VISCOELASTIC FRACTURE ANALYSIS USING A CONTINUUM DAMAGE MODEL

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ABSTRACT

Viscoelastic materials have their mechanical properties affected by microcracking. A continuum damage model is proposed to represent the behavior of the material and applied for the fracture analysis. It is considered that the crack grows when the damage in a point reaches a critical value.

RESUMO

Materiais viscoelásticos têm suas propriedades mecânicas degradadas devido à microfissuração. Assim sendo, propõe-se um modelo de dano contínuo para representar o comportamento do material e aplica-se este modelo para análise de fraturas. Considera-se que haja propagação da trinca quando o dano em um ponto atinge um valor crítico.

INTRODUCTION

In this paper we study the crack propagation problem for a material whose global behavior is linearly viscoelastic; the subject has great interest in the integrity analysis of structures made of materials as polymers and concrete.

The literature available on the subject is scant and sometimes contradictory, but from the classic theoretical and experimental studies of Knauss [5] and Schapery [8] we can learn:

1. A cracked solid of linear viscoelastic material has a critical stress σ_{re} under instantaneous loading that is determined using its instantaneous elasticity modulus and the equations of LEFM (linear elastic fracture mechanics). For a sustained stress $\sigma<\sigma_{re}$ we will have a delayed fracture; the time delay depends on the creep compliance of the material. Usually a stress σ_{rd} exists , such that for $\sigma<\sigma_{rd}$ no fracture occurs in a finite time.

2. In order to model theoretically this behavior it is necessary to postulate the existence of a process zone at the tip of the crack of initial length a. The material behavior in this zone is assumed as strongly non linear but other ways unspecified; the rupture condition is given indistinctly by stress, strain or energy limit criteria. On this setting, using several simplifying hypotheses and a special viscoelastic law, Schapery [8] obtained a relation between the propagation speed $\dot{a}$ and the acting stress intensity factor K .

It would be useful to have an equivalent model amenable to numerical (i.e. finite elements) analysis. Then we could model arbitrary geometrical, loading and support conditions and real viscoelastic relations. No such procedure has yet been developed.

Experimental observations tell of intense microcracking at crack tip, and thus a strongly nonlinear viscoelastic behavior, that can be represented using Continuous Damage Mechanics [11], [12]. For this zone, it is proposed a formulation developed by Saraiva e Creus [7] that couples viscoelasticity with continuum damage mechanics.

This model was incorporated into a finite element program for viscoelastic analysis and fracture [6] and several examples were analyzed. Damage accumulation at the crack tip is observed and the corresponding stress redistribution. When damage reaches the critical value at a given integration point, the crack propagates, say, to this integration point. At each time interval, a J integral determination could be performed. When the critical value J is attained, an elastic propagation condition should be expected.

THERMODYNAMIC FORMULATION

The Helmholtz energy for viscoelastic materials is [11]:

$$\Psi = \frac{1}{2}\,\varepsilon : C^0 : \varepsilon - q : \varepsilon + \Theta(\varepsilon^v) \tag{1}$$

where ε is the total strain, C^0 the elastic tensor, q the viscoelastic pseudo-stress e Θ a viscoelastic potential, function of viscoelastic strain ε^v. q is given by:

$$q = C^0 : \varepsilon^v \tag{2}$$

The Helmholtz complementary energy is:

$$\Phi = \frac{1}{2}\,\sigma : (C^0)^{-1} : \sigma + \varepsilon^v : \sigma + \Omega(\varepsilon^v) \tag{3}$$

where σ is the stress tensor and Ω a viscoelastic complementary potential.

Stress and strain are forces thermodynamically associated:

$$\sigma = \frac{\partial\Psi}{\partial\varepsilon} = C^0 : \varepsilon - q = C^0 : (\varepsilon - \varepsilon^v) \tag{4}$$

$$\varepsilon = \frac{\partial\Phi}{\partial\sigma} = (C^0)^{-1} : \sigma + \varepsilon^v = \varepsilon^e + \varepsilon^v \tag{5}$$

CONTINUUM DAMAGE MODEL FOR VISCOELASTIC SOLIDS

The damage is assumed to be isotropic and to modify only elastic material properties. Helmholtz and Helmholtz complementary energies are given by:

$$\Psi = \frac{(1-d)}{2}\, \varepsilon : C^0 : \varepsilon - (1-d)\, q : \varepsilon + \frac{(1-d)}{2}\, \varepsilon^v : C^0 : \varepsilon^v \qquad (6)$$

$$\Phi = \frac{1}{2\,(1-d)}\, \sigma : (C^0)^{-1} : \sigma + \varepsilon^v : \sigma \qquad (7)$$

where d is the damage variable. From both expressions we obtain the viscoelastic stress-strain relationship:

$$\sigma = \frac{\partial\Psi}{\partial\varepsilon} = (1-d)\, C^0 : (\varepsilon - \varepsilon^v) \qquad (8)$$

$$\varepsilon = \frac{\partial\Phi}{\partial\sigma} = \frac{1}{(1-d)}\, (C^0)^{-1} : \sigma + \varepsilon^v \qquad (9)$$

The relation obtained using Helmholtz energy resulted the same as the one obtained using Helmholtz complementary energy, showing the consistency of the formulation.

