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FORCE ACTUATOR AND PIEZOACTUATOR PLACEMENT STUDY THROUGH SVD ANALYSIS ON THE SIMPLY SUPPORTED PLATE

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Abstract. *The piezoelectric actuator and sensor, have received lot of attention by researcher. The reason for this, it is because these devices present the piezoelectricity effect. This effect is the conversion between mechanical energy in electric energy and vice versa. So this effect is very useful in active vibration control, AVC, and its results are more effective than passive vibration control. The intelligent structures are the units' compound by: actuator, sensor, controller and structures (Lima Jr, 1999) and (Oliveira, 2003). Intelligent structure good design, the actuators and sensors placement are fundamental parts, because misplacement can cause lack of system controllability and observability. So this paper intends to propose force and momentum piezoactuators placement technique, through index obtained from singular value decomposition of control matrix [B]. Although the most common in AVC is piezoactuator, we have simulated force actuator for comparative analysis. This situation is possible, because we have used Kirchhof plate model and Melosh square element with four nodes and three degrees of freedom every node. The structure about the study is a simply supported plate and the results check with done simulation in finite elements.*

1 INTRODUCTION

Vibrations control new aim of the flexible structures has been received more attention by many researchers. According to new aim, the control is more effective if we use active elements. So integrating elements such as: sensor, actuator and controller, the mechanical vibrations could be minimized better than the use of passive elements. Nowadays these systems joining sensors, actuators, controllers and structures, are called as intelligent structures (Lima Jr, 1999).

Several technologies were proposal and investigated by researchers. Among these technologies are the piezoelectric elements. These elements, present the piezoelectricity effect that permit the conversion between electric and mechanical energy and vice versa. The piezoelectric direct effect was discovered by Curie brothers and piezoelectric inverse effect was deduced by Lippman (Rao & Sunar, 1994). Among these elements, there are the piezoelectric materials, especially the ceramics, PZT – piezoelectric lead zirconate titanate and polymer films, PVDF – piezoelectric vinylidene fluoride (Lima Jr. & Arruda, 1999). The ceramics have high stiffness, therefore they are used as actuators. While that the polymer films are handler than ceramics and can be produced in complex geometric shapes, for this reason, they are used as sensors. (Lima Jr, 1999). Piezoelectric materials are small, lightweight and resilient against adverse working environments. Moreover piezoelectric materials have been used as both actuators and sensors, (Wang, 2001).

One of the pioneers in using piezoelectric actuators as elements of intelligent structure was (Crawley & De Luis, 1987). He worked with an aluminum beam with piezoelectric actuator attached and he worked also, with graffiti/ epoxy beam and glass/epoxy beam. It was used, velocity proportional feedback controller in his work

The intelligent structure design is divided in three areas, such as: Modeling in finite element method (FEM); Actuators and sensors placement; System controller. In a good intelligent structure design, actuators and sensors placement study is a fundamental part to avoid undesirable effects in structure under active control, such as: Lack of observability and controllability system. This paper purpose is to suggest, optimum force and momentum piezoelectric actuators placement, in a flexible structure, using modal and spatial controllability measurements. To quantify the controllability index, we intend to use the singular value analysis of the [B] control matrix.

2 KIRCHHOFF PLATE MODELING

According to Kirchhoff hypothesis, showed in the figure 1, the field displacement *u*, *v*, and *w* can be express such as (Lima Jr, 1999):

$$\begin{cases} u = -z\dfrac{\partial w}{\partial x} \\ v = -z\dfrac{\partial w}{\partial y} \\ w = w(x.y) \end{cases} \tag{1}$$

Where: *x* and *y* are Cartesian system coordinator placed in the plate medium surface and *z* is direction along of plate thickness.

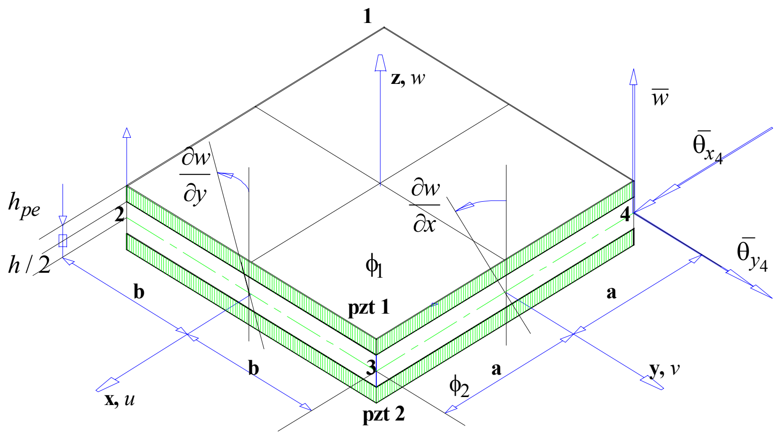

Figure 1 - Plate Elements (Lima Jr, 1999).

