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AN ANALYTICAL SOLUTION FOR HYDROGEN/NITROGEN JET DIFFUSION FLAMES

Greice da S. Lorenzzetti[a], **Francieli A. Vaz**[b] **and Álvaro L. de Bortoli**[a,b]

[a]*Graduate Program in Chemical Engineering, Federal University of Rio Grande do Sul, Street Engenheiro Luiz Englert, s/n - Building 12204, Porto Alegre, RS, Brazil, ppgeq@enq.ufrgs.br, http://www.ppgeq.ufrgs.br/*

[b]*Graduate Program in Applied Mathematics, Federal University of Rio Grande do Sul, Av. Bento Gonçalves, 9500 - Building 43111, Porto Alegre, RS, Brazil, ppgmap@mat.ufrgs.br, http://www.mat.ufrgs.br/~ppgmap/*

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Abstract. The combustion processes induce pollution and environmental problems; therefore, the accurate control of turbulent flames appears as a real challenge. These processes are highly complex, and involve turbulent mixing, heat transfer, radiation and chemistry. Analytical solutions for sets of mixing-reacting equations help us to understand the mechanisms of nonlinear effects. Therefore, the aim of this work is the development of an analytical solution for H_2/N_2 jet diffusion flame for moderate Reynolds and high Damköhler. The Reichardt's equation is used to approximate the flow, and consequently the mixture fraction. All thermochemical variables are determined from the mixture fraction since flames at moderate and high Reynolds are turbulent and the velocity/mixture fraction can be reasonably approximated by a diffusion equation. The analytical results compare reasonably with the experimental data showing that the method contributes to solve high temperature jet flames in a closed form.

1 INTRODUCTION

Combustion corresponds to a complex sequence of chemical reactions between a fuel and an oxidizer releasing heat and sometimes producing light too. Combustion is in general fast compared to molecular mixing, occurring in layers thinner than the typical length scales of turbulence (Peters, 2006), specially for high Damköhler values. We have a high Damköhler when the characteristic time for mixing is much longer than the chemical reaction time (Kuo, 2005).

If the fuel and the oxidizer velocities are small (low Reynolds) the mixture between fuel, oxidizer and products of combustion will be basically by diffusion, establishing a laminar diffusion flame. However, if the velocity is high, for high Reynolds, the mixture occurs due to the transport of mass characterizing the turbulent flux.

The flames can be classified as premixed, nonpremixed and partially premixed, being laminar or turbulent. Jet diffusion flames, among numerous types of turbulent flames, are of particular interest because of its wide application in engineering as in jet engines, diesel engines, and hydrogen-oxygen rocket motors (Warnatz et al., 2006). In a jet diffusion flame the fuel and the oxidizer enter the chamber in separate streams. The burn occurs only in the regions where the amount of fuel and oxidizer is stoichiometric.

The length scale and time-scale disparities in turbulent combustion are high (Poinsot and Veynante, 2001) and, at least theoretically, all scales must be well predicted to obtain an adequate mathematical solution of a jet diffusion flame. Due to the exothermicity of combustion reactions, the temperature increases since the chemical reactions become faster, then the length scale is short.

Jets and jet flames seem to be simple; however, their structure is complex because a jet usually starts with a potential zone of 2 to 10 diameters length and ends with a turbulent plume which is difficult to be obtained by the conventional numerical techniques. The majority of jet and jet flame simulations found in the literature reveals a turbulent structure which is coarser than that seen in the experiments.

Due to the inherent numerical difficulty for solving jet diffusion flames, many approximate models are found in the literature (Williams, 1985; Liñán, 1991; Veynante and Vervisch, 2002; Fernández-Tarrazo et al., 2006; Warnatz et al., 2006). The hypothesis of a two-layer flame structure considers a very thin fuel consumption layer and a much thicker oxidation layer, hypothesis normally adopted for stoichiometric and lean fuel conditions. There is also the flamelet model, whose basic idea is that an instantaneous flame element embedded in a turbulent flow has a structure of a laminar flame (Peters, 1992). The flamelet equations correspond to a balance between the unsteady changes, the diffusive effects and the chemical reactions. Then the species mass fraction depends of the mixture fraction, of the scalar dissipation rate and of the time.

The element mass fractions are not changed by reactive processes, they are changed by mixing and such is governed by diffusion. The mixture fraction value decreases along the burner axis similarly to the velocity (Kuo, 2005); such decrease depends on the mixing.

Biofuels such as biohydrogen, biomethanol, bioethanol and biodiesel may be the future sources of energy for transportation. H_2/N_2 flames are in nature similar to methane/hydrogen mixture flames. Hydrogen is the most common element in the universe and is a sustainable, non-polluting source of power. When it is used as a fuel, the main emission from fuel is potable water. Moreover, hydrogen is highly flammable and burn in air at a very wide range of concentrations between 4% and 75% by volume.