Introducing equations (8) and (9) into general viscoelastic relation [1], we obtain:

$$\varepsilon(\tau) = \frac{1}{(1-d)}\, \sigma(t) : (C^0)^{-1} + \int_{\tau_0}^{t} \frac{\partial D(t,\tau)}{\partial\tau}\, \sigma(\tau)\, d\tau \qquad (10)$$

$$\sigma(\tau) = (1-d)\, C^0 : \varepsilon(t) + (1-d) \int_{\tau_0}^{t} \frac{\partial E(t,\tau)}{\partial\tau}\, \varepsilon(\tau)\, d\tau \qquad (11)$$

where $D(t,\tau)$ and $E(t,\tau)$ are the creep and relaxation functions, respectively.

For the finite elements formulation we use, as usual, the virtual work principle and the interpolation relations:

$$\underset{\sim}{u} = \underset{\sim}{N}\, \underset{\sim}{u}^i \qquad (12)$$

where $\underset{\sim}{u}$ is the displacement vector inside the element, u^i is the nodal displacement vector and $\underset{\sim}{N}$ is the shape functions matrix. From the strain-displacement relation we also have:

$$\underset{\sim}{\varepsilon} = \underset{\sim}{B}\, \underset{\sim}{u}^i \qquad (13)$$

where matrix notation is used throughout. The balance equation is then written:

$$\int_V (1-d)\, \underset{\sim}{B}^T\, \underset{\sim}{C}^0\, \underset{\sim}{B}\, dV\, u^i = \underset{\sim}{P}_{ext} + \int_V (1-d)\, \underset{\sim}{B}^T\, \underset{\sim}{C}^0\, \underset{\sim}{\varepsilon}^v\, dV \qquad (14)$$

where P is the vector of external loads. The numerical solution of the integral equation above is simplified using a state variable approach [1]. Although the inclusion of damage introduces nonlinearity in the viscoelastic relationship, the usual incremental solution is used as a first approach. An iterative procedure could be used to control numerical error.

DAMAGE EVOLUTION LAW

The damage evolution law is defined as [10]:

$$\dot{d} = G(\varepsilon_{eq}, d)\ \dot{\varepsilon}_{eq} \tag{15}$$

where G is the damage potential and ε_{eq} the equivalent strain, that we define as:

$$\varepsilon_{eq} = (\langle\varepsilon\rangle : C^{0} : \langle\varepsilon\rangle)^{1/2} \tag{16}$$

where < > are the McAuley brackets ($\langle x\rangle = (x + |x|)/2$) is used, so that the damage is associated to tension strains.

The damage criterion is given by:

$$g = \varepsilon_{eq} - r_t \le 0 \tag{17}$$

where g is the damage function and r_t is the damage parameter, given by:

$$r_t = \max\left\{ r_0, \max_{s\in(-\infty,t)} \varepsilon_{eq}(s) \right\} \tag{18}$$

The damage potential used in this work is that proposed by Kachanov [3] to creep damage:

$$G(\varepsilon_{eq}, d) = A\left[\frac{\varepsilon_{eq}}{(1-d)}\right]^{\alpha} \tag{19}$$

where A e α are material parameters.

FRACTURE ANALYSIS FOR VISCOELASTIC MATERIALS

For several stress-strain relationships, J-integral is used as a critical parameter and evaluated as:

$$J = \int_{\Gamma} \left(\phi\, n_1 - T : \frac{\partial u}{\partial x_1} \right) ds \tag{20}$$

where Γ is an arbitrary contour whose beginning and end lay over the crack faces and follows a counterclockwise sense, n_1 is the component in the x_1

direction of a unit vector normal to Γ, and u is the displacement vector.

The position and orientation of x_1 and x_2 axes are shown in Figure 1.

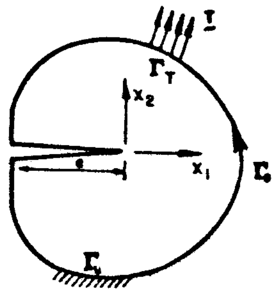


Figure 1: J integral

There are direct relations between J and the stress intensity factor K in the linear elastic case or the crack opening displacement (COD) in the general case. J approach is used for approximate analysis of viscoelastic materials [9].

