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DYNAMIC STABILITY OF IMPERFECT CABLE STAYED MASTS

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Abstract. Cable stayed masts are used in several engineering applications. In this work, the non-linear finite element method, using an updated Lagrangian formulation, is used to study the effect of initial geometric imperfections on the non-linear vibrations of cable stayed masts subjected to axial time dependent loads. The non-linear equations are solved using the Newton-Raphson method associated to an arc-length technique and the Newmark method is used to calculate the time responses of the system. Validation examples are presented and the influence of initial geometric imperfections and cable tensioning is studied when stayed towers are subjected to different types of axial loads. The Budianski´s criterion is used to study the loss of stability under sudden and harmonic loads. Obtained numerical results show the great influence of both cable tensioning and cable positioning on the non-linear behavior of the system and could be used as a tool for an analysis of the nonlinear dynamics of the structure previous to design.

1 INTRODUCTION

Cable-stayed truss and tube masts and towers are widely used in several engineering areas with applications in civil, off-shore, mechanical, telecommunications and aero-space engineering. The efficiency of these structures to support axial loads is due to the stay cables and their behavior is characterized by large displacements associated with high load bearing ratios. The cables must not only have sufficient capacity to carry the dead loads, but must also have enough reserve capacity to carry the live loads. As cable-stayed structures show large displacements, high non-linearities are associated with their static and dynamic behavior. Therefore, the knowledge of their non-linear behavior is of interest to engineers and scientist.

Due to their efficiency and different engineering applications, the analysis of cable-stayed structures has been object of several investigations in the last decades. In Madugula (2002) it is possible to obtain a complete revision related to the analysis of towers and stayed masts.

In this work, only a few papers will be cited. Neves (1990) presented a finite element program to study the non-linear static and dynamic behavior of cable-stayed bridges. Using a three-dimensional model, he showed that, due to their non-linearity, the cables strongly influence the structural response of the system. Kahla (1997), using a three dimensional finite element model, studied the non-linear response of cable-stayed towers. The obtained results demonstrated that, during the non-linear response, the structure failed due to the compression forces generated by the cables.

Xu et al (1997) proposed a three dimensional finite element model to study the dynamic response of the stayed towers of the Tsing Ma bridge. They showed that there is a high dynamic interaction between towers and cables which affects the natural frequencies of the system.

Wahba et al (1998) and Madugula et al (1998), using the nonlinear finite element method and an analytical model, studied the non-linear static and dynamic response of stayed towers. Obtained results showed that the analytical model presents lower displacements if compared with those obtained by the finite element method. Experimental models were also studied and reliable agreement with the numerical model was observed.

Kahla (2000) studied the effect of cable failure on the dynamic response of stayed towers concluding that, if the failure occurs in certain cables, the chance of failure of the whole structure is increased. Millar and Barghian (2000), using two finite element codes, studied the static and dynamic response of structures that displays dynamic jumps. They concluded that non-linear static problems can be analyzed as dynamic systems without damping.

Cheng et al. (2002), using an advanced non-linear finite element formulation, studied the aerostatic stability of stayed bridges showing the high non-linearity of the response due to lateral winds. Chan et al. (2002) performed a second order analysis of imperfect stayed columns and showed that the buckling load of the columns can be increased by the pre-tensioning of the stay cables. Yan-Li et al. (2003), using a discrete model, analyzed the vibrations of stayed masts under wind loads, finding a good agreement between experimental and numerical results.

Pasquetti (2003) studied the buckling and vibration characteristics of cable-stayed towers using a simplified SDOF model. Freire et al. (2005) studied the non-linear effects of a stayed bridge using both linear and non-linear finite elements. The obtained results show that the cable curvature increases the non-linearity of the system mainly when large displacements generate axial tensions on the cables. Orlando (2006) studied the non-linear dynamics and control of a tower-pendulum system under harmonic loads. A detailed parametric analysis of the non-linear oscillations showed that a non-linear pendulum absorber can increase or decrease the vibration amplitudes of the tower.

The post-critical behavior of perfect stayed masts was studied in detail by Saito and Wadee (2008) and Carvalho (2008). They showed that the post-critical behavior is strongly influenced by the cables and its initial tensioning and, depending on the system geometry, the post-critical paths could change completely.

Using real scale experiments and a finite element modeling, Araujo et al (2008) studied the static behavior of stayed masts, looking for the optimal structural geometry. A parametric analysis showed that cables increased the critical load and, for small imperfection levels, the load bearing capacity of stayed mast is higher than that of the non-stayed mast.

Recently, Carvalho (2008) and Saito and Wadee (2009a) using the finite element method analyzed the dynamic behavior of imperfect stayed structures. They concluded that, depending on the load characteristics, the system is imperfection sensitive. Saito and Wadee (2009.b) using the finite element method showed that the interactive buckling response of stayed masts could lead to a critical load reduction. Using the Budianski criterion, Del Prado et al (2010) analyzed the dynamic instability of perfect stayed masts under sudden and harmonic loads. The numerical results showed the strong influence of load and cable positioning on the critical load of the system.

