# A GENERAL FORMULATION FOR COMPUTATION OF SUBSONIC REGION OF A BLUNT BODY PROBLEM WITH TIME DEPENDENT APPROACH

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## RESUMEM

Este artigo trata de la investigación del campo de flujo sobre cuerpo chato por la resolución de las equaciones tiempo dependientes de Euler. La solución es obtenida por la marcha en el tiempo una hasta alcanzar su estado permanente. El choque es capturado como una parte de la solución. El programa de computación desarrollado es aplicable a flujo continuo y puede ser facilmente modificado para considerar los efectos de la eonización y de la disociación en los cuerpos de reentrada. Muestra de los resultados de flujo sobre una esfera localizada en una corriente supersónica es presentada y comparada con otros resultados.

## ABSTRACT

The paper deals with the flow field investigation over a blunt body solving the time dependent Euler's equations. The solution is marched in time till the steady state is reached. The shock is captured as a part of the solution. The computer code developed is applicable to a continuum flow and can be easily modified to consider the ionization and the dissociation effects in a reentry-body. Sample results over a sphere kept in a supersonic stream are presented and compared with other results.

## INTRODUCTION

There has been considerable interest in the solution of the subsonic region of a blunt-body problem since the last three decades or so when it was originally solved by Belotser kovski |1| using the method of integral relations. Subsequently, various attempts have been made |2 - 5| to investigate this problem over a sphere by a time-dependent approach. In the present paper, the formulation, using time-dependent approach |5|, for a general configuration is presented. The equations are written in the conservation form of Lax |3|.

The time derivative is replaced by a forward and the spatial derivatives by a central difference schemes. The solution is obtained by marching in time till the convergence is reached. Various studies have been made to accelerate the convergence by controlling the time step as the CFL stability criteria, in certain cases, did not yield a convergent solution.

The shock is captured as a part of the solution. Sample results over a sphere along with its computer schlieren are presented and compared. The comparisions show a good agreement with other theoretical/experimental data.

The computer code developed is applicable to continuum flow but can be easily applied to consider the ionization and the dissociation effects in a re-entry-body.

## DIFFERENTIAL EQUATIONS

The well-known basic equations of gasdynamics include those for conservation of mass, momentum and energy in addition to the equation of state.

Considering that the gas is inviscid and non-conduting these non linear partial differential equations can be written in a compacted form, in terms of four independent variables namely the time plus the spatial coordinates:

$$\sum_{\ell=1}^{4} \theta^{\ell}_{x_\ell} = \theta^5$$

where the subscript $x_1$ is time and $x_2$, $x_3$, $x_4$ are spatial derivatives, depending upon the coordinates sistem choosen. The $\theta^i$ are column matrices defined by

$$\theta^1 = h_1 h_2 h_3 \begin{vmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ e \end{vmatrix} \qquad \theta^2 = h_2 h_3 \begin{vmatrix} \rho \\ p+\rho u^2 \\ \rho u v \\ \rho u w \\ (e+p)u \end{vmatrix}$$

$$\theta^3 = h_1 h_3 \begin{vmatrix} \rho v \\ \rho u v \\ p+\rho v^2 \\ \rho v w \\ (e+p)v \end{vmatrix} \qquad \theta^4 = h_1 h_2 \begin{vmatrix} \rho w \\ \rho u w \\ \rho v w \\ p+\rho w^2 \\ (e+p)w \end{vmatrix}$$

and

$$\theta^5 = \begin{bmatrix} \rho u v h_3 \dfrac{\partial h_1}{\partial x_2} + \rho u w h_2 \dfrac{\partial h_1}{\partial x_3} - (p+\rho v^2)\ h_3 \dfrac{\partial h_2}{\partial x_1} - (p+\rho w^2)\ h_2 \dfrac{\partial h_3}{\partial x_1} \\ \rho v w h_1 \dfrac{\partial h_2}{\partial x_3} + \rho u v h_3 \dfrac{\partial h_2}{\partial x_1} - (p+\rho w^2)\ h_1 \dfrac{\partial h_3}{\partial x_2} - (p+\rho u^2)\ h_3 \dfrac{\partial h_1}{\partial x_2} \\ \rho u w h_2 \dfrac{\partial h_3}{\partial x_1} + \rho v w h_1 \dfrac{\partial h_3}{\partial x_2} - (p+\rho u^2)\ h_2 \dfrac{\partial h_1}{\partial x_3} - (p+\rho v^2)\ h_1 \dfrac{\partial h_2}{\partial x_3} \end{bmatrix}$$