Due the fact of shear effect isn't taken in to consideration, the deformation field can be writing in function of displacement such as:

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} = -z\dfrac{\partial^2 w}{\partial x^2} \\ \varepsilon_y = \dfrac{\partial v}{\partial y} = -z\dfrac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = -2z\dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{2}$$

2.1 ACTUATOR EQUATION

The contribution of piezoelectric material can be divided in two classes, called inside, material, and outside, forces and momentum. The inside contribution is due structure material propriety, such as: Mass, stiffness and damping and is present although no electric potential is apply. While the outside contribution is due induced deformation when a potential electric is apply in PZT (Tzou & Fu, 1994 and Banks & Wang, 1995). The deformation amplitude induced in PZT is:

$$\varepsilon_{pe} = (\varepsilon_x)_{pe} = (\varepsilon_y)_{pe} = \frac{d_{31}}{h^a}\phi^a \tag{3}$$

Where: ε_{pe} is the induced deformation, d_{31} the piezoelectric constant (m/V), ϕ electric potential applies in the actuator (V). The individual stress, σ_x and σ_y (Gpa), in PZT is:

$$(\sigma_x)_{pe} = (\sigma_y)_{pe} = \frac{-E_{pe}d_{31}}{1-\mu_{pe}h_{pe}}\phi \tag{4}$$

Where: E and μ are piezoelectric Young module and Poisson coefficient. Integrating the voltage under element face, the results force and outside momentum, due PZT individual activation, can be writing like this:

$$(N_x)_{pe} = (N_y)_{pe} = \frac{-E_{pe}h_{pe}}{1-\mu_{pe}}\varepsilon_{pe} \tag{5}$$

$$(M_x)_{pe} = (M_y)_{pe} = -\frac{1}{8}\left(\frac{E_{pe}}{1-\mu_{pe}}\right)\left[4\left(\frac{h}{2}+h^a\right)^2 - h^2\right]\varepsilon_{pe} \tag{6}$$

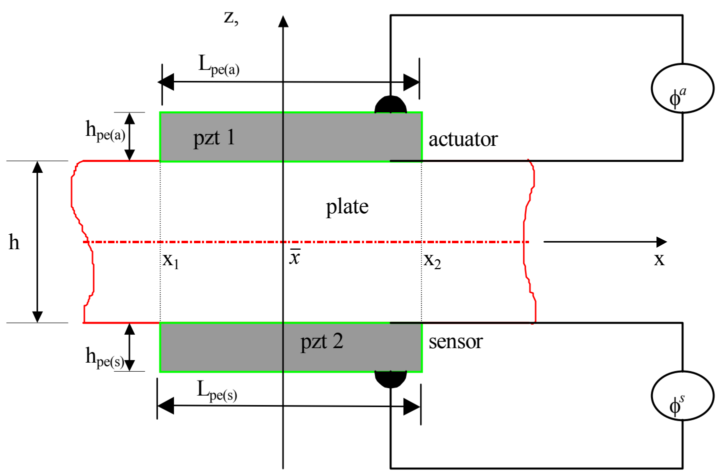

Figure 2 – Plate with piezoelectric elements attached (Lima Jr & Arruda, 1999).

