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DYNAMIC CHARACTERISTICS OF 1 AND 2 DEGREES-OF-FREEDOM SYSTEMS ACTING AS DYNAMIC VIBRATION ABSORBERS ON CONTINUUM SYSTEMS

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Abstract. Dynamic vibration absorbers are used to reduce the vibration amplitude of systems near their resonance frequencies. Although its operating principle is easy to understand, its practical use needs some considerations about the coupling between the primary system and the absorber. In this work, the dynamic characteristics of 1 and 2 degrees-of-freedom (DOF) systems acting as dynamic vibration absorbers are analyzed. The discrete structures (absorbers) are added on a beam type structure (primary system) and the frequency response of the compound system is calculated under the variation of some of the absorber's parameters, for instance: mass, damping, stiffness, location on the beam. Also, the differences between having 2 1-DOF systems and 1 2-DOF systems located at the same positions are analyzed and compared. As a result, we show what type of absorber reveals the best effectiveness on the reduction of the vibration amplitude of a given point on the primary system.

1 INTRODUCTION

The traditional dynamic vibration absorber is made by a spring-mass system which causes to reduce the motion of a structure to which it is attached. This behavior is due to the fact that the vibrations of the whole system are distributed in a different way when more degrees of freedom are added to the primary system. The most interesting change happens when a perfectly tuned 1-DOF system is attached to the primary structure (both natural frequencies are selected to be equal). Then, if the this structure is excited at its resonant frequency, the attached system or dynamic vibration absorber (DVA) absorbs the vibration on the primary structure by an increment of its amplitude. This useful application was firstly described by Ormondroid and DenHartog (1928) and since then, it has been used in many applications Snowdon et al. (1984); duPlooy et al. (2005) and discussed in many other (Brennan and Dayou (2000); Ozer and Royston (2005)).

It is well known that a perfectly tuned undamped DVA of 1-DOF (only a mass and spring) reduces drastically the vibrations at one natural frequency of the primary structure. However, it produces an increase of the vibrational levels at two closed frequencies. For this is reason, it is necessary to add damping to the traditional DVA to overcome this difficulty. Another desirable effect of this addition is the fact that it produces the widening of the new peaks over the frequency axis making it possible to be used as mechanism for vibration suppression.

The most important application of a DVA is the passive control of vibration. In this area, it is very frequently to use algorithms to optimize its parameters Febbo and Vera (2008) prior to put it into the primary structure. However, this task is analytically rather involved and several approximations are needed to obtain a practical solution. Sometimes these mathematical solutions are unrealistic and it is necessary first to understand the dynamic behavior of the couple system (structure + DVA) in order to select the set of parameters that satisfy the reduction requirements.

The objective of this paper is along the above idea. First, we present a mathematical model, based on Lagrange's equations, that allows us to study the vibrations of continuous systems with discrete systems attached. This analytical model is suitable to be applied to structural elements, as beams o plates, with several discrete systems attached. To gain physical insight into the mechanisms of control of vibration we analyze a simply supported supported beam with multiple 1 DOF DVAs and a 2 DOF DVA of varying parameters. Finally, we end with a summary of the results that establishes the main characteristics of these systems for vibration suppression.

2 MATHEMATICAL MODEL

In this section, the mathematical formulation for multiple degree of freedom DVAs coupled to a beam is presented. The beam-type structure and the dynamic absorbers can be modelled separately and then coupled together using Lagrange's multipliers approach (see Dowell (1979)).

Figure1 describes the transverse vibrations of a beam with internal viscous damping and a 2 DOF attached to it. Its displacement amplitude at position x and time t is assumed to be a superposition of modal amplitudes multiplied by normal mode shapes $\phi_i(x)$ (of the beam)

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) \tag{1}$$

where $q_i(t)$ are generalized coordinates, or modal amplitudes which depend on t. The total kinetic energy, strain energy and dissipation function can be written as

$$T = \frac{1}{2}\sum_{i=1}^{n} M_i \dot{q_i}^2 + \frac{1}{2}\frac{m_e}{(a_1+a_2)^2}\left(\dot{z}_{m1}a_2 + \dot{z}_{m2}a_1\right)^2 + \frac{1}{2}\frac{I_e}{(a_1+a_2)^2}\left(\dot{z}_{m2} - \dot{z}_{m1}\right)^2 \tag{2}$$

$$V = \frac{1}{2}\sum_{i=1}^{n} \omega_i^2 M_i q_i^2 + \frac{1}{2}\sum_{\nu=1}^{r} k_\nu (z_{m\nu} - z_\nu)^2 \tag{3}$$

$$D = \frac{1}{2}\sum_{i=1}^{n} c_{vi}\dot{q_i}^2 + \frac{1}{2}\sum_{\nu=1}^{r} c_\nu (\dot{z}_{m\nu} - \dot{z}_\nu)^2 \tag{4}$$

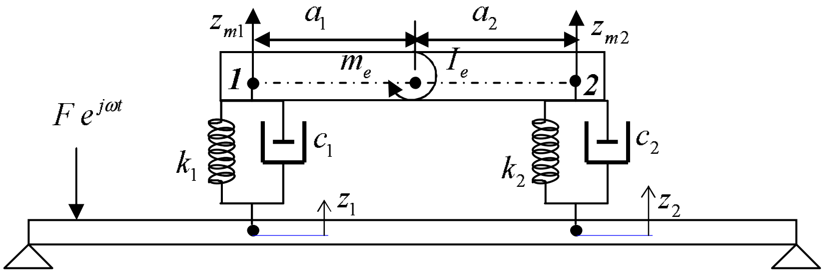


Figure 1: Schematic representation of the 2-DOF system attached to a beam.

M_i, c_{vi} and ω_i are the *i*th modal mass, damping constants and natural frequencies of the beam ; k_ν, c_ν and m_ν are the stiffness, damping constants and masses of the DVA's respectively. Lagrange's equations that include the dissipation function D and a primary harmonic excitation at the point $x = x_f$, $Fe^{j\omega t}\delta(x - x_f)$, are expressed by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{s}_k}\right) + \frac{\partial D}{\partial \dot{s}_k} + \frac{\partial V}{\partial s_k} = Q_k + \sum_{\nu=1}^{r} \lambda_\nu \frac{\partial f_\nu}{\partial s_k} \qquad k = 1, ...n + 2r \tag{5}$$

where λ_ν are Lagrange's multipliers and subscript r is the number of restrictions of the problem. Q_k is the generalized force $Q_k = Fe^{j\omega t}\phi_k(x_f)$. The equations that represent the restrictions are

$$f_\nu = \sum_{i=1}^{n} \phi_i(x_\nu) q_i(t) - z_\nu = 0 \qquad \nu = 1, ...r \tag{6}$$

which allow to eliminate the redundant coordinates of the problem. Taking these expressions into account and replacing the expression of T, V D and Q_k in Eq. (5), Lagrange's equations can thus be written as

$$\begin{aligned} M_i\ddot{q}_i + 2\xi_i M_i \omega_i \dot{q}_i + M_i\omega_i^2 q_i &= F\phi_i(x_f)e^{j\omega t} + \sum_{\nu=1}^{r}\lambda_\nu \phi_i(x_\nu) \qquad i = 1, ...n; \\ m_\nu \ddot{z}_{m\nu} + c_\nu(\dot{z}_{m\nu} - \dot{z}_\nu) + k_\nu(z_{m\nu} - z_\nu) &= 0 \qquad \nu = 1, ...r \\ c_\nu(\dot{z}_{m\nu} - \dot{z}_\nu) + k_\nu(z_{m\nu} - z_\nu) &= \lambda_\nu \qquad \nu = 1, ...r \end{aligned} \tag{7}$$

where $\xi_i = \frac{c_{vi}}{2M_i\omega_i}$. Finally, equations (7) put in matrix can be expressed by

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f_p} + \mathbf{f_s} \tag{8}$$

to produce a system of n equations by n unknowns where $\mathbf{M}_{n\times n}$ results in a diagonal matrix whose elements are the modal masses, $\mathbf{C}_{n\times n}$ is the modal damping matrix and $\mathbf{K}_{n\times n}$ is the modal stiffness matrix. $\mathbf{f_p}$ represents the external forces and $\mathbf{f_s}$ are the forces applied by the DVA. Assuming all variables harmonic: $q_i = \overline{q}_i e^{j\omega t}$, $z_{m\nu} = \overline{z}_{m\nu} e^{j\omega t}$ and $z_\nu = \overline{z}_\nu e^{j\omega t}$, it is possible to obtain the modal amplitude vector $\bar{\mathbf{q}}$

$$\bar{\mathbf{q}} = (\mathbf{K_d} + \phi \mathbf{K_{abs}} \phi^{\mathbf{T}})^{-1} F \phi(x_f) \tag{9}$$

where $\mathbf{K_d} = (\mathbf{K} - \omega^2 \mathbf{M} + i\omega \mathbf{C})$ is the dynamic stiffness of the beam and $\phi_{n\times\nu}$ is the matrix of modal amplitudes whose components are the modal amplitudes of the *i*th mode evaluated at the νth absorber's position. ϕ is the n-vector of mode shapes.