where $h_i = \partial c_i/\partial x_i$ are metric coefficients, whence $c_i$ are the arc length evaluated in the $x_i (2 \leqslant i \leqslant 4)$ diretion, keeping the other two independent variables fixed.

In equation (1), $\rho$ represents the density, p is the pressure and u, v, and w are velocity components in the $x_i$ directions respectively and finaly e represent the total energy per unit volume related by

$$e = |h + (u^2+v^2+w^2)/2| - p/\rho \qquad (2)$$

whence h is the static enthalpy.

Depending upon the choice of $h_1$, $h_2$ and $h_3$ the different coordinate system, table 1, can be used

| System | coordinate | spatial increment | metric |
|---|---|---|---|
| | $x_m$* | $\Delta x_m$ | $h_m$ |
| Cartesian | x, y, z | $\Delta x$, $\Delta y$, $\Delta z$ | 1, 1, 1 |
| Cylindrical | r, $\phi$, z | $\Delta r$, $\Delta\phi$, $\Delta z$ | 1, r, 1 |
| Spherical | r, $\theta$, $\phi$ | $\Delta r$, $\Delta\theta$, $\Delta\phi$ | 1, r, r $\sin\theta$ |

* m = 1, 2, 3

## FINITE DIFFERENCE EQUATIONS

To choose a coordinate system, we use those metric of the table 1. We expand and rearrange the equation (1). How this Euler equations is written in a conservation form, we use the classical FTCS (Forward Time Central Space) approach obtaining the first order derivatives resulting in the general explicit finite difference scheme given by

$$\frac{1}{\Delta t}(\bar{\theta}^{\ell}_{i,j,k} - \theta^{\ell}_{i,j,k}) + 0\ (\Delta t) = \sum_{m=1}^{3} \{S_m \Omega^m + 0|(\Delta * m)^2|\} \qquad (2)$$

The value of $\theta^{\ell}_{i,j,k}$ (n -time level) is obtained of average put into effect upon the mesh point surrounding to a generic mesh point (i,j,,k). May be included in the expression that evolve $\theta^{\ell}_{i,j,k}$ terms that depend on the coordinate system, which finally assure that flow in the free stream region remain uniform during computation. The $\bar{\theta}^{\ell}_{i,j,k}$ means the calculation in (n + 1) time level.

The $S_m$ is central difference operator which when applied to generic function $\Omega^m$ (dependent of the variables p, $\rho$ and v) has the form $(1/2\Delta x_m)\ |\Omega^{m}{}_{+1} - \Omega^{m}{}_{-1}|_{i,j,k}$.

The spatial increment is represented by $\Delta x_m$ and the time level increment obtained of the CFL condition is given by

$$(\Delta t/h_m \Delta x_m) < |1/(|\dot{x}_m| + a)| \qquad (4)$$

whence a is the local sound velocity and $h_m$ the scale factor. As the CFL condiction is necessary but not sufficient to ensure the stability requiriment in the computation process, manipulations in $\Delta t$ (CFL) value was accomplished.

## METHOD OF SOLUTION

As the value of the flow quantities are advanced in time, the solution of the difference analogues of Eqs (3) is straight forward. At t = 0, the initial conditions are that of the free-stream. The boundary conditions in the entire computing region are specified as in Ref.|4|. The grid is entended one mesh width inside the solid body. None of the variables is computed there but assigned a suitable value to satisfy the exact flow tangency condition on the body.