3 FINITE ELEMENTS APROXIMATION

We consider four nodes in the rectangular plate element, according to plate classic theory (Bathe, 1996), in each node it has three degrees of freedom such as: $\overline{w}$ displacement in direction *z*, $\overline{\theta}_x$ rotation in relation to axe *x* and $\overline{\theta}_y$ rotation in relation to axe *y*. So the displacement function, *w*, is:

$$w(x_i, y_i) = d_1 + d_2 x_i + d_3 y_i + d_4 x_i^2 + d_5 x_i y_i + d_6 y_i^2 + d_7 x_i^3 + d_8 x_i^2 y_i + d_9 x_i y_i^2 + d_{10} y_i^3 + d_{11} x_i^3 y_i + d_{12} x_i y_i^3 \tag{7}$$

Where:

$$\begin{cases} i = 1,2,\ldots,4 \\ x_1 = -a;\ y_1 = -b;\ x_2 = a;\ y_2 = -b \\ x_3 = a;\ y_3 = b;\ x_4 = -a;\ y_4 = b \end{cases} \tag{8}$$

In the matrix form, the equation (8) is:

$$w = \{P\}^T \{d\} \tag{9}$$

The vector $\{q_i\}$ is defined such node displacement field, in the rectangular element, such as:

$$\{q_i\} = \left\{ \overline{w}_1 \quad \overline{\theta}_{x_1} \quad \overline{\theta}_{y_1} \quad \ldots \quad \overline{w}_4 \quad \overline{\theta}_{x_4} \quad \overline{\theta}_{y_4} \right\}^T \tag{10}$$

4 PIEZOELETRIC VARIATIONAL EQUATION

The behavior of piezoelectric material, there are mechanics and electric effects which can be written in the matrix form, (Lima Jr e Arruda, 1997) and (Lima Jr., 1999) such as:

$$\begin{aligned} &\iiint_V \rho \{\delta u\}^T \{\ddot{u}\} dV + \iiint_V \{\delta\varepsilon\}^T \{\sigma\} dV - \iiint_V \{\delta E\}^T \{D\} dV = \\ &\iiint_V \{\delta u\}^T \{\bar{f}_V\} dV + \iint_{S_F} \{\delta u\}^T \{\bar{f}_S\} ds - \iint_{S_q} \delta\phi \sigma_q dS \end{aligned} \tag{11}$$

The linear piezoelectricity constructive equation is:

$$\begin{aligned} \{\sigma\} &= \left[c^E\right]\{\varepsilon\} - [e]\{E\} \\ \{D\} &= [e]^T \{\varepsilon\} - \left[\xi^\varepsilon\right]\{E\} \end{aligned} \tag{12}$$

Where:

$$\begin{aligned} [e] &= \left[c^E\right][d] \\ \left[\xi^\varepsilon\right] &= \left[\xi^\sigma\right] - [d]^T \left[c^E\right][d] \end{aligned} \tag{13}$$

Where: $\{\sigma\}$- stress tensor; $\{\varepsilon\}$- deformation tensor; $\{E\}$- electric field vector; $\{D\}$- electric displacement vector; $[C^E]$- elasticity matrix for constant electric field; $[e]$- piezoelectric constants matrix; $[\xi^\varepsilon]$- dielectric constants tensor for constant deformation $[\xi^\sigma]$- dielectric constant matrix for constant stress; $[d]$- constant matrix of piezoelectric deformations.

The variational principle equation for piezoelectric material (Lima Jr, 1999), is obtained putting equation (8) in (7), so it is given by:

$$\iiint_V \rho\{\delta u\}^T\{\ddot{u}\}dV + \iiint_V \{\delta\varepsilon\}^T\left[c^E\right]\{\varepsilon\}dV - \iiint_V \{\delta\varepsilon\}^T[e]^T\{E\}dV - \iiint_V \{\delta E\}^T[e]\{\varepsilon\}dV$$
$$-\iiint_V \{\delta E\}^T[\xi]\{E\}dV = \iiint_V \{\delta u\}^T\{\bar{f}_V\}dV + \iint_{S_f}\{\delta u\}^T\{\bar{f}_S\}dS - \iint_{Sq}\delta\phi\sigma_q dS \tag{14}$$

From of Hamilton principle and eletromechanic variational principle to piezoelectric materials and applying it in the rectangular plate, we obtain the mass matrix of structure without or with the piezoelectric element attached, given by:

$$\left[m_{st}\right] = \rho_{st}h\iint_{A_{st}}\left[N_w\right]^T\left[h_{st}\right]\left[N_w\right]dA_{st} \tag{15}$$

$$\left[m_{pe}\right] = \rho_{st}h_c\iint_{A_{pe}}\left[N_w\right]^T\left[h_{pe}\right]\left[N_w\right]dA_{pe} \tag{16}$$