In this work, the cases of a 2-DOF coupled and uncoupled DVA are considered, see figure 2. The dynamics of each type of DVA is included in $\mathbf{K_{abs}}_{\nu\times\nu}$ matrix

(a) uncoupled 2-DOF (2 1 DOF DVA) $\mathbf{K_{abs}} = diag(K_i)$ where

$$K_i = \frac{-\omega^2 m_i (1 + i2\xi_{ai}(\omega/\omega_{ai}))}{1 - (\omega/\omega_{ai})^2 + i2\xi_{ai}(\omega/\omega_{ai})} \tag{10}$$

where $\xi_{ai} = \frac{c_i}{2m_i\omega_{ai}}$ and $\omega_{ai} = \sqrt{k_i/m_i}$

(b) coupled 2-DOF

$$\mathbf{K_{abs}} = \begin{pmatrix} k_1^* - \omega^2 \frac{(m_e a_2^2 + I_e)}{(a_1+a_2)^2} & \omega^2 \frac{(I_e - m_e a_1 a_2)}{(a_1+a_2)^2} \\ \omega^2 \frac{(I_e - m_e a_1 a_2)}{(a_1+a_2)^2} & k_2^* - \omega^2 \frac{(m_e a_1^2 + I_e)}{(a_1+a_2)^2} \end{pmatrix} \tag{11}$$

where $k_i^* = k_i + j\omega c_i$

The displacement of the structure at point x_a can be written as

$$W(x_a) = \sum_{i=1}^{n} \phi_i(x_a) \bar{q}_i \tag{12}$$

where $\bar{q}_i$ is given by (9)

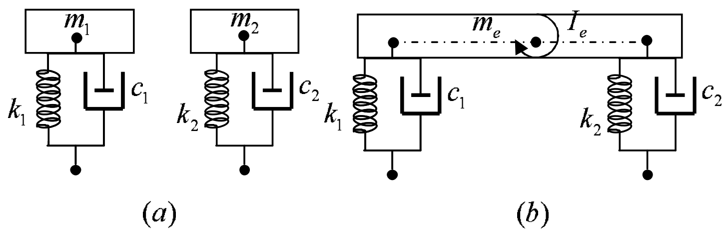


Figure 2: (a) Uncoupled 2-DOF system (2 1-DOF systems) and (b) coupled 2-DOF system.

3 DYNAMIC CHARACTERISTICS OF THE BEAM WITH 1 AND 2-DOF SYSTEMS ATTACHED TO IT

The aim of this section is to study the displacement amplitude of a beam with 1 and 2-DOF systems (coupled and uncoupled) as a function of the frequency of the excitation ω. For the sake of clearness of the presentation, the cases under study has been divided into three.

Case 1 a 1-DOF system attached to the beam.

Case 2 two 1-DOF systems (uncoupled 2-DOF) attached to the beam.

Case 3 a 2-DOF system (coupled 2-DOF) attached to the beam.

For all cases, a simply supported rectangular beam, with physical dimensions given by table 1 is employed. Obviously, any other boundary condition could have been used instead. The beam is excited by a harmonic force $Fe^{j\omega t}$ located at $x_f = 0.1L$.

Table 1: Physical dimensions of the employed beam

$\rho[kg/m^3]$ density	$L\ [m]$ length	$E[N/m^2]$ Young's modulus	$h\ [m]$ thickness	$b\ [m]$ width	$I\ [m^4]$ moment of inertia	$m_b\ [kg]$ mass	ξ [adim] damping coeff
7.86×10^3	1	208×10^9	0.00635	0.0381	$8.1295\ \times10^{-10}$	1.9016	0.01

With these physical constants, the first two natural frequencies of the beam results: $f_1 = 14.8123$ Hz and $f_2 = 59.2493$ Hz. We shall name "bare beam" the beam without attachments.

3.1 Case 1

Figures 3(a) to (d) show the displacement amplitude of the beam vs ω as the parameters of the 1-DOF system attached to it are changed (damping, position, mass, tuning). The selected parameters are modified one at a time, leaving the rest of them fixed. Usually, a discrete system (1-DOF system in this case) mounted on the beam in this way is called a dynamic vibration absorber or simply absorber, because it can be thought as a vibration absorber device Ormondroid and DenHartog (1928). For the cases shown in Figs. 3 (a) to (c) the tuning ratio (or tuning) α, which is the ratio between the natural frequency of the absorber and any natural frequency of the beam (in this case the first) is $\alpha = \frac{\omega_a}{\omega_1} = 1$.

In Fig. 3(a) we vary the damping coefficient of the absorber. There, it can be observed that the displacement amplitude of the beam is nearly zero for its first natural frequency whereas two new peaks appear at frequencies below and above it. Clearly, this is produced by the addition of a 1-DOF system to the beam. From the same figure, it can be deduced that, as the damping coefficient is increased the height of the peaks are reduced and widened, leaving unaltered the location of them.

In Fig. 3(b) the location of the absorber on the beam is modified. The results show that the maximum reduction of the displacement amplitude for the tuning frequency is attained when the absorber is located at an antinode of the bare beam. Figure 3(c) shows how the mass of the absorber affects the amplitude of maximum reduction. This time, the mass ratio ($\mu = m_1/m_b$, m_1 mass of the absorber, m_b mass of the beam) is modified changing the stiffness constant of the 1-DOF, leaving unalter the tuning ratio. This variation produces a shift of the peaks and a decrement of the minimum amplitude as the mass of the absorber is increased. It is straightforward to obtain an expression for the shift of the peaks Brennan and Dayou (2000):

$$\Delta\Omega = \omega_1 - \omega_2 = \omega_n|\phi_n(x_\nu)|\mu^{1/2} \quad (13)$$

where $\phi_n(x_\nu)$ is the mode shape at the position of the absorber and ω_n the natural frequency of the beam at this mode. Notably, it can be stated that changing the position of the 1-DOF (leaving fixed all other parameters) or varying its mass produces the same effect on the dynamic behavior of the beam. This may be understood with the help of the effective mass concept Febbo (2006) since when the mass or location of the absorber changes, its effective mass changes as well.

Another interesting feature of this variation (not shown in the figure for the sake of clearness) is obtained for values of $\mu > 0.01$. When the mass of the absorber is high enough, further increases of it produces no variation of the amplitude of maximum reduction (at the tuning frequency). This implies that there exists a definite value of μ (this time $\mu = 0.01$) above which no further reductions of the amplitude is possible (this fact was also pointed out by Brennan and Dayou (2000)).

Finally, Fig.3 (d) presents the results for different tuning ratios. The mass ratio and damping coefficient for this case are set to $\mu = 0.05$ and $\xi_m = 0.001$ respectively. It is observed that the amplitude of maximum reduction is different for different tuning ratios which demonstrates that the efficiency of the absorbers is strongly dependent on this.