The magnitude of the time-step, $\Delta t$, used for the marching is controlled by the CFL stability criteria |4|. But certain difficulties were encountered, with this $\Delta t$, to obtain a stable solution. However a stable solution could be possible for a reduced time-step value of $\Delta t/N$ where N = 600. Actually, different values of N are attempted for a stable solution and the largest of $\Delta t/N$ is retained since the spread of the shock transition is primarily proportional to $(\Delta r)^2/\Delta t$.

In the case of a sphere, the ray $\theta = 0$ is treated in a different manner |4| to avoid problems associated with the vanishing of $\sin\theta$ in the governing equations written in spherical coordinate system.

## RESULTS AND DISCUSSIONS

The present computer code developed at IAE is very general to consider different configurations. However a few sample results are presented only for a sphere to demonstrate the good comparision obtained with experimental data. In fig. 1, the pressure distribution and the location of the sonic point ever a sphere for different Mach nos 3,5 and 10 are presented. The comparision with the experimental data is very good. Fig.2 shows a sample of the development of a steady state solution from the initial value of the free stream at the $\theta = 0^0$ over a sphere kept in a free stream of Mach no. = 4.03 in Fig. 3, a computer schlieren for M = 4.03 over a sphere is presented. It can be seen that the shock is not very sharply cpatured in the present case. However, its spread can be reduced by increasing the mesh points near the earlier calculated shock regions.

The solution presented was carried out for an ideal gas. However, by adding the equations of chemical kinetics, it is

PRESSURE AT STAGNATION POINT ( M=4,0 , N=600 )