Where: $[m_{st}]$ is the structure mass matrix and $[m_{pe}]$ is the piezoelectric mass matrix.
The structural and piezoelectric stiffness matrixes are:

$$\left[k_{qq}\right] = \iint_{A_{pe}}\left[B_K\right]^T\left[c_k^{pe}\right]\left[B_k\right]dA_{pe} \tag{17}$$

$$\left[k_{q\phi}\right] = h_b^2\iint_{A_{pe}}\left[B_k\right]^T[e]\left[B_\phi\right]dA_{pe} \tag{18}$$

$$\left[k_{\phi q}\right] = h_b^2\iint_{A_{pe}}\left[B_\phi\right]^T[e]^T\left[B_k\right]dA_{pe} \tag{19}$$

$$\left[k_{\phi\phi}\right] = -h_c\iint\left[B_\phi\right]^T\left[\xi^\varepsilon\right]\left[B_\phi\right]dA_{pe} \tag{20}$$

Finally the force and electric loads outside vectors are:

$$\{f_s\} = \iint_{A_{pe}}\left[N_w\right]^T\{\bar{f}_s\}dA \tag{21}$$

The elements of each matrix are assembled in order to obtain a global matrixes system that is given by:

$$\begin{cases}\left[M_{qq}\right]\{\ddot{q}_i\} + \left[k_{qq}\right]\{q_i\} + \left[k_{q\phi}\right]\{\phi_i\} = \{F_S\} + \{F_{el}\} \\ \left[k_{\phi q}\right]\{q_i\} + \left[k_{\phi\phi}\right]\{\phi_i\} = \{Q_s\}\end{cases} \tag{22}$$

In the piezoelectric sensor there isn't voltage apply ($Q_s = 0$). So the electric potential yield by sensor is:

$$\{\phi_s\} = -\left[K_{\phi\phi}\right]^{-1}\left[K_{\phi q}\right]\{q_i\} \tag{23}$$

Replace the Eq. (23) in the Eq. (22), we get the equation global system for a beam with actuator attached, that is:

$$\left[M_{qq}\right]\{\ddot{q}_i\} + \left[K^*_{qq}\right]\{q_i\} = \{F_S\} + \{F_{el}\} \tag{24}$$

Where:

$$\left[K^*_{qq}\right] = \left[K_{qq}\right] - \left[K_{q\phi}\right]\left[K_{\phi\phi}\right]^{-1}\left[K_{\phi q}\right] \tag{25}$$

$$\{F_{el}\} = -\left[K_{q\phi}\right]\{\phi_a\} \tag{26}$$

Where F_{el} is due the momentum applies by piezoactuator and F_s is due force applies by force actuator in the flexible structures.

5 CONTROLLABILITY INDEX

The system controllability comes of from the modern control theory. It is used to determine if a system can be controlled there being a controller. The decomposition of singular matrix [S] yields a measure quantity of system controllability. This index shows the energy that is need in the actuator to control a given input. The Eq. (22) can be writing in the state space:

$$\begin{aligned}\{\dot{x}\} &= [A]\{x\} + [B]\{u\}\\ \{y\} &= [C]\{x\}\end{aligned} \tag{27}$$

Where:

$$[B] = \begin{bmatrix}[0]\\ \left[M_{qq}\right]^{-1}\{F_{el}\}\end{bmatrix}\{y\} = \{\phi_S\}[C] = \left[\left([0]_i - \left[K_{\phi\phi}\right]_S^{-1}\left[K_{\phi q}\right]_S\right)\quad [0]\right] \tag{28}$$

The rank of state space matrixes, depend of modes numbers that are considered and the actuators number in the structure. From Eq. (27), the control force applied can be written, such as:

$$\{f_c\} = \{F_{el} + F_s\} = [B]\{u\} \tag{29}$$

Where {u} is the electric potential vector, we have that:

$$\{\mathrm{f_c}\}^{\mathrm{T}}\{\mathrm{f_c}\} = \{\mathrm{u}\}^{\mathrm{T}}[\mathrm{B}]^{\mathrm{T}}[\mathrm{B}]\{\mathrm{u}\} \tag{30}$$

Writing:

$$[B] = [M][S][N]^T \tag{31}$$

Using singular analysis value, where:

$$[M],[N] \in R^n \ e \ [M]^T[M] = [I] \ e \ [N]^T \ [N] = [I] \tag{32}$$

Where:

$$[\mathrm{S}] = \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_k \end{bmatrix} \tag{33}$$

The biggest value σ_i, show the optimum place for actuator.