In this work, the non-linear finite element method, using an updated Lagrangian formulation, is used to study the non-linear vibrations of imperfect cable-stayed masts subjected to axial time dependent loads. The non-linear equations are solved using the Newton-Raphson method associated to an arc-length technique and the Newmark method is used to calculate the time responses of the system.

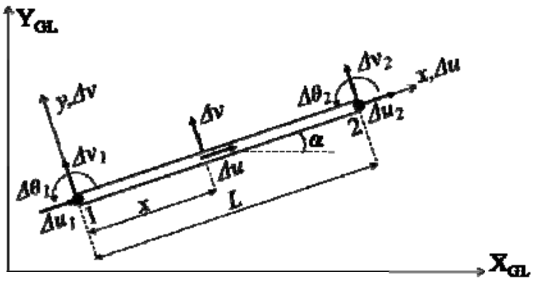
Validation examples are presented and the influence of initial geometric imperfections and cable tensioning is studied when stayed towers are subjected to different dynamic loads. Special attention is given to the influence of geometric imperfections on natural modes and natural frequencies and the Budianski´s criterion is used to study the loss of stability under sudden and harmonic loads. Obtained numerical results show the great influence of both cable tensioning and cable positioning on the non-linear behavior of the system and could be used as a tool for an analysis of the nonlinear dynamics of the structure previous to design.

2 MATHEMATICAL FORMULATION

The present formulation is based on previous works by Silveira (1995), Galvão (2000), Oliveira (2002), Campos Filho (2004) and Carvalho (2008) who implemented finite element models to analyze the geometric non-linear behavior of plane structural frames and cable system.

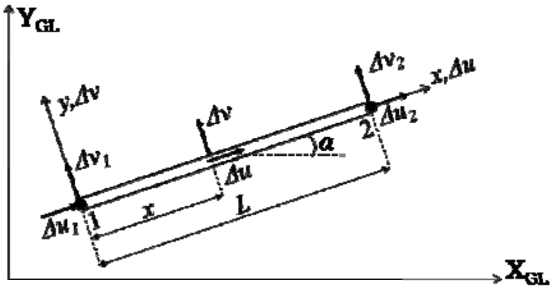
Consider a plane straight beam-column with Young modulus E, area A, and inertia moment I and density ρ, as shown in Figure 1. The finite element is limited by nodes 1 and 2. The local system coordinates are denoted by x and y and global coordinates by X_{GL} e Y_{GL}. For each node, the nodal displacement and rotations are denoted by Δu_i, Δv_i, and $\Delta\theta_i$ with i = 1,2.

For the cable finite element, it is considered a truss plane element with Young modulus E, area A, density ρ and limited by two nodes with local coordinates x and y and global coordinates X_{GL} e Y_{GL}, as shown in Figure 2. The transversal and axial nodal displacement are given by Δu_i and Δv_i with i =1,2, respectively.



(a)

Figure 1: Beam-column finite element.



(b)

Figure 2: Cable finite element

Using variational principles and a Lagrangian updated referential, the non-linear dynamic equilibrium equations are written as:

$$\mathbf{M}\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\left[\mathbf{K}_L+\mathbf{K}_\tau+\frac{1}{2}\mathbf{K}_1+\frac{1}{6}\mathbf{K}_2\right]\Delta\mathbf{u}+\mathbf{Fi}(u)=\lambda\,\mathbf{Fr}(t) \tag{1}$$

where $\mathbf{M}$ is the consistent mass matrix, $\mathbf{C}$ is the viscous damping matrix, $\mathbf{K}_L$ i the linear stiffness matrix, $\mathbf{K}_\tau$ is the initial stress matrix, $\mathbf{K}_1$ and $\mathbf{K}_2$ are the non-linear displacement dependent matrixes, $\mathbf{Fi}(u)$ is the displacement dependent internal force vector, $\mathbf{Fr}(t)$ is the time dependent load vector, $\ddot{\mathbf{u}}, \dot{\mathbf{u}}$ and $\Delta\mathbf{u}$ are the acceleration, velocity and displacement vectors respectively, and λ is a load parameter.

3 NUMERICAL RESULTS

Consider a perfect clamped-free column with internal diameter = 0.475 m, external diameter = 0.500 m and elasticity modulus E = 1.18e8 kN/m^2. The column is clamped at the base and supported by two inclined cables with $\alpha = 60°$, cross-section diameter = 0.018 m, elasticity modulus E = 1.0e8 kN/m^2 and a pre-tensioning force T = 10 kN. The column is subjected to an axial load P and a perturbing moment M as shown in Figure 3(a).

Figure 3(b) shows the influence of stay cables on the post-critical behavior of an axially loaded column and u is the transversal displacement at the top of the column. When no cables are considered in the analysis, after the critical load, the system displays a stable post-critical path with a small initial curvature. If the two cables are considered, the value of the critical load increases more than seven times, but the system displays in this case an unstable post-buckling behavior with a sharp decrease in the load carrying capacity, being the minimum post-critical load, associated with a fold bifurcation, lower than the critical load of the column without cables. So, when the column reaches the critical loads it jumps to a post-buckling configuration associated with large displacements. Figure 3(c) shows the post-critical path of the cable stayed column as a function of the axial displacement at the top of the column *v*. So, based on the theory of elastic stability, high imperfection sensitivity is expected for this structural system.