Next, we study how the damping of the beam affects the dynamic behavior the whole system and the so called fixed points. These points (when exist) are points whose locations on the frequency response curves are independent of the value of the damping coefficient of the absorber, ξ_m. This fact is used for absorber's optimization purposes. Jacquot (1978) calculated that a value of $\xi_m = 0.1846$ makes the frequency response curve near the resonance of the beam to be flat and minimum (see Figs. 4 (a) and (b)) satisfying a predetermined DenHartog (1956) optimization criterion. Despite of the fact that this is an interesting point, here we concentrate on the study of the dynamic behavior of the considered system and not on the optimization procedure.
For the calculations leading to Figs. 4 (a) and (b), the absorber is tuned with $\alpha = 0.9$, $\mu = 0.05$ and it is located at $x_1 = 0.5L$. Several damping coefficients for the absorber are considered. It is seen that the fixed points only appear when the damping of the beam is absent.

3.2 Case 2

We analyze the case when 2 1-DOF systems are attached to the beam. Figures 5 (a) and (b) show the displacement amplitude of an undamped beam ($\xi = 0$) as the damping coefficients of the 2-DOF systems are changed. The 2 absorbers are tuned to the first and second natural frequency of the beam, ($\alpha_1 = \frac{\omega_{a1}}{\omega_1} = 1$, $\alpha_2 = \frac{\omega_{a2}}{\omega_2} = 1$) and the observation point is set at $x_a = 0.4L$. The 2-DOF systems are located at $x_1 = 0.4L$ and $x_2 = 0.6L$ and their mass ratio are set to $\mu_1 = 0.05$ and $\mu_2 = 0.05$.
Fig. 5 (a) show that for frequencies near the first resonance of the beam, the effect of damping is to reduce the maxima of the peaks and to increase the minima. This is the same effect observed as for case 1. The existence of the fixed points is also evidenced.
However, for frequencies near second resonance of the bare beam (Fig. 5 (b)) these fixed points can not be identified anymore. It is important to notice that this characteristic is due to the addition of a second absorber and not to the consideration of an undamped beam.

Figures 6 (a) and (b) show the same cases as in Figs 5 (a) and (b) but this time for a damped beam ($\xi = 0.01$). Both figures (6 (a) and (b)) represent similar characteristics to Figs. 5 (a) and (b) and for this reason they will not be discussed in detail. Also, the fixed points are not

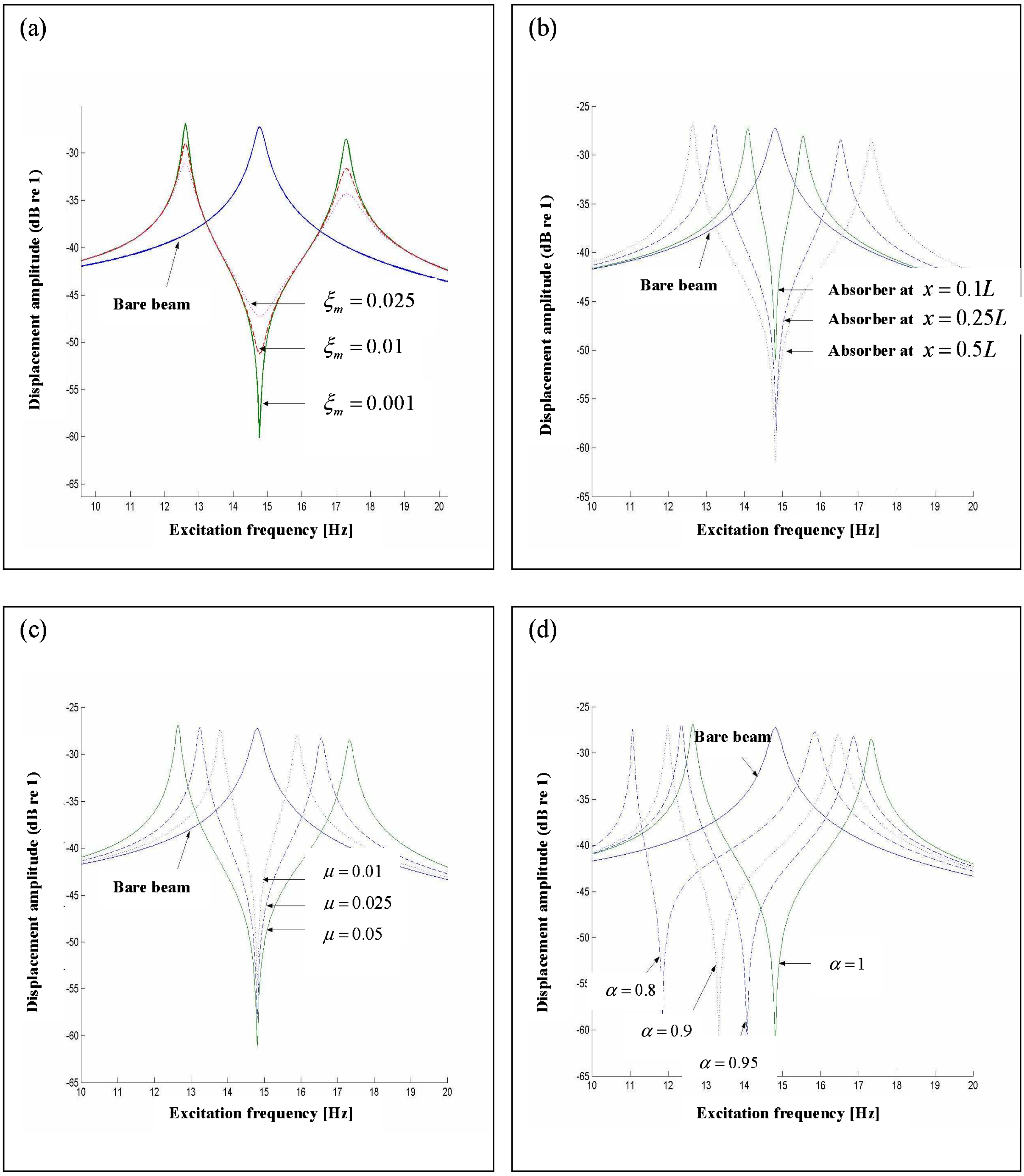


Figure 3: Frequency response of a 1-DOF system (absorber) attached to a beam. (a) absorber with different damping coefficients ($\mu = 0.05$, $x_1 = 0.5L$), (b) absorber at different locations ($\mu = 0.05$, $\xi_m = 0.001$), (c) absorber with different mass ratios ($x_1 = 0.5L$, $\xi_m = 0.001$), (d) absorber with different tuning ratios ($\mu = 0.05$, $x_1 = 0.5L$, $\xi_m = 0.001$).