In this way, this work develops an analytical solution for a confined jet diffusion flame. The method is based on the solution of the diffusion Reichardt's equation for the velocity, and consequently for the mixture fraction. The experimental data found for hydrogen/nitrogen jets (Sandia National Laboratories, 2010) are used to check/compare the analytical results. The main contribution of this work is to use the Reichardt equation, already known, with the problem of jet diffusion flames.

2 MODEL FORMULATION

Instead of solving the time-consuming set of reacting Navier-Stokes equations, consider the application of the momentum transfer law due to Reichardt. He discovered that the velocity profiles under consideration can be approximated successfully by a Gaussian function, or by its integral, the error function (Schlichting, 1979).

In two-dimensional flow the momentum equation turns

$$\frac{\partial}{\partial x}\left(\frac{\bar{p}}{\rho}+(\bar{u})^2\right)+\frac{\partial}{\partial y}(\bar{uv})=0, \tag{1}$$

where ρ is the fluid density, $\bar{p}$ the mean pressure, and $\bar{u}$, $\bar{v}$ the mean velocity components. Reichardt made the assumption that $\bar{uv}=-\lambda\frac{\partial\bar{u}^2}{\partial y}$, where λ is the momentum transfer length, which states that the flux of the *x*-component of momentum is proportional to the transverse gradient of momentum. Eliminating $\bar{uv}$, for constant pressure, one obtains the Reichardt's fundamental equation

$$\frac{\partial\bar{u}^2}{\partial x}=\lambda\frac{\partial^2\bar{u}^2}{\partial y^2}. \tag{2}$$

In this way, the distribution of momentum in free turbulent flow is governed by a generalized heat conduction equation type. The solution of this equation can be given by

$$\bar{u}^2=\exp\left\{-2\left[ax^2+b\left(\frac{y}{x}\right)^2\right]\right\}, \tag{3}$$

where a and b are related with λ by the expression

$$\lambda=\frac{ax^5-bxy^2}{bx^2-4b^2y^2}. \tag{4}$$

This solution does not considers the density changes, which are relevant at flame region; but it approximates the solution of a jet diffusion flame. The mixture fraction (Bilger, 1979) is assumed to be equal to the velocity obtained from Eq. (2) because one considers the existence of two jet invariants (Kuo, 2005):

- Conservation of momentum $I_u=\int_0^\infty u^2rdr=\frac{1}{\eta}(u_0^2r_0^2/2)$,
- Conservation of mass of fuel specie $I_Z=\int_0^\infty uZrdr=\frac{1}{\eta}(u_0r_0^2/2)$,

where η is the viscosity, and u_0 and r_0 the inflow jet velocity and the jet radius, respectively. Moreover, the relation $\frac{Z.x}{I_Z}=\frac{U.x}{I_u}$, results in $Z=U$ for $u_0=1$, where Z is the mixture fraction and U the jet velocity. All thermochemical variables may be determined from the mixture fraction since at high Damköhler such approximation turns reasonable.

Consider a one-step global reaction given by $\mu_{H_2}[H_2]+\mu_{O_2}[O_2]+\mu_{N_2}[N_2] \rightarrow \mu_{H_2O}[H_2O]+\mu_{N_2}[N_2]$, where μ_i is the stoichiometric coefficient of species i. When the mass fraction of O_2 is zero, then the mass fraction of fuel Y_F is given by (Peters, 2006),

$$Y_F = Y_{F,1}\,\frac{Z - Z_{st}}{1 - Z_{st}}, \tag{5}$$

and the temperature by

$$T = T_0 + QY_{O_2,2}\,\frac{1 - Z}{C_{p_{O_2}}\mu_{O_2}W_{O_2}}, \tag{6}$$

and if $Y_F = 0$, it results for Y_{O_2} and T, respectively

$$Y_{O_2} = Y_{O_2,2}\left(1 - \frac{Z}{Z_{st}}\right), \tag{7}$$

$$T = T_0 + \frac{QY_{F,1}Z}{C_{p_F}\mu_F W_F}. \tag{8}$$

where $Y_{F,1}$ and $Y_{O_2,2}$ are the inlet mass fractions of the fuel and the oxidizer, respectively, Z_{st} the stoichiometric mixture fraction, T_0 the mixing temperature (without combustion), Q is the heat release, C_p the specific heat, and W_i the molecular mass of species i. As the element mass fraction does not change during combustion (is conserved) one obtains the burned quantities for:

$Z \leq Z_{st}$,

$$Y_{H_2O,b} = \frac{nW_{H_2O}Y_{F,1}Z}{2W_F}, \tag{9}$$

$Z \geq Z_{st}$,

$$Y_{H_2O,b} = n\frac{W_{H_2O}}{2W_F}Y_{F,1}Z_{st}\frac{1 - Z}{1 - Z_{st}}. \tag{10}$$

Using these equations for mass fraction and the equation for the velocity (Reichardt's equation), we determine the profile of a H_2/N_2 jet diffusion flame.

3 RESULTS

In a jet diffusion flame the fuel is injected from a tube with diameter d, as shown in Fig. 1. Inside a burner the Mach is usually low, the pressure remains almost constant and the heat losses to the walls are assumed to be small (Poinsot and Veynante, 2001).