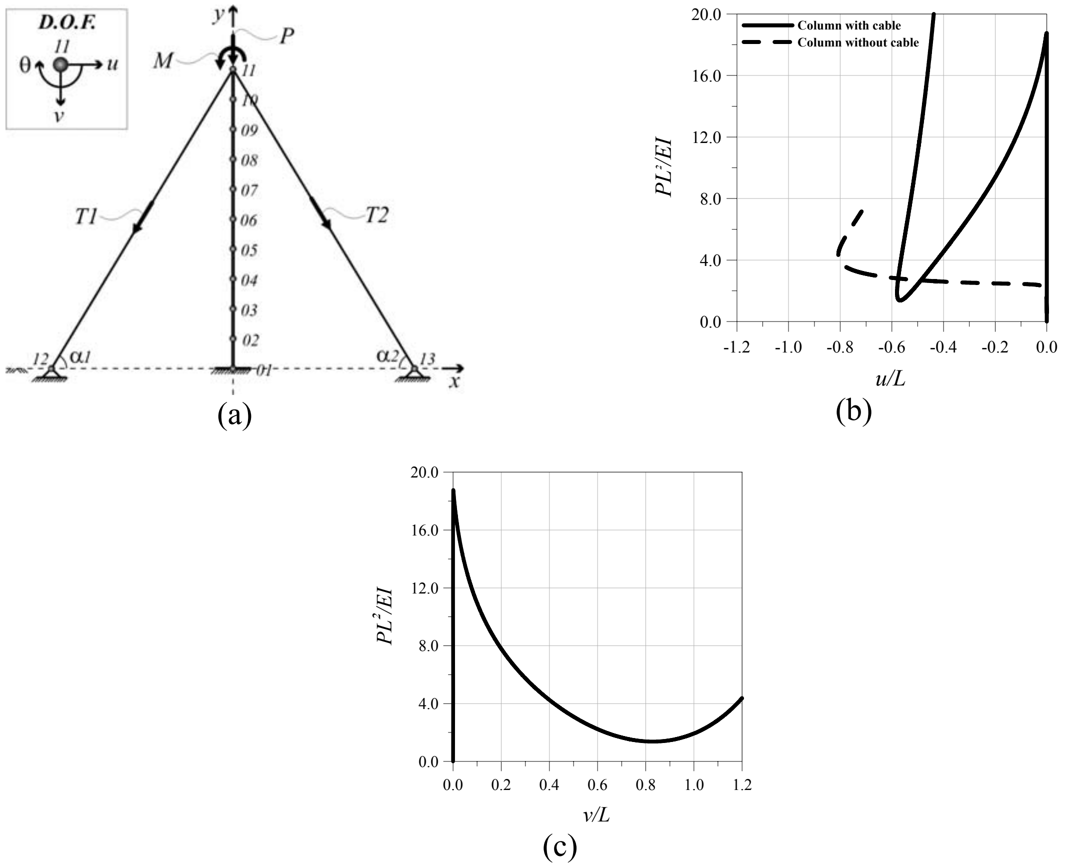


Figure 3: Perfect cable-stayed column. (a) Geometric characteristics. (b) Post-critical paths as a function of the lateral displacement. (c) Post-critical paths as a function of the vertical displacement.

Figure 4 displays the initial deformed configuration of the column due to different initial cable tensioning. This induces an initial geometric imperfection in the column. In this work two different initial geometric imperfections (Case 1 and Case 2) are considered. Table 1 shows the nodal coordinates corresponding to both imperfection cases. As can be observed, the initial imperfections for Case 2 are a little bit higher than Case 1.

Figure 5(a) shows the variation of the load parameter with the lateral displacement at the top of the column, *u*, while Figure 5(b) shows the variation of the load parameter with the vertical displacement, *v*. When compared with the perfect case (equal cable tensioning) the nonlinear equilibrium path shows a rather different behavior. The column looses stability at a limit point which is much lower than the critical load of the perfect system. The decrease in the critical load is of about 60%. This illustrates the high imperfection sensitivity of this structural system and the deleterious effect of asymmetric cable tensioning on the non-linear response.

Comparing the post-critical paths for Cases 1 and 2, it is possible to observe the effect of the size of the initial imperfections. The limit point for Case 1 is higher than that for Case 2 with a reduction of about 11% which means that small variations in the level of imperfections leads to a large variation in the load carrying capacity.

Table 1: Nodal coordinates of initial geometric imperfections for Cases 1 and 2.