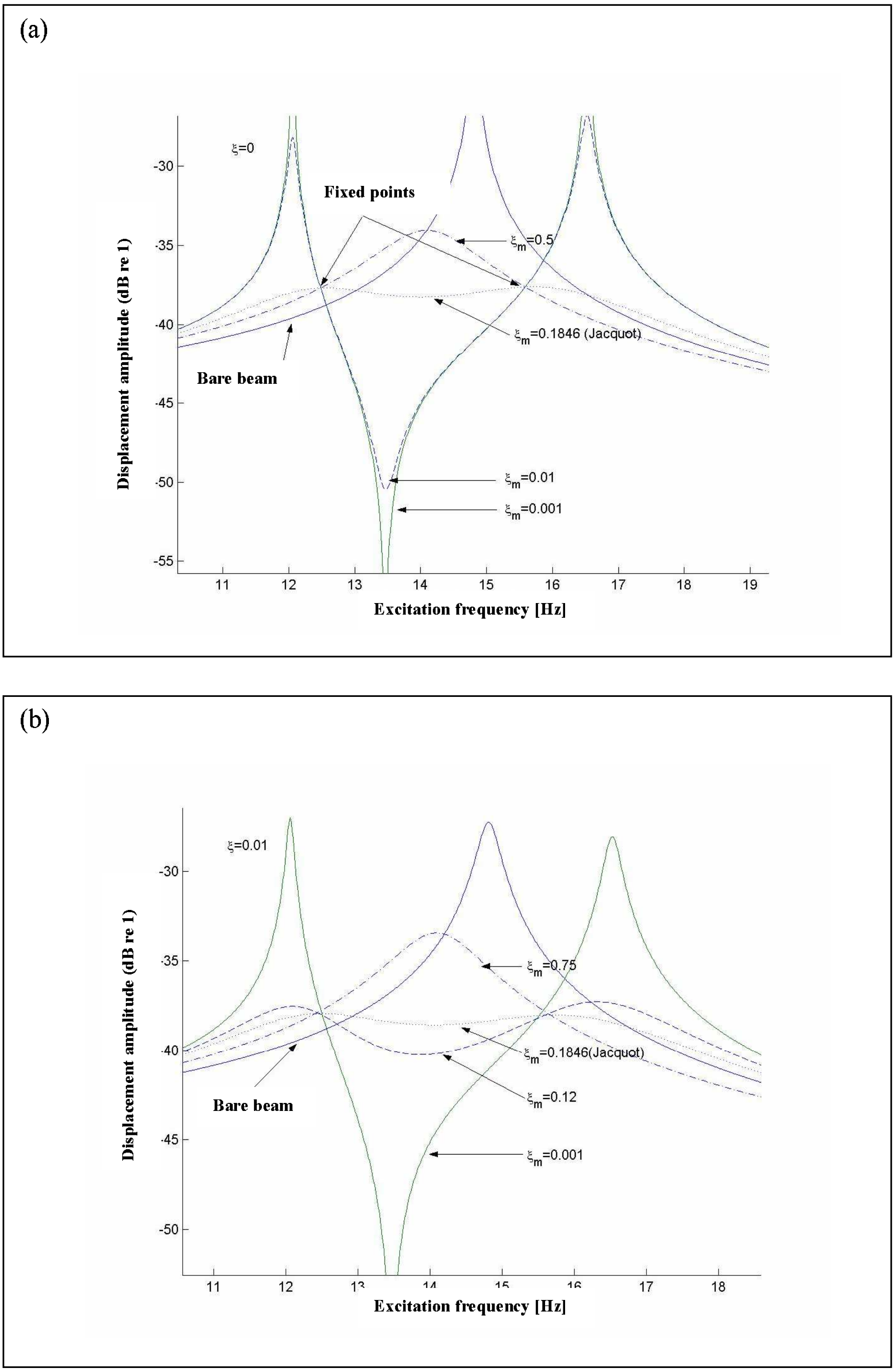


Figure 4: Frequency response of a 1-DOF system mounted to a beam. (a) Beam without damping ($\xi = 0$), $\mu = 0.05$, $x_1 = 0.5L$ (b) idem (a) Beam with damping $\xi = 0.01$.

present in the frequency response curves. Finally, from Fig. 6 (b) it is interesting to point out that, considering the case of $\xi_m = 0.001$, the frequency of minimum amplitude is not the second natural frequency of the beam. This sets a difference on the dynamic behavior of the beam if two 1-DOF systems are added instead of one.

In Figs. 7 (a) to (c) we study the frequency response of a damped beam ($\xi = 0.01$) when the mass, tuning ratio and location of the 2 1-DOF systems are changed. Only the frequencies near the second resonance of the beam are shown in Figs. 7 (a) and (c) since the curves for frequencies near the first resonance are of the same characteristics. Figure 7 (a) illustrates the situation when the location of the absorbers are changed. For a tuning ratio of $\alpha_i = 1$ (perfect tuning), a mass ratio of $\mu_1 = \mu_2 = 0.05$ and $\xi_{m1} = \xi_{m2} = 0.001$ it is observed that the amplitude for the frequency of minimum amplitude can be significantly reduced if the location of the absorbers are modified.

Figure 7 (b) show the modifications introduced into the system when the mass of both absorbers is modified. The other parameters of the absorbers are the same as for Fig. 7 (a). The location of the absorbers are set at $x_1 = 0.4L; x_2 = 0.6L$ arbitrarily. It can be observed that, as the mass of the absorbers are increased the amplitude for the frequency of minimum amplitude is also reduced. Another effect of the changing mass is that also the value of the frequency for minimum amplitude is modified, despite of the fact the tuning ratio is the same for all cases. Also, an interesting point appears for this case too. If values of $\mu > 0.01$ are considered, the amplitude for the frequency of minimum amplitude remains unalter. This means that if greater masses are used for amplitude reduction, no further decrements are expected for these masses.

Finally, in Fig. 7 (c) we vary the tuning ratio of the absorbers (for $\mu_1 = \mu_2 = 0.05$ and $\xi_{m1} = \xi_{m2} = 0.001$). Both absorbers are located at $x_1 = 0.4L; x_2 = 0.6L$. Here, it is possible to observe that different tuning ratios can substantially modify the amplitude for the frequency of minimum amplitude for the selected location.

3.3 Comparison between Case 1 and Case 2 on the dynamic behavior of the beam

In Fig. 8 we make a comparison between adding a 1-DOF system or adding 2 1-DOF systems of the same total mass ($\mu = \mu_1 + \mu_2$). The damping constants are selected identical for all cases $\xi_m = \xi_{m1} = \xi_{m2}$ and a damped beam of $\xi = 0.01$ is chosen. All the absorbers are tuned to the first natural frequency of the beam $(\alpha_i = 1)$ and the locations of the absorbers are indicated on Fig. 8. It is possible to observe that the effect of adding another absorber to the beam, leaving constant the total mass do modify neither the frequency for maximum reduction nor its amplitude. Consequently, this means that the amplitude of maximum reduction is only affected by the amount of mass and not affected by the number of absorbers if they have the same total mass. However, it is remarkable to see that the same reduction is attained for the three selected cases despite of the fact the absorbers are located at different positions.

3.4 Case 3

This case reveals the dynamic characteristics of adding a 2-DOF system to a damped beam. The 2-DOF absorber is tuned to the first and second natural frequency of the beam and the observation point is located at $x_a = 0.4L$. The stiffness constants of the 2-DOF to make a perfect tuning to the beam are $k_1 = 20016.1$ y $k_2 = 5447.6$.

Figures 9 (a) to (d) show the displacement amplitude of the beam for frequencies near its second natural frequency. The results for the first frequency show analogous curves and they