The jet flame is chosen because it is a representative of the class of nonpremixed flames. Here, one employs a tube of diameter $D_e = 1$ and the fuel is injected from a small tube of diameter d (see Fig. 1). The domain spans a region of $110D \times 10D$ in the axial and the radial directions, respectively. When the burner dimensions are much larger than the fuel jet diameter, of a jet diffusion flame, the heat losses to the walls are usually small and the contribution due to radiation turns negligible; radiation is more important in furnaces, spreading of fires in buildings and forest fires (Law, 2006).

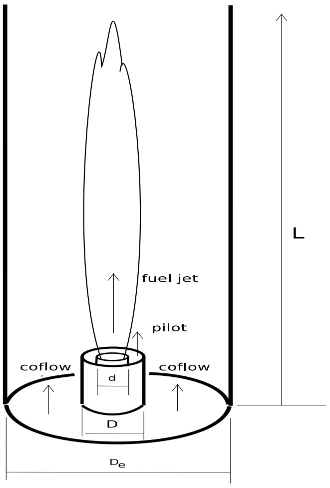

Figure 1: Burner sketch

The analytical result found in this paper for velocity is compared with the experimental values obtained by researches of the Sandia National Laboratories (2010). The result for velocity, and consequently the mixture fraction, shown in Fig. 2 indicates the decay of the centerline velocity with downstream distance from the fuel jet. The results are satisfactory, since there is a good agreement with the experiment. Note that the axial velocity distribution can be reasonably approximated by an exponential function.

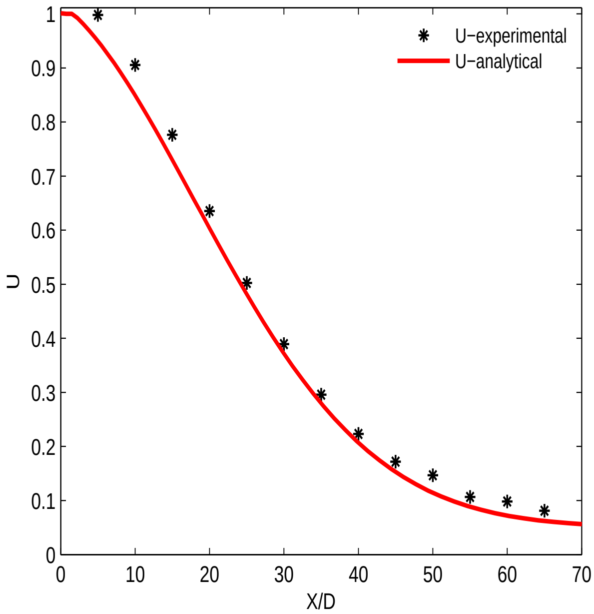

Figure 2: Comparison of the axial velocity for H_2/N_2 jet with experiment (Sandia National Laboratories, 2010).

Based on the Burke-Schumann analytical solution (Warnatz et al., 2006), and the equations (5), (7), (9), (10) we obtain the mass fractions of H_2, O_2 and H_2O, as shown in Fig. 3. The results are analyzed along the burner centerline, where the mass fraction of fuel decreases, and the oxidizer increases. The nitrogen in the fuel, as in the air, acts as an inert, not affecting the flame. Although of the use of a simple equation, Eq. (2), the flame characteristics seem to be adequately captured.

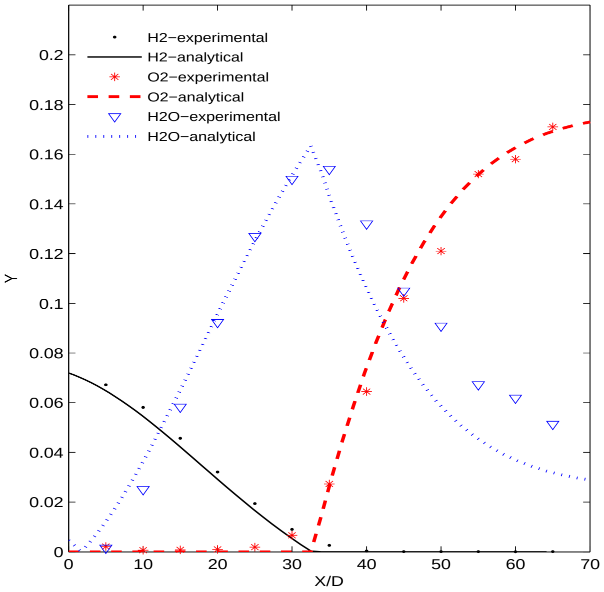

Figure 3: Mass fraction of fuel H_2, oxidizer O_2, and water vapor H_2O along the burner centerline.

4 CONCLUSIONS

The present work developed an analytical method for the solution of a jet diffusion flame based an the solution of Reichardt's equation. The results are in reasonable agreement with experimental data found in the literature.

The appropriate model simplification allows to reduce the cost needed when solving simple jet diffusion flames. Moreover, the understanding of such simplifications is important when obtaining analytical solutions for complex flames. The contribution of the present work goes in this direction.

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