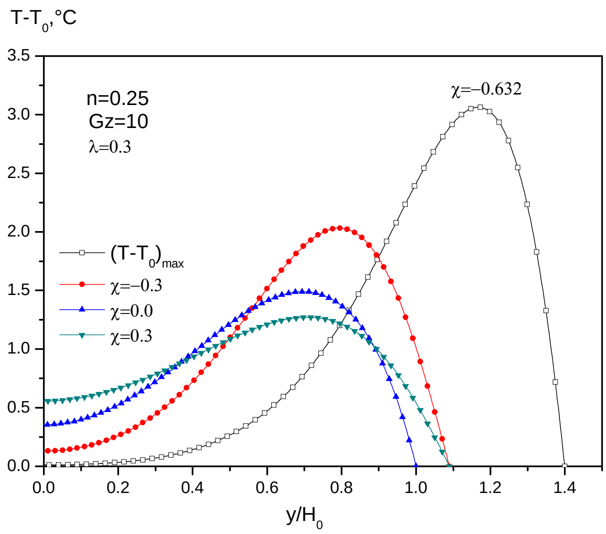

Figure 8. Transversal Temperature profile for the calendaring process with a power law index $n = 0.25$ and Graetzt number $Gz = 10$ with a non-dimensionless flow rate $\lambda = 0.3$ in the case of Non-Newtonian fluid

In the figure 8, we can see the influence of the non-dimensionless flow rate on the temperature profile for a Non-Newtonian fluid. When the flow rate decreases, the temperature decrease, because the velocity decrease too.

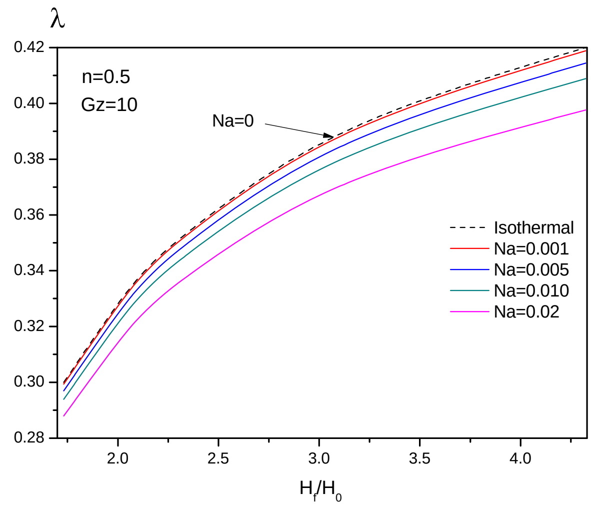

Figure 9. Calendering Thickness, in terms of λ, as a function of H_f / H_0 for a power law index $n = 0.5$.

From the previous figures it is apparent that under certain operating conditions the temperature rise due to viscous dissipation can be high enough to be detrimental for such temperature sensitive materials as PVC and robbers. It is important to note that the maximum exit temperature at the exit plane can be substantially smaller than the absolute maximum temperature throughout the whole flow field.

In the present study we can see the influence of the power law index n, on the exiting sheet thickness and temperature profiles at non-Newtonian fluids, how was predicted by a note on the text book by Middleman, he developed an order of magnitude of the sensitivity of calendered thickness to temperature fluctuations, where a 3° variation in temperature will cause more than a 20 percent variation in calendered thickness, this can be shown on figure 9. Since the λ increases the value of the $-\chi_f$, increases too, so the viscous dissipation is less than the Newtonian case, i.e., the Newtonian material are heated more than a pseudoplastic material, and then, the material will exit before than the Isothermal case. On the other hand, we determine the point where the rolls bite the sheet for some values of the non-dimensionless flow rate λ. The pseudoplastic pressure profiles are higher than for the Newtonian fluid.

5 CONCLUSIONS

In this work we have obtained the pressure and temperature profiles on calendaring process on a non-Newtonian fluid. The pressure profiles were influenced by the power law index. For the Non-Newtonian fluid the thickness sheet at the outlet was higher than in the Newtonian case. In the upstream of the pseudoplastic material, the point where the roll bites the material for the first is before in comparison to Newtonian fluids. The point of the maximum pressure is located in $\chi = -\lambda$.

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NOMENCLATURE

B	Geometry parameter
c_p	Heat capacity
h_0	Distance between rolls (nip)
H_e	Inlet thickness
$h(x)$	Roll variable curvature length
K	Consistency index
k	Thermal conductivity
n	Power law index
p	Pressure in physical units
$\bar{P}$	Non-dimensionless pressure
Q	Volumetric flow in physical units
Q^*	Non-dimensionless volumetric flow
Q_c	Characteristic volumetric flow
R	Roll radius
T	Temperature in the fluid in physical units
T_0	Temperature at the inlet of the fluid, in physical units
T_w	Roll temperature
U	Roll velocity
$\bar{u}$	Axial velocity of the fluid in physical units
u	Non-dimensionless axial velocity of the fluid in physical units
$\bar{v}$	Transversal velocity of the fluid in physical units
v	Non-dimensionless transversal velocity of the fluid
x	Axial coordinate in physical units
y	Transversal coordinate in physical units
Y	Non-dimensionless computational transversal coordinates

Greek letters

χ	Non-dimensionless axial coordinate
χ_f	Non-dimensionless axial coordinate at the inlet
ρ_f	Density of the fluid
τ_{xy}	Shear stress
λ	Non-dimensionless flow rate
η	Non-dimensionless transversal coordinate
θ	Non-dimensionless temperature

Non-dimensional numbers

Gz	Graetz number
Pe	Peclet number

Re^* Modified Reynolds number

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