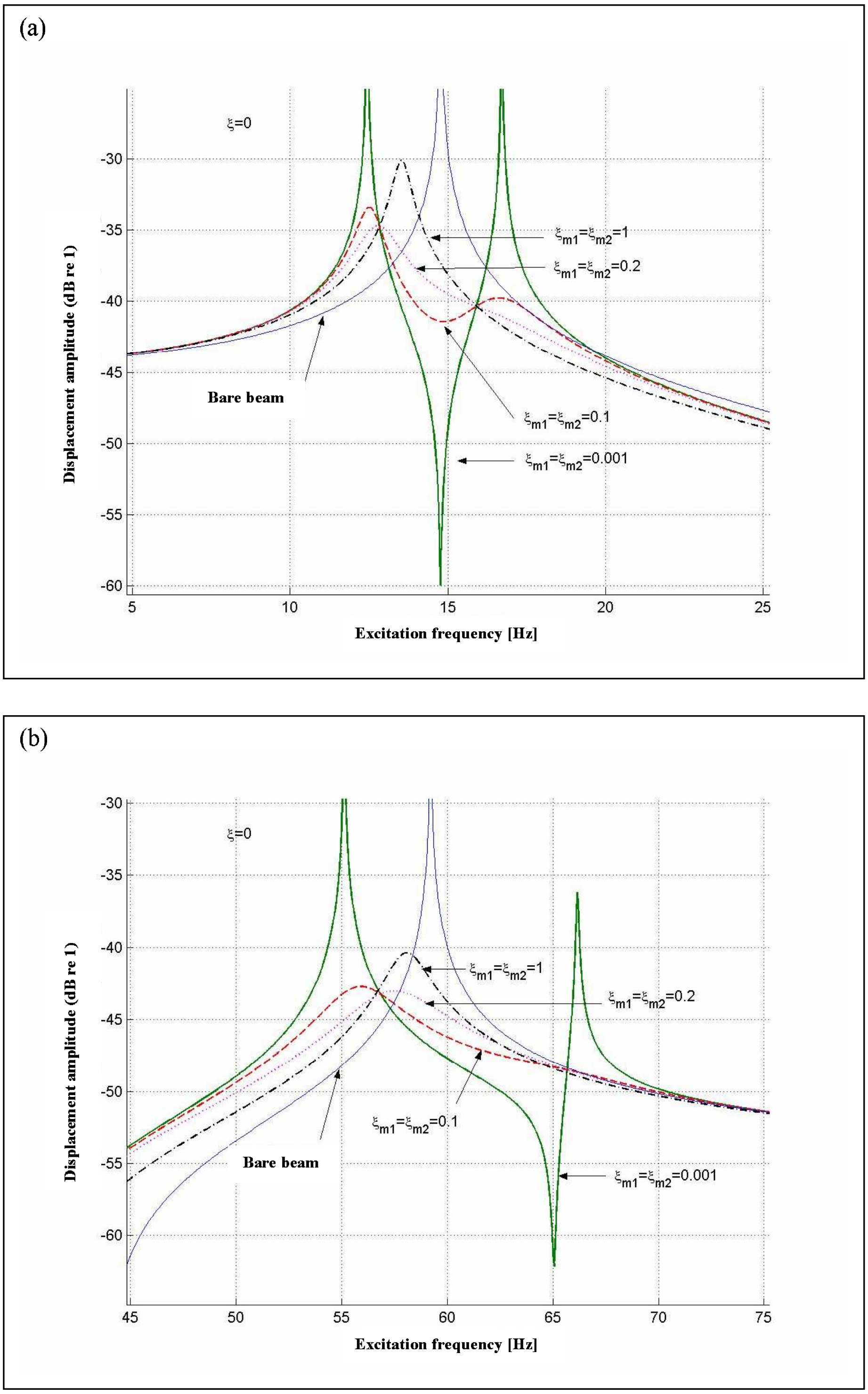


Figure 5: Frequency response of 2 1-DOF systems attached to an undamped beam ($\xi = 0$). (a) Amplitudes near the first resonance of the beam; $\mu_1 = \mu_2 = 0.05$, $x_1 = 0.4L$, $x_2 = 0.6L$. (b) Idem (a) for amplitudes near the second resonance of the beam.

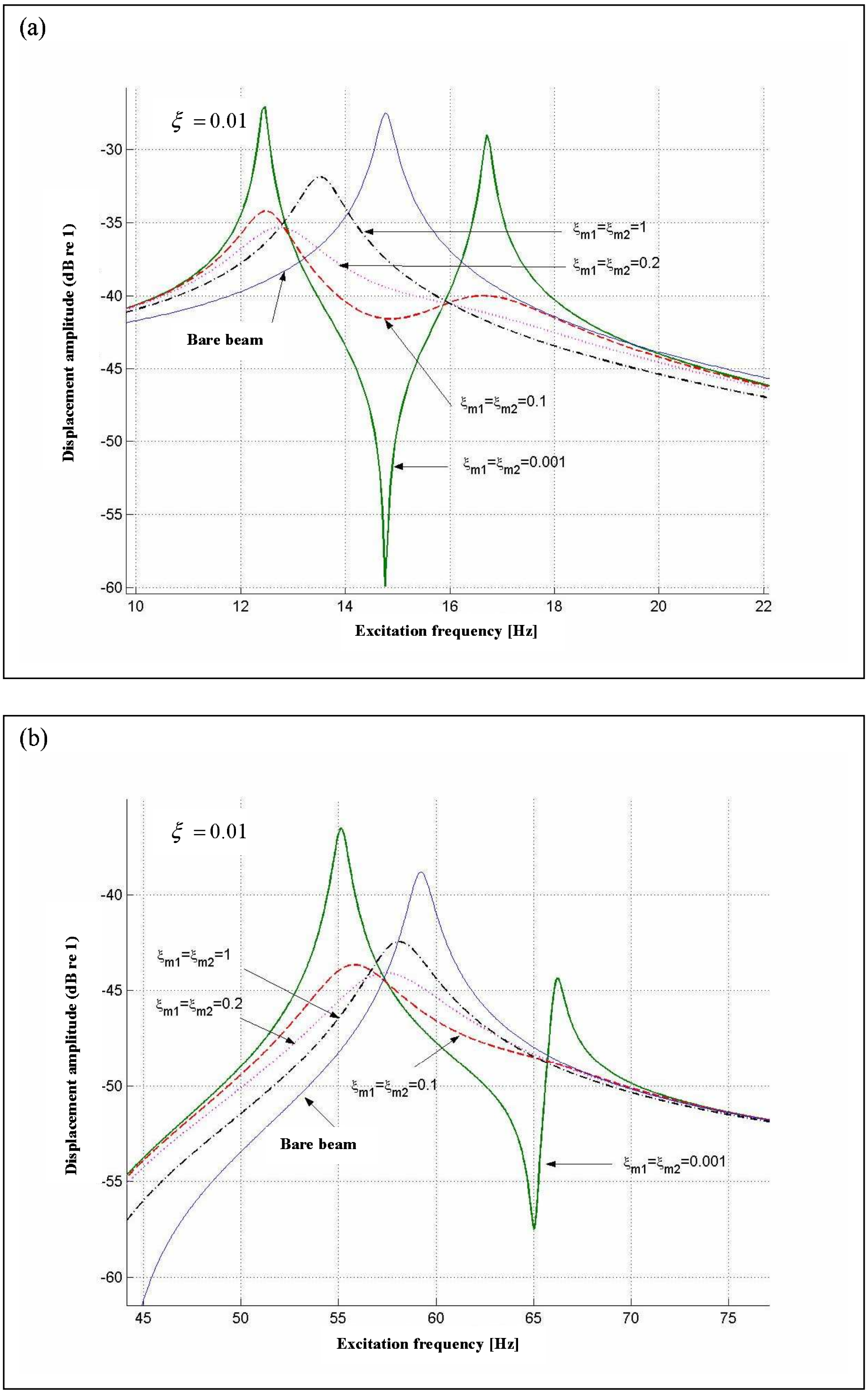


Figure 6: Frequency response of 2 1-DOF systems attached to a damped beam ($\xi = 0.01$). (a) Amplitudes near the first resonance of the beam, $\mu_1 = \mu_2 = 0.05$, $x_1 = 0.4L$, $x_2 = 0.6L$. (b) Idem (a) amplitudes near the second resonance of the beam.