6 NUMERIC SIMULATION

We simulated a simply supported plate according to following table 1.

Table 1: Properties of simulated simply supported plate

Parameters	Value	Unit
Length	1.5	*m*
Width	1.0	*m*
Thickness	0.075	*m*
Elasticity Module (*E*)	210	*GPa*
Specific density (ρ)	7800	*kg/m³*

After simulations, the first set of vibration modes are presenting by figures 3 and 4, above:

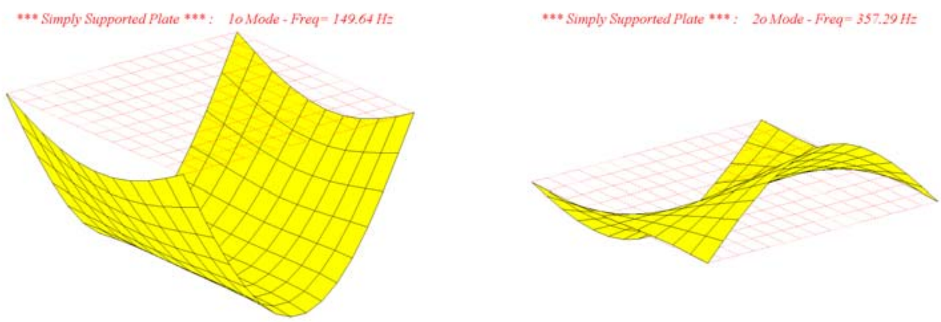

Figure 3 – Vibration shapes (a) 1º Mode (b) 2º Mode.

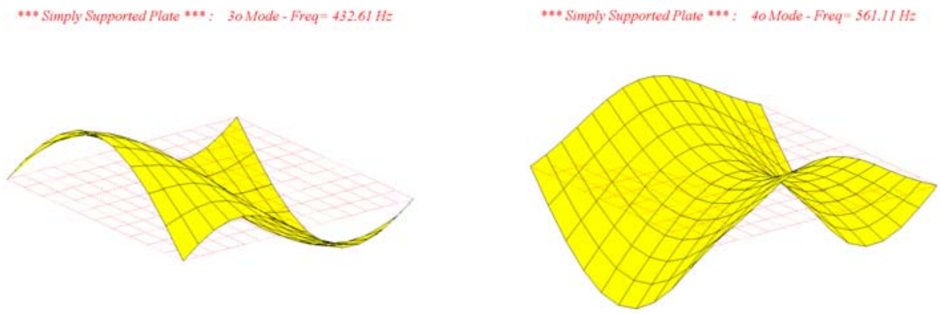

Figure 4 – Vibration shapes (a) 3º Mode (b) 4º Mode.

We simulated in both situation, force actuator and piezoactuator for 1º mode, for analysis comparisons. The results are showed as following:

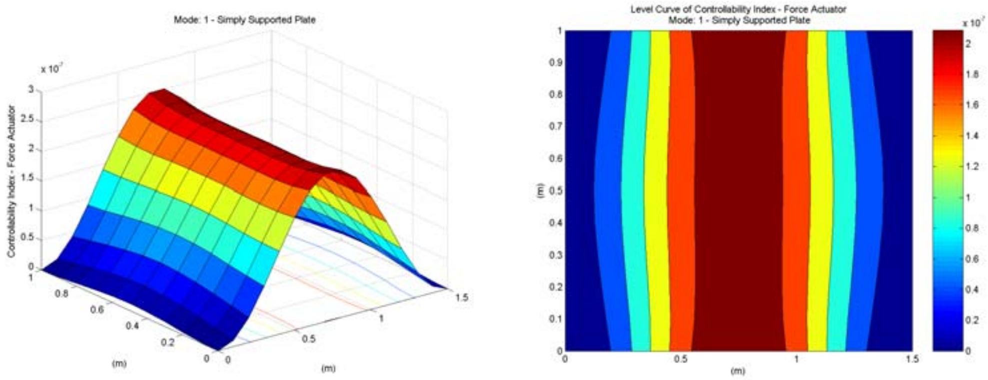

Figure 5 – Force actuator placement index for 1º mode (a) Surface (b) Level contour.