	Case 1		Case 2	
Node	$x\ (m)$	$y\ (m)$	$x\ (m)$	$y\ (m)$
01	- 0.000	0.000	- 0.000	0.000
02	- 0.480	9.988	- 0.533	9.987
03	- 1.724	19.910	- 1.916	19.900
04	- 3.343	29.777	- 3.714	29.752
05	- 4.882	39.657	- 5.424	39.619
06	- 5.903	49.605	- 6.559	49.561
07	- 6.055	59.603	- 6.728	59.559
08	- 5.156	69.562	- 5.729	69.513
09	- 3.232	79.374	- 3.591	79.304
10	- 0.511	88.996	- 0.568	88.884
11	- 2.643	98.485	- 2.937	98.317
12	- 57.735	0.000	- 57.735	0.000
13	- 57.735	0.000	- 57.735	0.000

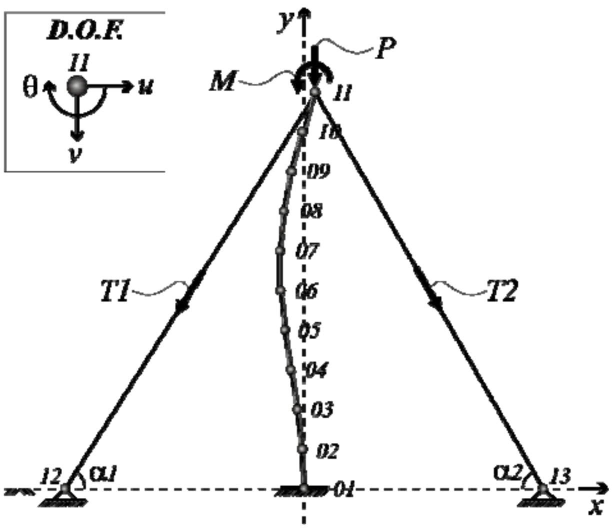


Figure 4: Imperfect cable stayed column

Now consider the perfect column with two stay cables and subjected to a suddenly applied axial load P as illustrated in Figure 6(a). Figures 6(b), 6(c) and 6(d) display the time responses of the stayed column for increasing values of axial load given as a fraction of the critical static axial load P_{cr}. Zero initial conditions are considered in the analysis. As shown in Figure 6(b), for certain values of P, the damped response converge to a pre-buckling static configuration. As P increases, see Figures 6(c) and 6(d), the column displays increasing vibration amplitudes and jumps to a post-buckling configuration. Figure 6(e) shows the loss of stability of the cable-stayed column for increasing values of axial load, using the Budianski's criterion. For this type of load the critical load is $P/P_{cr} \approx 0.95$.

Now consider the perfect column with two cables and subjected to a harmonic axial load P with amplitude P_a and frequency Ω. Figures 7(a), 7(b) and 7(c) show the time response of the lateral displacement for increasing values of the axial load amplitude P_a considering a forcing frequency equals to the natural frequency of the stayed column (ω_o). These time responses represent the lateral displacement of the top of the column.

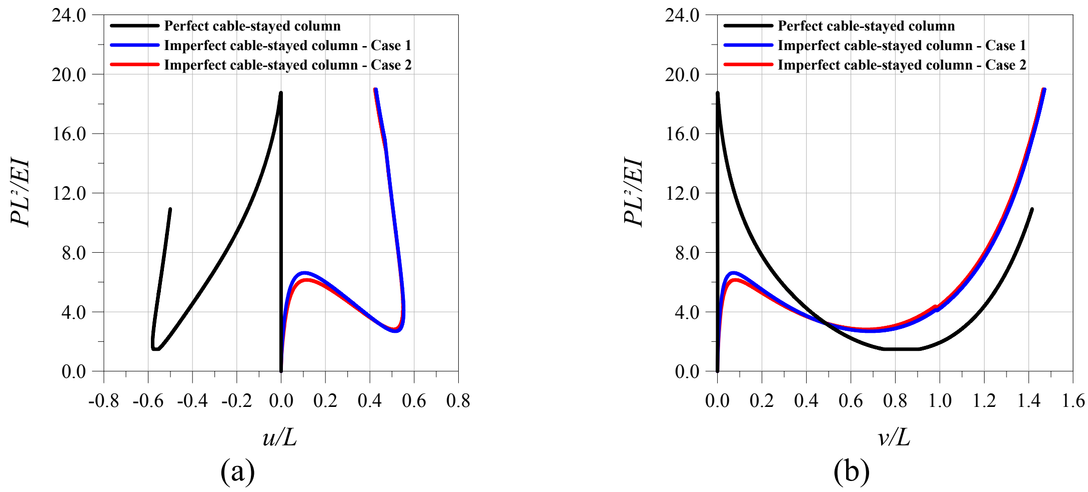


Figure 4: Comparisons of post critical paths of the perfect and imperfect cable stayed column. (a) Lateral displacement. (b) Vertical displacement.

For small values of the forcing amplitude, the column displays small amplitude lateral oscillations. However, in Figure 7(c), for $P_a = 2.40\ P_{cr}$, the lateral displacement of the column grows exponentially, indicating loss of instability of the Mathieu type. Figure 7(d) shows the loss of stability curve of the cable-stayed column for increasing values of the amplitude of the axial harmonic force for $\Omega = \omega_o$. Using the Budianski's criterion, the critical load is $P_a = 2.30\ P_{cr}$.