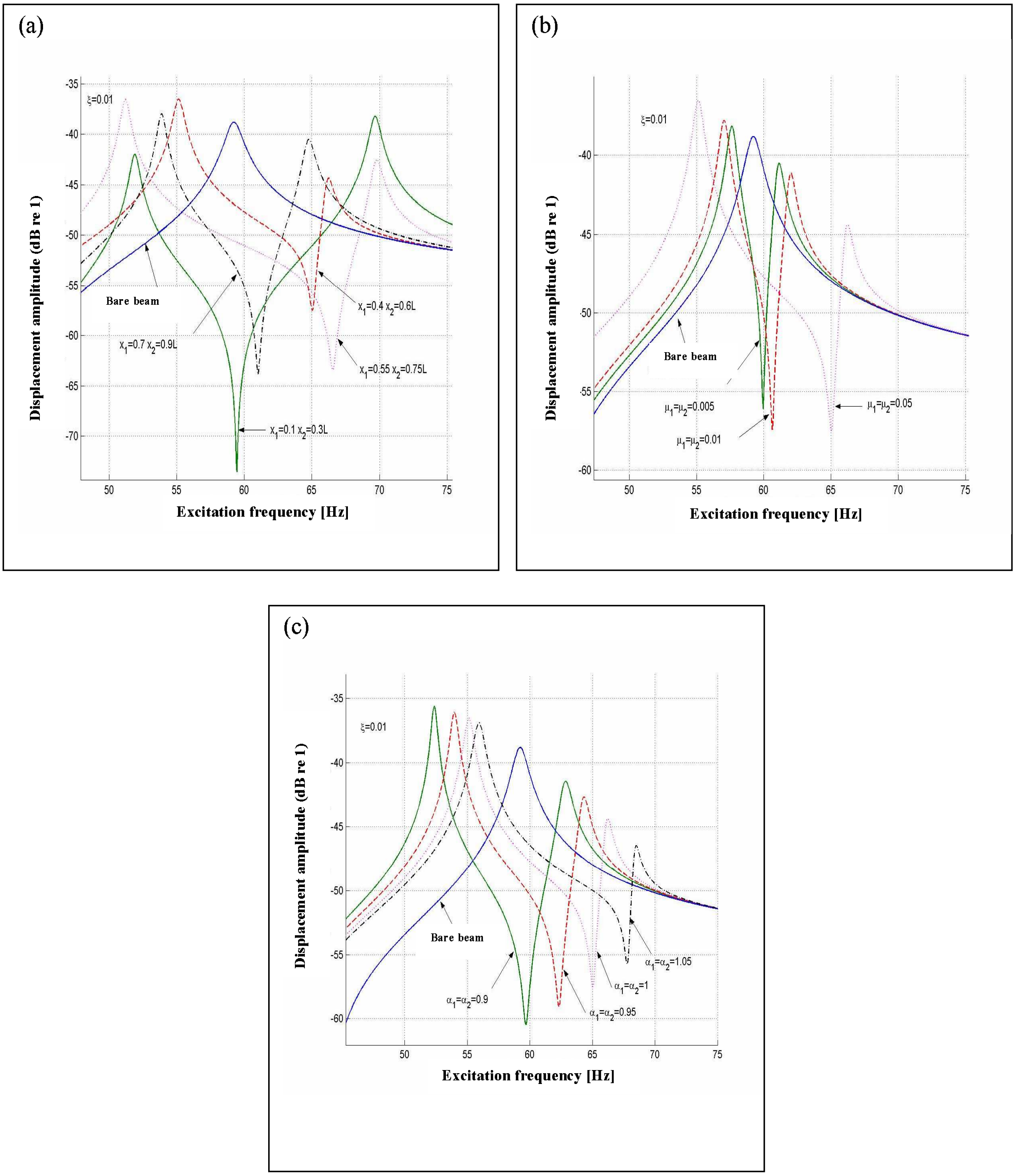


Figure 7: Frequency response of 2 1-DOF system attached to a beam. (a) Amplitudes for different locations of the absorbers, (b) for different mass ratios, (c) for different tuning ratios.

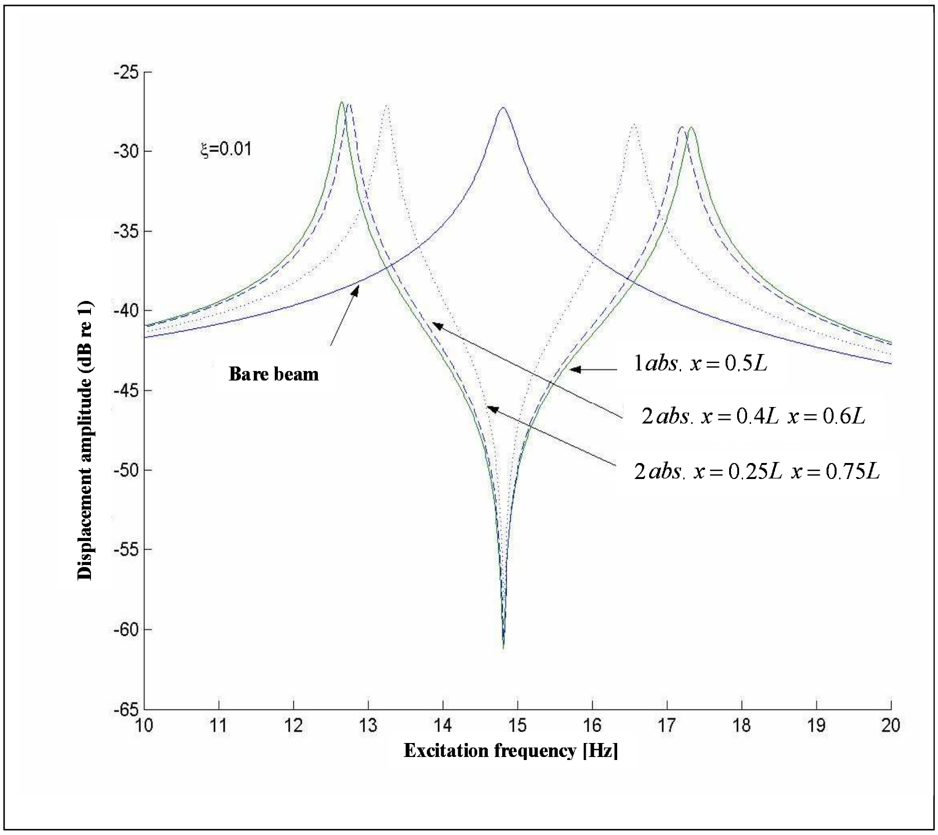


Figure 8: Frequency response of a beam with one 1-DOF system or two 1-DOF systems attached to it. The total mass of the two 1-DOF systems is equal to the mass of the 1-DOF system for a meaningful comparison.

are not shown to avoid unnecessary repetitions. Fig. 9 (a) presents the results considering different damping coefficients for the 2-DOF absorber, ξ_{m1} and ξ_{m2}. The other fixed parameters are the mass ratio $\mu_e = \frac{m_e}{m_b} = 0.1$; mass moment of inertia $I_e = 0.1 m_e$; $a_1 = 0.1L$; $a_2 = 0.1L$ (considering a rectangular parallelepiped, see Fig 1) and location $x_1 = 0.4L$ $x_2 = 0.6L$. As it has been pointed out for cases 1 and 2, it can be seen that the minimum amplitude decreases as the damping diminishes and the new peaks also increase their amplitudes. Noticeably, the frequency of minimum amplitude is not the second natural frequency of the beam but another very near (the same as for case 2).

In Fig. 9 (b) we alter the location of the absorber on the beam for $\xi_{m1} = \xi_{m2} = 0.001$ and $\mu_e = 0.1$ ($I_e = 0.1 m_e$). The results show that the location of the absorber results crucial for vibration suppression at the frequency of maximum reduction.

In Fig. 9 (c) we modify the mass ratio of the absorber leaving constant its (two) natural frequencies. As usual, this has been achieved by varying their stiffness constants k_1 and k_2. The absorber is located at $x_1 = 0.4L$ and $x_2 = 0.6L$ and $\xi_{m1} = \xi_{m2} = 0.001$. The results show that the more the mass of the absorber is, the greater the amplitude reduction results. Again, the frequency of minimum amplitude is slightly shifted of the natural frequency of the beam.

Finally, Fig. 9 (d) depicts the modification of the displacement amplitude of the beam as the tuning of the absorber is modified. For this case, $\mu_e = 0.1$; $\xi_{m1} = \xi_{m2} = 0.001$; $x_1 = 0.4L$ and $x_2 = 0.6L$ is selected. Then, it is possible to observe that the minimum amplitude and frequency of minimum amplitude are markedly modified. It is also observed that the height of the peaks are not symmetrical.

3.5 Comparison between Case 2 and Case 3 on the dynamic behavior of the beam

Here, we compare the frequency response of a damped beam with 2 1-DOF systems (Case 2) with the frequency response of the beam with a 2-DOF system (Case 3).
In Fig. 10 (a) and (b) the absorbers are at $x_1 = 0.4L$ and $x_2 = 0.6L$ and the observation point is at $x_a = 0.4L$. The mass of the 2-DOF system is the same as the sum of the masses of the 2 1-DOF systems ($\mu_e = \mu_1 + \mu_2 = 0.1$). Both absorbers are tuned to the first and second natural frequencies of the beam ($\alpha_1 = \alpha_2 = 1$) and the damping constants are set to $\xi_{m1} = \xi_{m2} = 0.001$ for both systems. In the next lines, we will refer to the 2 1-DOF systems as case 2 and to the 2-DOF system to case 3, to avoid unnecessary repetitions

Fig. 10 (a) show the results for frequencies near the first resonance of the beam. One of the differences resides in that the separation of the peaks for case 2 and case 3 are different. A minor separation can be found for case 3 and also a poorer reduction of vibration amplitude. This may be due to the distributed inertia for case 3. For frequencies near the second resonance of the beam, Fig. 10 (b) show that the separation of the peaks are larger for case 2. However, a better reduction of the amplitude at the frequency of maximum reduction is observed for case 3 instead.