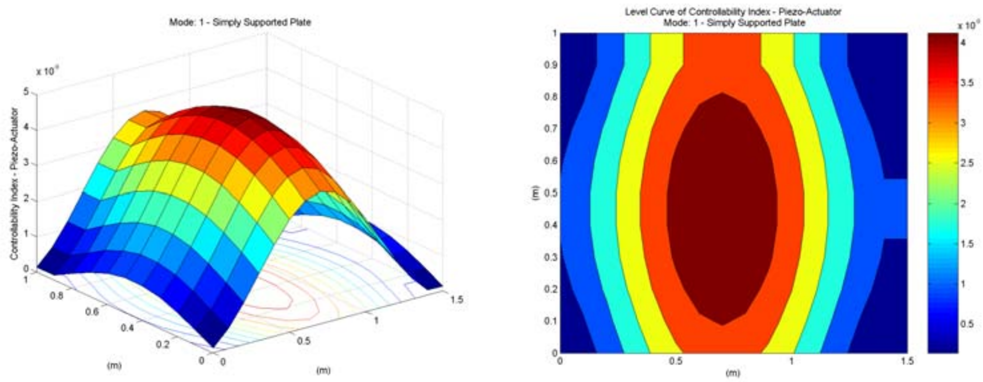

Figure 6 – Piezoactuator placement index for 1º mode (a) Surface (b) Level contour

According to figure 5, the best places to put the force actuator are the line with coordinates: (0.75,0) *m* and (0.75,1) *m* from of the origin. The best place to put the piezoactuator, figure 6, is in the center of plate with coordinate (0.75,0.5) *m* from of the origin.

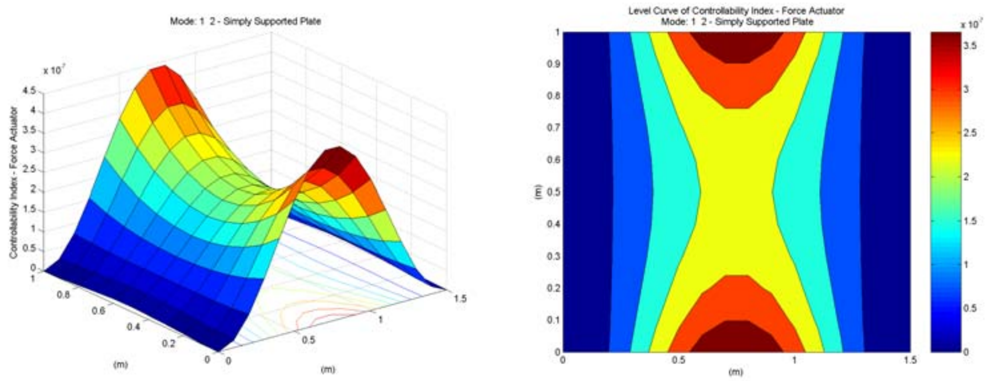

Figure 7 – Force actuator placement index for 1 and 2 modes (a) surface (b) Contour level.

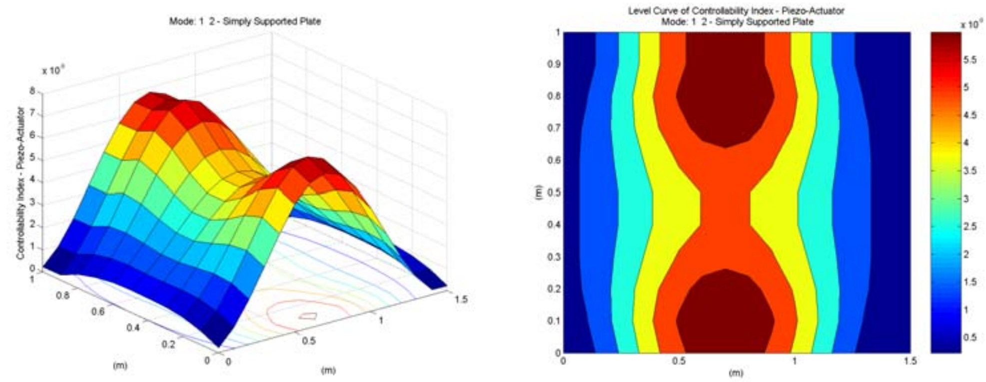

Figure 8 – Piezoactuator placement index for 1 and 2 modes (a) Surface (b) Contour level.