Figures 8(a), 8(b) and 8(c) show the lateral time responses for increasing values of the axial load amplitude P_a considering a forcing frequency equals to two times the natural frequency of the stayed column (ω_o). The time responses show the variation of the lateral displacement at the top of the column.

For small values of the forcing amplitude, the column displays small amplitude lateral oscillations. However, in Figure 8(c), for $P_a = 0.80\ P_{cr}$, the lateral displacement of the column shows large amplitude vibrations. Figure 8(d) shows the loss of stability curve of the cable-stayed column for increasing values of the amplitude of the axial harmonic force for $\Omega = 2\omega_o$ and, according with Budianski's criterion, the critical load is $P_a/P_{cr} \approx 0.70$. This case corresponds to the main parametric instability region. When comparing Figure 7(d) with Figure 8(d), it is possible to observe that the dynamic buckling load for $\Omega = 2\omega_o$ is much lower than for $\Omega = \omega_o$.

Consider now, the cable stayed column with both Case 1 and Case 2 initial imperfections levels. Figure 9 display the loss of stability curves, using the Budianski's criterion, with the column under sudden axial load for Case 1 and Case 2. When comparing Figure 9(a) and 9(b) with Figure 6(e), the effect of initial imperfections on the load capacity is clearly observed. For Case 1 the instability load is $P = 0.0126\ P_{cr}$ and for Case 2 is $P = 0.002\ P_{cr}$. As can be verified, there is a strong reduction on the dynamic buckling load due to increasing initial imperfections.

Now, the effect of harmonic loads on imperfect stayed column is considered. Figure 10 shows the Budianski's curve for the imperfect column under harmonic axial load with $\Omega = \omega_o$. For Case 1 the instability load is $P_a = 0.02\ P_{cr}$ and for Case 2 is $P_a = 0.0085\ P_{cr}$. Again, comparing these values with those of the perfect column, a high imperfection sensitivity is observed.