The results for a different location of the absorbers are considered in Figs. 11 (a) and (b) ($x_1 = 0.1L$ and $x_2 = 0.3L$). All other parameters remain the same. As stated previously, the fact of changing the position of the absorbers leads to a drastically change of their performances. For instance, for frequencies near the first resonance of the beam (Fig. 11 (a)) case 3 performs a much better reduction than case 2. On the contrary this situation tends to equilibrate

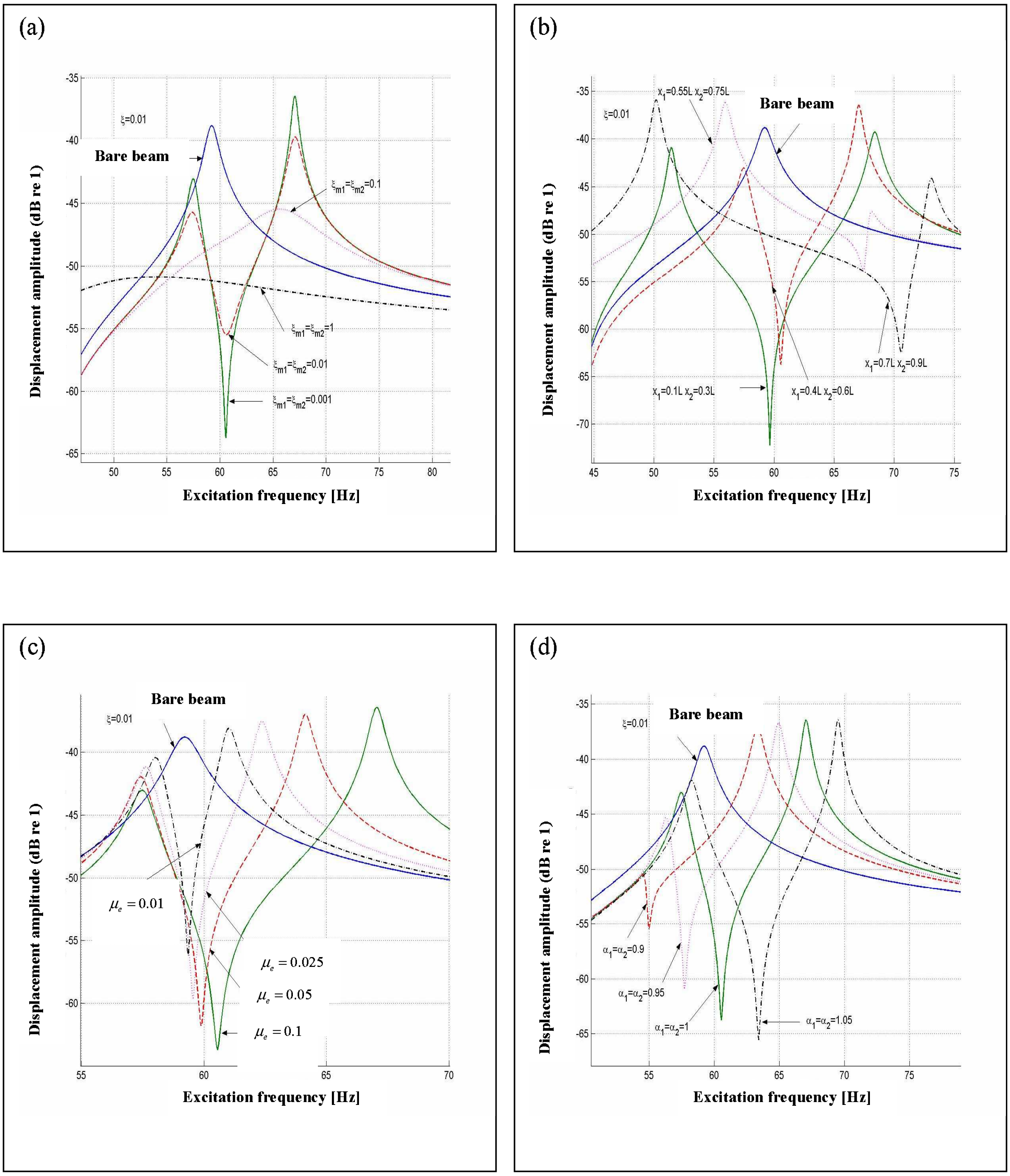


Figure 9: Frequency response curves of a beam with a 2-DOF system attached to it; frequencies near the second resonance of the beam.(a) Absorber with different damping coefficient; (b) at different locations; (c) with different mass ratios; (d) with different tuning ratios.

for frequencies near the second resonance of the beam, Fig. 11 (b).

Taking into account all the information provided above, it is possible to conclude that, if the separation of the peaks are the same, the amplitude at the frequency of minimum amplitude tends to be the same. Moreover, a large separation of the peaks leads to a better reduction of vibration amplitude provided the amplitude of the peaks are equal.

4 CONCLUSIONS

In this work, the dynamics of 2-DOF systems (coupled and uncoupled) mounted on beams has been presented. At the same time, their performances as functions of their masses, damping, locations and tuning have been analyzed. Generally speaking, the numerical calculations show that it is possible to tune them in such a way to effectively control the amplitude of vibration of a beam at a given point.

From the large number of cases studied, it is possible to ensure that the effect of damping is to increase the level of vibrations at the natural frequency of the beam and at the same time to widen the peaks that appear naturally in the coupled system (beam+absorber) due to the inclusion of additional degrees of freedom. Obviously, no damping means maximum reduction at the tuned frequency but larger amplitudes at the peaks.

It is interesting to note that **only** in the case of an undamped beam with a 1-DOF system, the fixed points exist. This is extremely important since the existence of fixed points is vital in the design of optimum absorbers DenHartog (1956), Febbo and Vera (2008).

Regarding to the effect of the location of the absorbers, this results crucial since the way the absorber acts on the beam is through the elastic forces made by the springs. Consequently, the closer these springs to an antinode locates, the larger the forces onto the beam result. Also, the nature of the 2-DOF systems is important for their effectiveness as absorbers. This can be understood since a coupled 2-DOF system has a rotational and a translational mode whereas an uncoupled 2-DOF system has only translational ones.

The effect of raising the mass of the DVA is to reduce the level of vibrations at the frequency of maximum reduction. However, our numerical examples demonstrate that this reduction has a limit, above which a further increment of the mass produces no effect.

Another interesting conclusion is that the behavior of one 1-DOF system and two 1-DOF system symmetrically placed from a beam antinode results very similar if the same total mass is considered.

From the comparison between 2 1-DOF systems and a 2-DOF system, it is possible to say that an equal separation of the peaks leads to a similar amplitude reduction at the frequency of minimum amplitude. On the other side, a large separation of the peaks leads to a better reduction of vibration amplitude provided the amplitude of the peaks are equal.

As a final remark, we wish to mention that the presented mathematical model can be regarded as a useful tool to modelling other structures with attached discrete systems (like plates or shells) provided their displacement amplitudes are written as a sum of mode shapes.