According to figure 7, the best places to put the force actuator are the positions in the middle of the plate and near of the edges where there are the lines with coordinates: (0.75,0) *m* and (0.75,1) *m* from of the origin. While that the best positions to put the piezoactuator, figure 8, are the coordinates: (0.75,0.2) *m* and (0.75,0.8) *m* from of the origin likes as force actuator position.

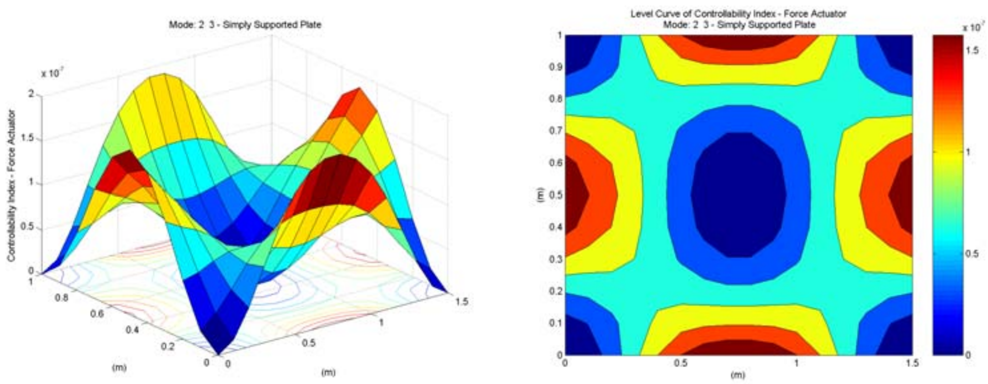

Figure 9 – Force actuator placement index for 2 and 3 modes (a) Surface (b) Contour level.

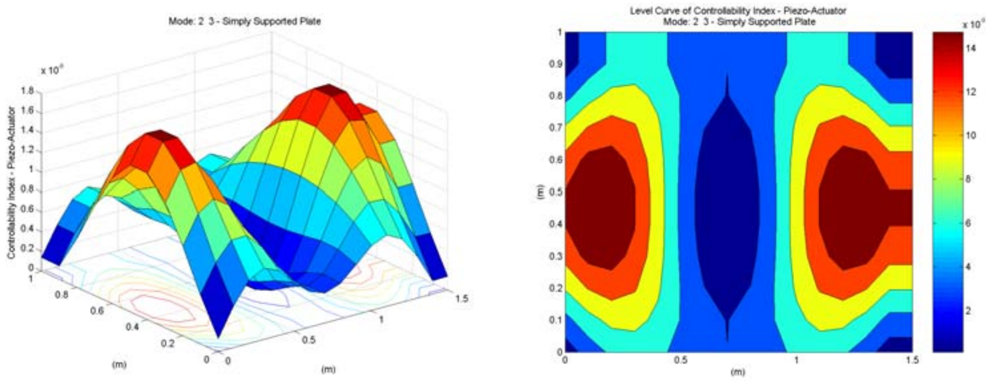

Figure 10 – Piezoactuator placement index for 2 and 3 modes (a) Surface (b) Contour level.

According to figure 9, the best places to put the force actuator are the coordinates: (0.75,0) *m*, (0.75,1) *m*, (0, 0.5) *m* and (1.5,0.5) *m* from of the origin. The best places to put the piezoactuator, figure 10, are the coordinates: (0.2,0.5) *m* and (1.3,0.5) *m* from of the origin.
The optimum place to put force actuator and piezoactuator are the peaks of controllability index graphics. In these points the singular values are maximum, as showed from figure 5 to figure 10.

7 CONCLUSIONS

We showed an index to quantify controllability system of the simply supported plate with piezoelectric attached, in this paper. With this index, it is possible to determine the optimum place to actuators, this way we minimizing the controller effort. We showed that the singular value decomposition, of the control matrix, could be used like measurement to quantify the energy supplied to actuators. The performance of this index was good and the contribution of this paper is extending of work of Wang 2001 from one dimension to two dimensions structure, that is, a simply supported plate. We can see the difference between force actuator and piezoactuator. The optimum places are different, because the singular value decomposition is not the same. The reason for this is the control matrix [B] different for force actuator and piezoactuator. In each node, the force is the first degree of freedom, the second and third degree of freedom are momentum. So in case the force actuator there is only the first position in each node. While that the piezoactuator, the second and third position.

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