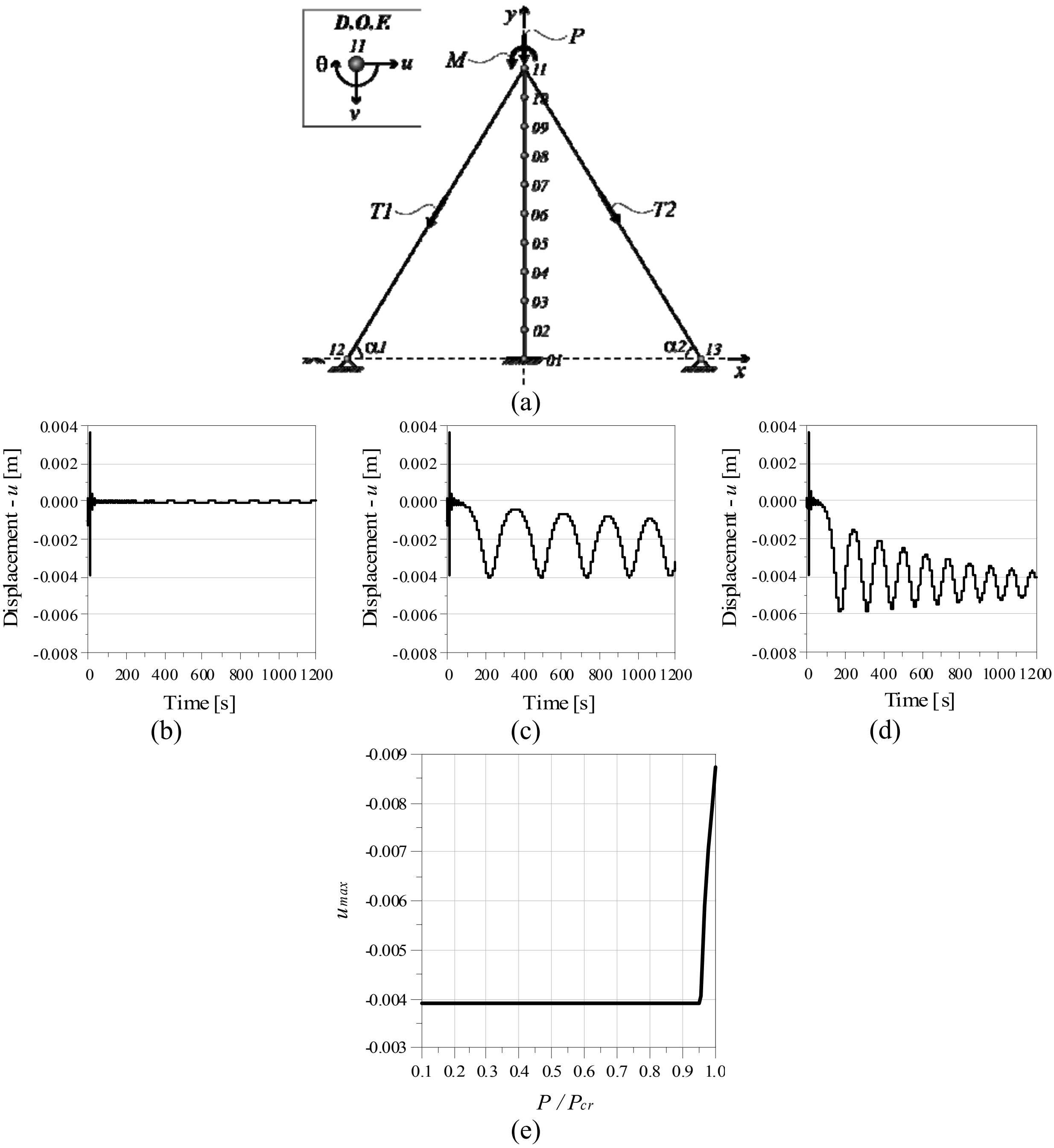


Figure 6: Dynamic instability of perfect cable stayed column under sudden axial load. (a) Geometric characteristics. (b) Time response for $P = 0.92P_{cr}$. (c) Time response for $P = 0.96P_{cr}$. (d) Time response for $P = 0.97P_{cr}$. (e) Budianski curve for column under sudden axial load.

Figure 11 display the Budianski's curve for imperfect column under harmonic axial load with $\Omega = 2\omega_o$. As the load increases the column maximum displacement grows gradually and, at a critical value, the displacement exhibits an exponential growth, indicating loss of stability. For Case 1 the critical load is $P_a = 0.035\ P_{cr}$ and for Case 2 the critical load is $P_a = 0.026\ P_{cr}$. Again these critical values are much smaller than those for perfect column showing the deleterious influence of geometrical initial imperfections on the load carrying capacity of the tower.

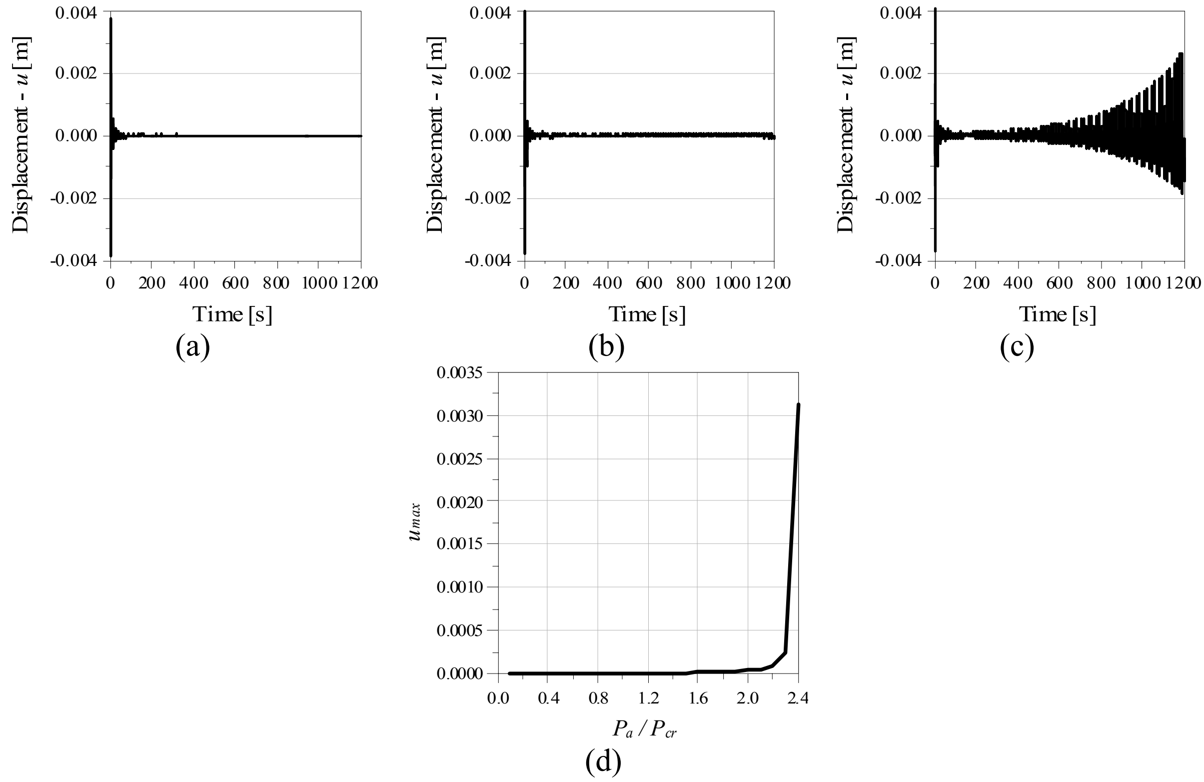


Figure 7: Dynamic instability of perfect cable stayed column under harmonic axial load with $\Omega = \omega_o$. (a) Time response for $P_a = 1.40P_{cr}$. (b) Time response for $P_a = 2.20P_{cr}$. (c) Time response for $P_a = 2.40P_{cr}$. (d) Budianski curve for column under harmonic load with $\Omega = \omega_o$.