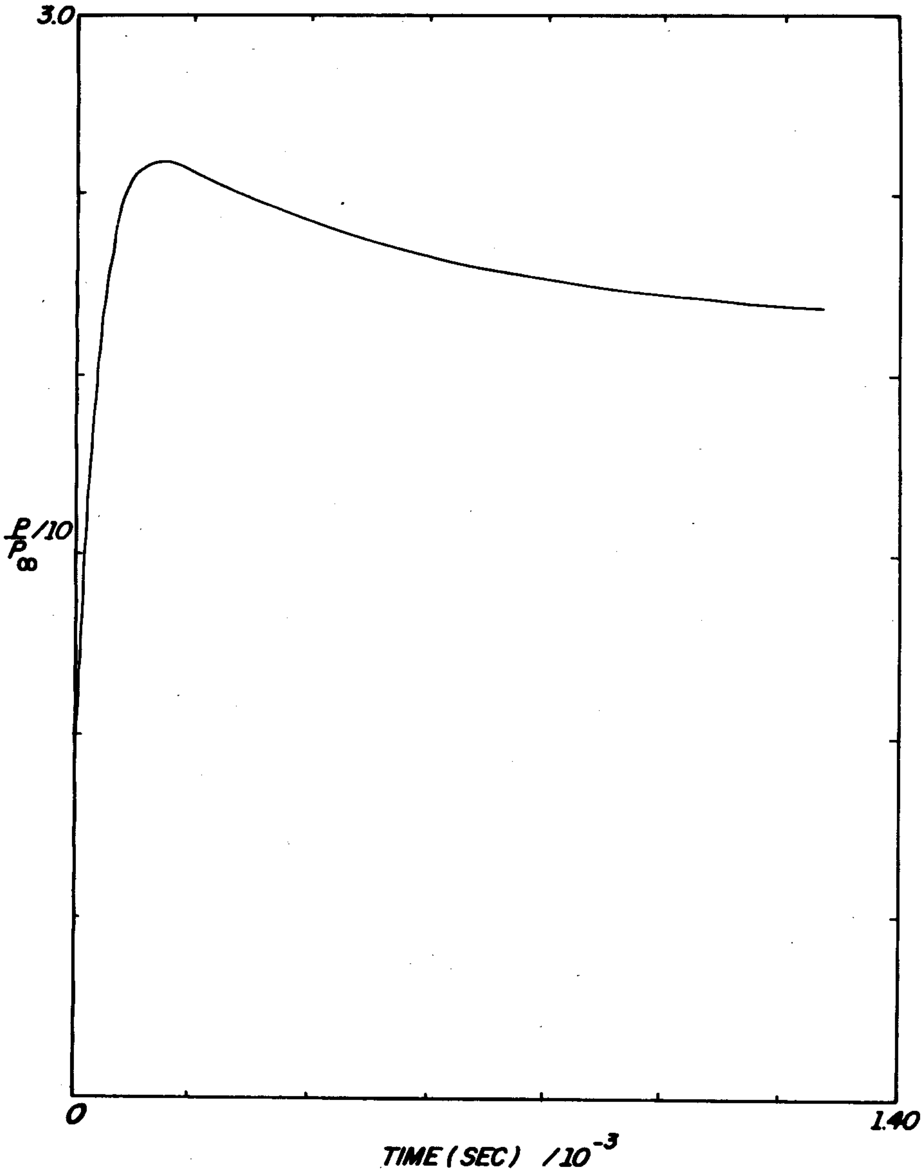


Fig:2 DEVELOPMENT OF STEADY - STATE CONDITION

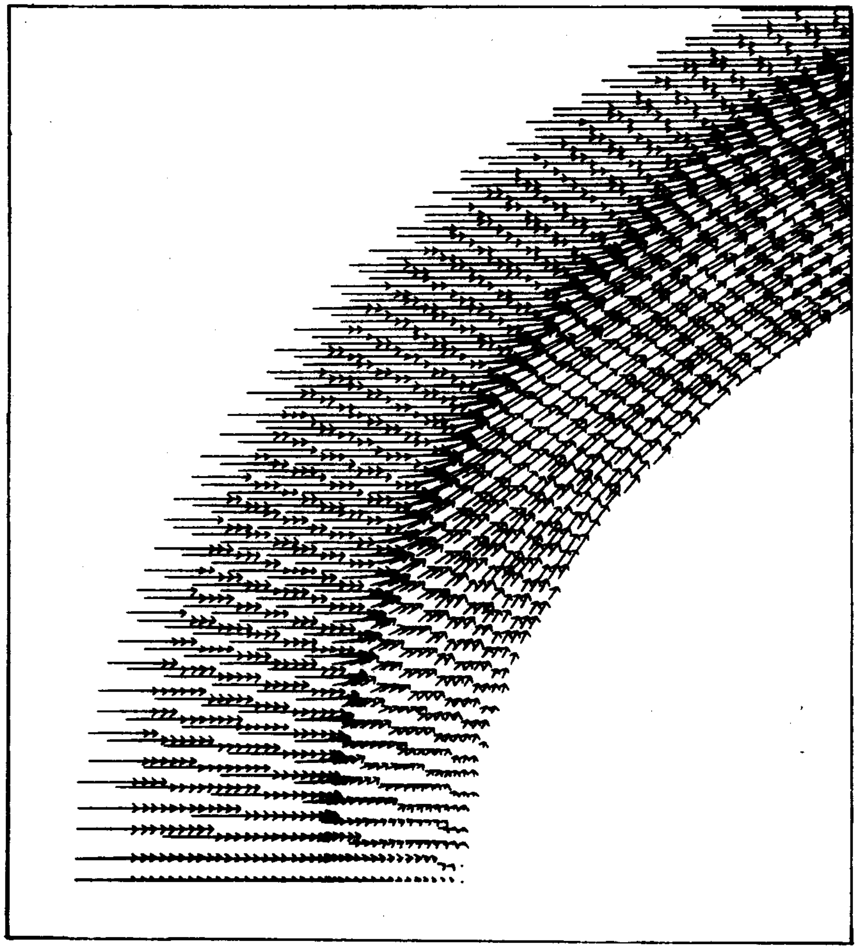

*Fig.3 COMPUTER SCHLIEREN OF THE SHOCK SHAPE FOR A SPHERE ($M_{\infty}$=4,03 ).*