The J integral is meaningful only for linear and non-linear elastic materials. For other stress-strain laws, its use is restricted to monotonically increasing loads.

Crack growth is predicted when J reaches a critical value J_c, characteristic of the material. A crack propagation study can be performed displacing the contour Γ whenever a rupture at the crack tip occurs, increasing the crack length. For a finite element analysis of this kind of problem, it is necessary an algorithm for mesh redefinition in order to follow the crack growth. A work on this subject was presented for the linear case by Ingraffea et al.[2].

In the present paper, we develop a simpler method that uses only Continuum Damage Mechanics. As the cracked body is modeled by a finite element mesh, localized failure occurs at the points under high stress and strain levels, mainly at the crack tip, due to the damage process.

The damage variable d is evaluated at the Gauss points of finite elements and it is considered that rupture occurs when d reaches the critical value d_{cr}, characteristic of the material. In these points, a residual stiffness is preserved in order to avoid numerical singularities in the stiffness matrix.

NUMERICAL ANALYSIS: EXAMPLE

We consider a center-cracked plate under traction, whose geometry is shown in Figure 2. All the parameters used are hypothetical, so no unit system is indicated.

The viscoelastic properties of the material are characterized by the functions:

$$K(t) = 50000 + 50000\ e^{-t/4}$$

$$G(t) = 50000 + 50000\ e^{-t/4}$$

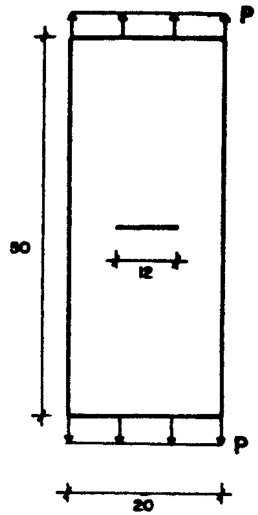


Figure 2: Center-cracked plate

The damage parameters are:

$$A = 0.1$$

$$\alpha = 0.01$$

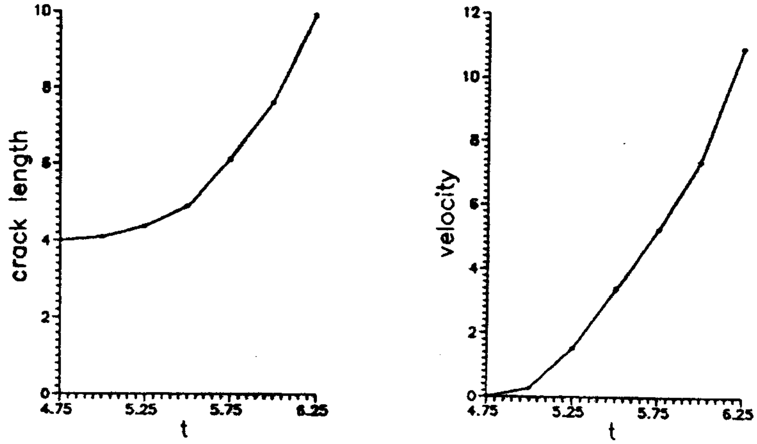
$$r_0 = 0$$

$$d_{cr} = 0.95$$

where d_{cr} is the critical value for damage. When an integration point reaches d_{cr} we assume a residual stiffness equal to 5% (1 - d = 0.05).

We consider an applied load of 400 units and taking advantage of symmetry we modeled a quarter plate using 8-nodes isoparametric elements for plane stress. The mesh has 119 elements and 406 nodes. Along the fracture line there exist 12 square elements of 0.5 units side length. The time interval used was 0.25 and the Gauss integration order 2x2.

Crack propagation began at t=5; Figure 3(a) shows a graphic of the crack length variation a with time; failure with complete division of the body occurs at t=6.25. Crack propagation velocity $\dot{a}$ was determined by interpolating a fifth order polynomial on the data of Figure 3(a) and differentiating this polynomial. The result is given in Figure 3(b), showing increasing velocity, as expected for this type of specimen.



(a) (b)

Figure 3: (a) a x t (b) $\dot{a}$ x t

CONCLUSIONS

The results obtained seem to validate the proposed method for the description of crack growth and propagation in viscoelastic media.

Computationally, the method is relatively simple, since no mesh redefinition or any special procedure to find crack propagation direction are required. Besides, this method allows the analysis of cyclical loads, instead of J integral, which is meaningful just for monotonically increasing loads.

Researches in the sense of evaluating the influence of the mesh and the time step over the convergence process are necessary. Preliminary tests show that problems may occur when coarse meshes or too great time intervals are employed.

Probably, the inclusion of an incremental-iterative solution algorithm will improve the process convergence and decrease the influence of the time interval.

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