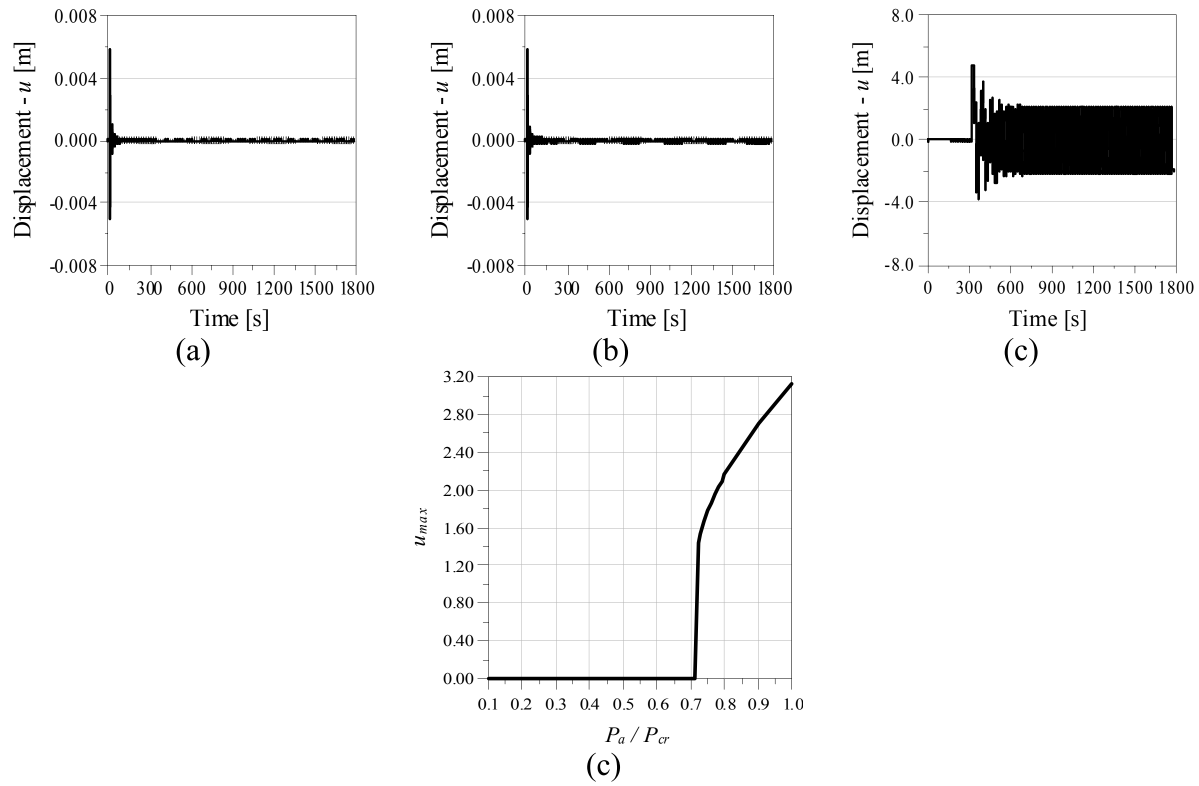


Figure 8: Dynamic instability of perfect cable stayed column under harmonic axial load with $\Omega = 2\omega_o$. (a) Time response for $P_a = 0.50P_{cr}$. (b) Time response for $P_a = 0.70P_{cr}$. (c) Time response for $P_a = 0.80P_{cr}$. (d) Budianski curve for column under harmonic load with $\Omega = 2\omega_o$.

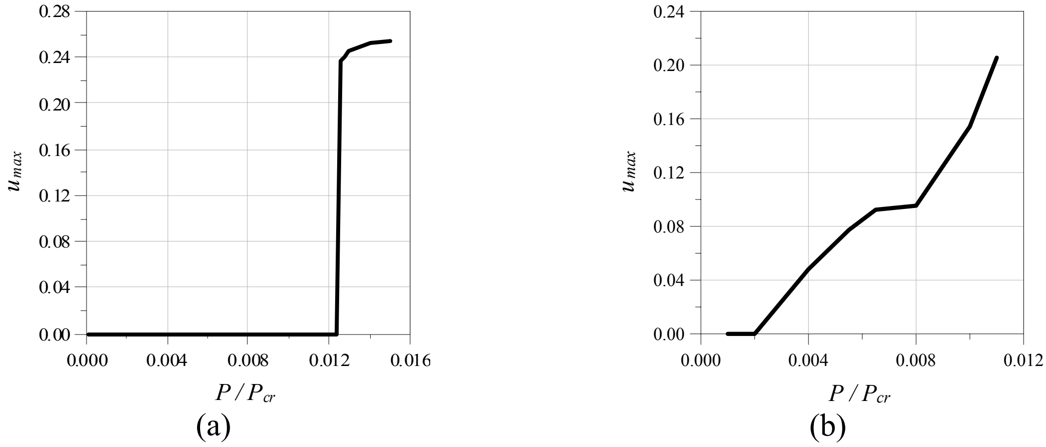


Figure 9: Dynamic instability curves of imperfect cable stayed column under sudden axial load. (a) Budianski's curve for Case 1. (b) Budianski's curve for Case 2.