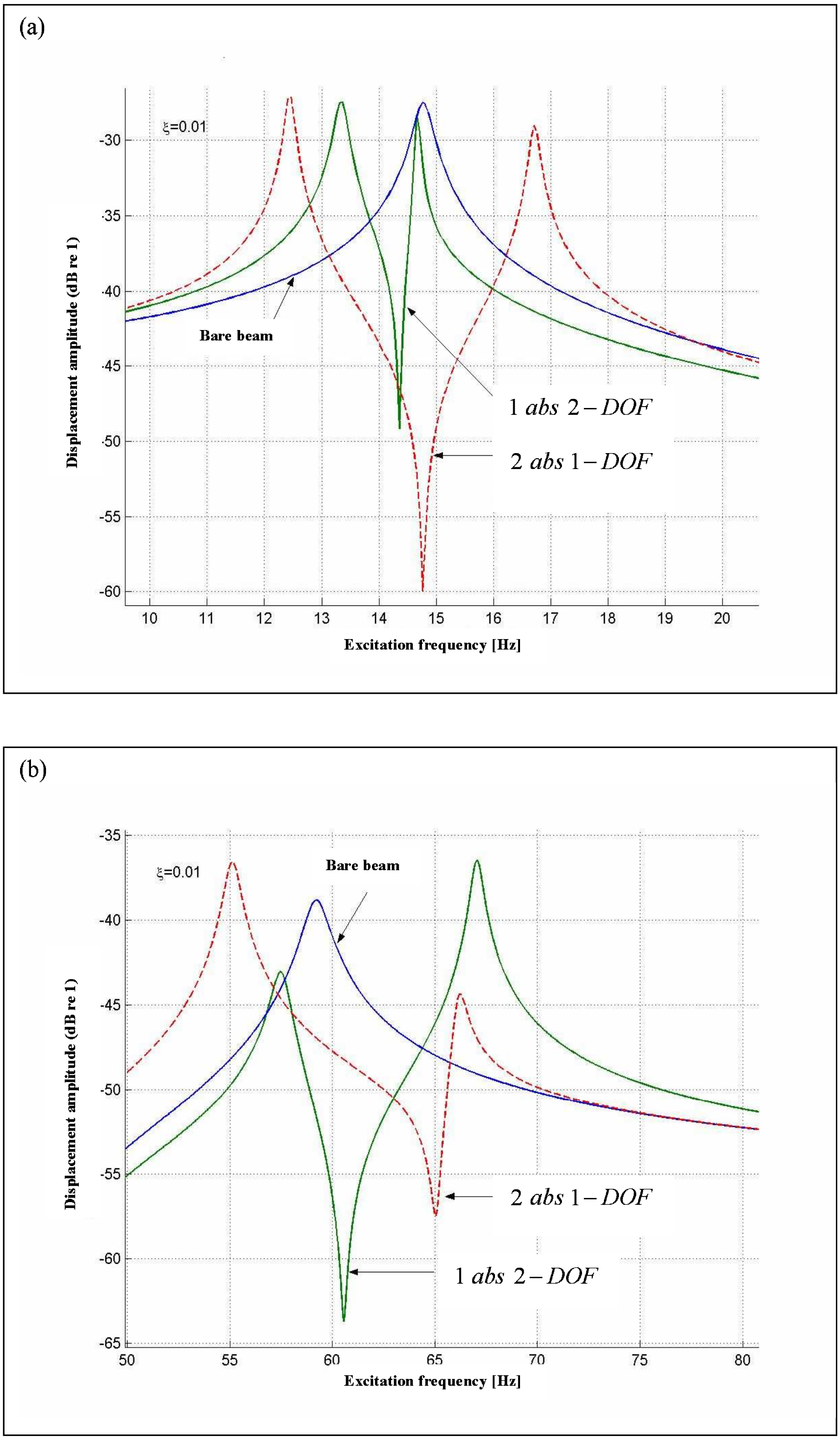
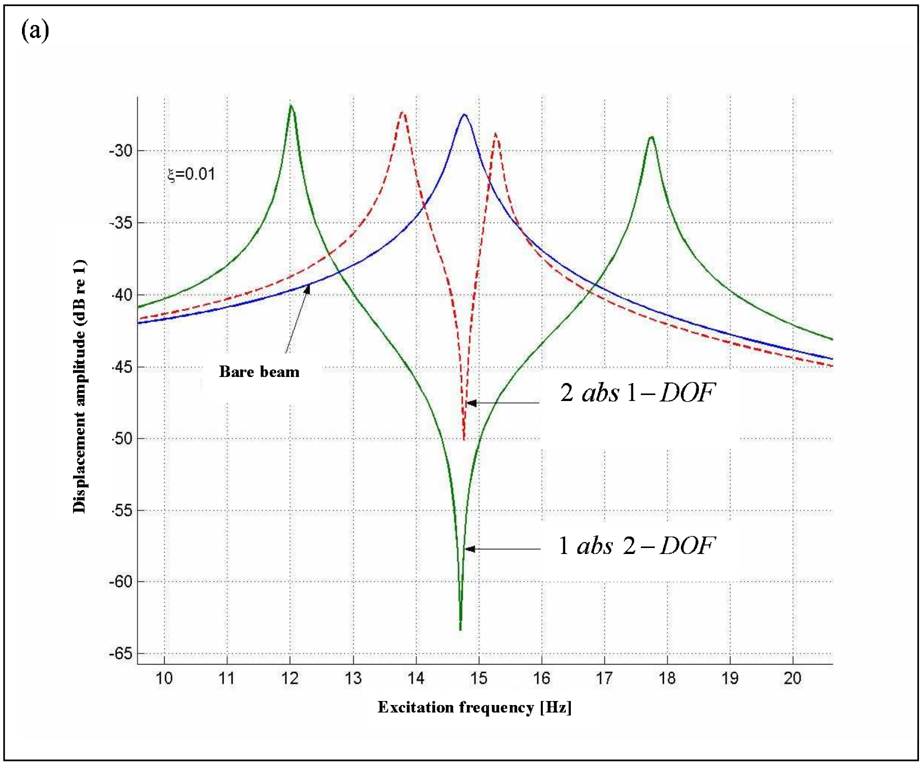


Figure 10: Frequency response of a beam with one 2-DOF system and two 1-DOF systems. Location of the absorbers $x_1 = 0.4L$ y $x_2 = 0.6L$, (a) frequencies near the first resonance of the beam; (b) frequencies near the second resonance of the beam.



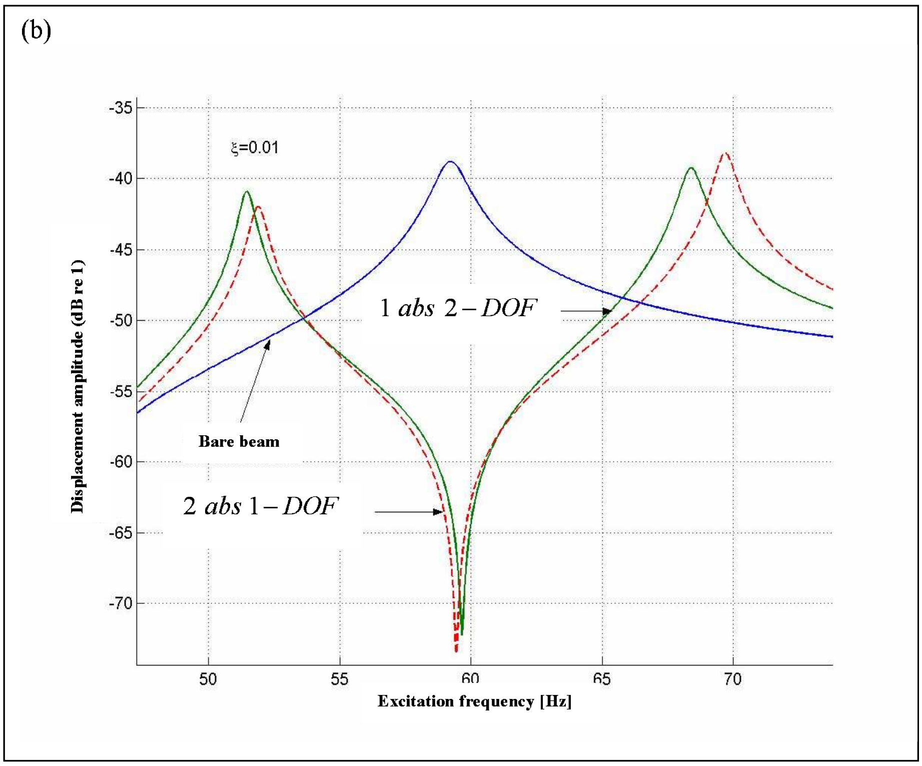


Figure 11: Frequency response of a beam with one 2-DOF system and two 1-DOF systems. Location of the absorbers: $x_1 = 0.1L$ y $x_2 = 0.3L$. (a) Frequencies near the first resonance of the beam; (b) frequencies near the second resonance of the beam.

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5 ACKNOWLEDGEMENTS

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