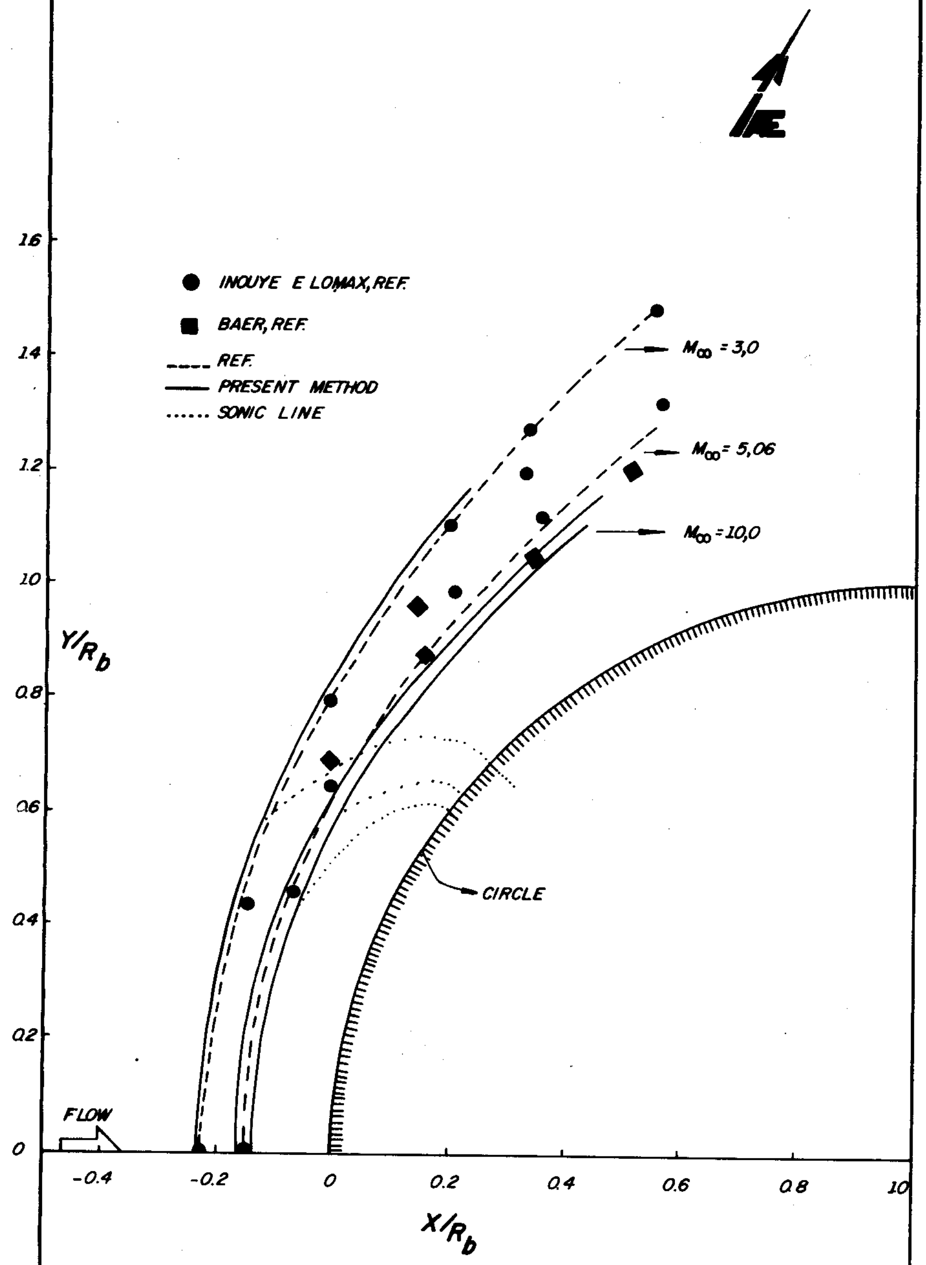


Fig: 1 SHOCK SHAPE IN A SPHERIC- BODY

possible to include the effects of dissociation and ionization on the flowfield as encountered in re-entry problem.

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