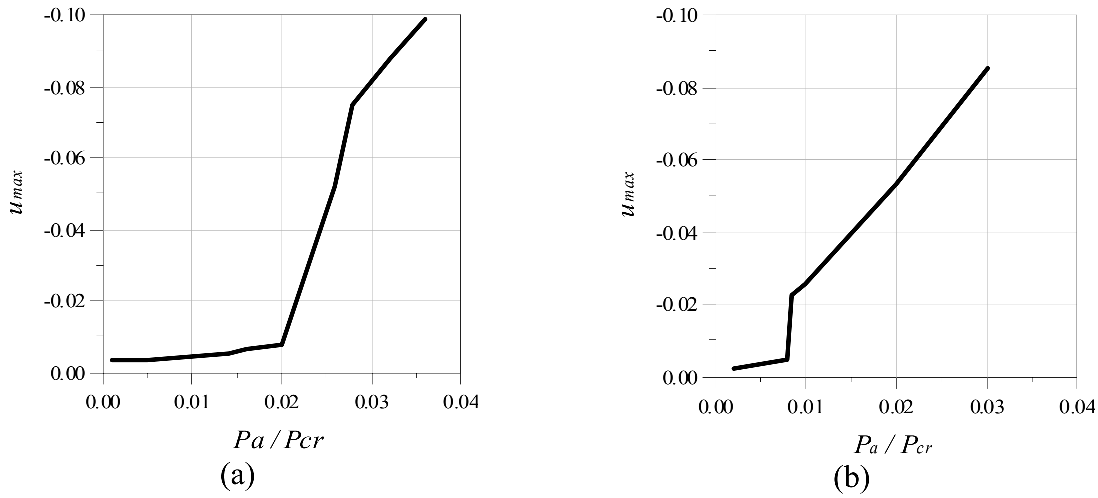


Figure 10: Budianski's curves for imperfect stayed column under harmonic load with $\Omega = \omega_o$. (a) Case 1. (b) Case 2.

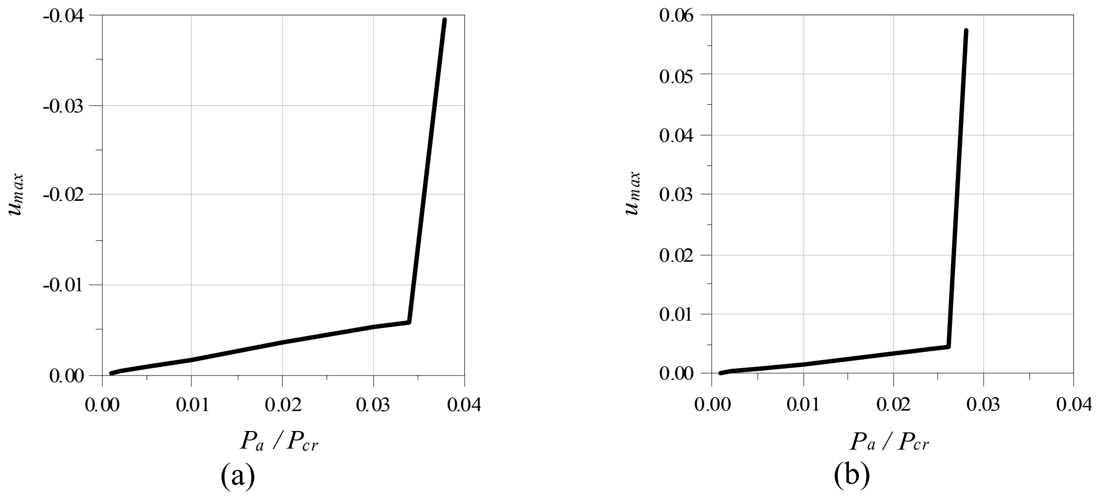


Figure 11: Budianski's curves for imperfect stayed column under harmonic load with $\Omega = 2\omega_o$. (a) Case 1. (b) Case 2.

4 CONCLUSIONS

In this work, the non-linear finite element method, using an updated Lagrangian formulation, is employed to study the non-linear vibrations of perfect and imperfect cable-stayed masts subjected to axial time dependent loads. The non-linear equations are solved using the Newton-Raphson method associated to an arc-length technique and the Newmark method is used to calculate the time responses of the system.

Using the Budianski´s criterion, the loss of stability under sudden and harmonic loads is also analyzed. As observed, the behavior of the system is highly influenced by cable tensioning, load characteristics and imperfection levels which generate lower or higher instability loads. Then numerical results show the great influence of both cable tensioning and initial geometric imperfections on the non-linear behavior and stability of the system.

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