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A NONLINEAR PANEL METHOD IN THE TIME DOMAIN FOR SEAKEEPING FLOW PROBLEMS

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RESUMEN

Un método de paneles no-lineal en el dominio del tiempo para problemas de "tenida a la mar" es brevemente descripto. Luego de una semi-discretización espacial, la resolución numérica de un sistema de Ecuaciones Diferenciales Ordinarias (EDO), en el dominio del tiempo, permite obtener tanto el potencial de velocidades como el desplazamiento normal en la superficie libre. Las condiciones de borde sobre la superficie libre, cinemática y dinámica, implican una restricción no-lineal en el sistema de EDO. Como una primera validación del método propuesto, se incluyen un par de ejemplos numéricos simples.

ABSTRACT

A non-linear panel method in the time-domain for seakeeping problems is outlined. After a spatial semi-discretization, the velocity potential and the normal displacement on the free surface, are obtained by means of numerical solution of an ODE's system in the time domain. The boundary conditions on the free surface (both kinematic and dynamic) are non-lineal restrictions over the ODE's system. As a first validation of the proposed method, a pair of simple numerical examples are shown.

INTRODUCTION

One of the subject of interest to naval hydrodynamics is the seakeeping problem, i.e. the stability properties of ship-like bodies under the influence of an incoming wave. Acceptance of a simulation method is related to its price to performance ratio, i.e. the computational effort for a simulation run in comparison with the physical relevance of its results. As in general, the price of a simulation method increases with the level of its sophistication. The panel method (or boundary element method) is a member of the CFD family which can be used in preliminary design stages [1]. As it is well known, this method assumes a potential flow model and it is a practical tool for predicting the pressure field over rather complicated hull geometries [2], or when a parametric study involves an extensive set of numerical test cases, for example, the transfer functions plots, or the Response Amplitude Operators (RAO). Then, it is also possible to shift to another fluid dynamics description level, e.g. Euler and Navier-Stokes solvers, or use mixed strategies, as the viscous-inviscid interaction techniques. The panel method strategies are based in the discretization and solution of boundary integral equations and they are closely related with the Green function theory. For ship hydrodynamics, the Rankine and Kelvin kernels are widely used [3]. The Kelvin source-kernel has mathematical properties rather elaborated and, in a classical sense, it is only useful for linearized boundary conditions on the free surface, and the radiation boundary conditions at infinity are satisfied, so only the wetted hull surface must be discretized with the corresponding reduction in computer time. Nevertheless, it requires the computation of specialized functions via numerical integration in the time-domain [4-6], as well as in the frequency-domain [7]. Furthermore, in the frequency-domain, it suffers from spurious frequencies, i.e. a certain number of frequencies for which the linear system is singular. These frequencies are often outside the range of interest, but the accuracy of the method is globally affected. Another possibility is to combine finite elements with DNL absorbing boundary conditions [8-10]. This alternative does not suffer spurious oscillations but is more expensive in CPU time. On the other hand, the Rankine source-kernel has rather simple properties, but it does not satisfy the boundary conditions at the free surface. Then, the free surface must be also discretized in order to impose the two first boundary conditions by a collocation technique, while the radiation boundary conditions can be imposed in another indirect way. Aside of these drawbacks, a great advantage of the Rankine source-kernel emerge in hydrodynamic flow problems with non-linear conditions [11]. In this work, a report of a non-linear panel method in the time-domain is done. The basic strategy is to solve an ODE's system for the velocity potential and displacement on the free surface. Both kinematic and dynamic boundary conditions on the free surface are taken into in account, which are a non-lineal restriction over the ODE's system. Stability and consistency for the numerical resolution are addressed and some strategies are outlined for its resolution.

THE STEKHLOV OPERATOR AND THE PANEL METHOD

Let us to consider a rigid ship-like body performing small oscillations around its hydrostatic equilibrium position, on the free surface of an inviscid, incompressible and irrotational fluid. Such flow problem is related with the seakeeping one considered in ship hydrodynamics. An hydrodynamic standard analysis in space $\mathbf{x} = (x, y, z)$ and time t domains gives: the Laplace equation for the velocity potential $\phi(\mathbf{x}, t)$ in the instantaneous flow domain $\Omega(t)$, and the kinematic and dynamic boundary conditions on the instantaneous free surface $\Gamma_S(t)$, for the wave-height $\eta(\mathbf{x}, t)$ and the flux $\sigma(\mathbf{x}, t)$, that is,

$$\begin{cases} \nabla^2\phi = 0 & \text{in } \Omega(t); \\ -\eta_{,t} + n_z^{-1}\sigma = 0 & \text{at } \Gamma_F(t); \\ \phi_{,t} + \frac{1}{2}|\nabla\phi|^2 + g\eta = 0 & \text{at } \Gamma_F(t); \end{cases} \tag{1}$$

where $\mathbf{n}$ is the unit normal of the free surface, n_z is its projection on the z-axis, positive upwards, and g is the gravity acceleration. For a numerical resolution in the time-domain t, we choose a low order panel method. First, a semi-discretization in the spatial variable $\mathbf{x}$ is done for a instantaneous geometry, which has a free surface, a wetted hull surface and another fixed surfaces (e.g. the bottom one). In this way, we arrive to a system of Ordinary Differential Equations (ODE) for the time t. Neglecting the gradient term in Eqn. (1.c), the implicit form

$$\begin{cases} \mathbf{H}(\boldsymbol{\eta}_S)\mathbf{u}_1 + \mathbf{G}(\boldsymbol{\eta}_S)\mathbf{u}_2 = \mathbf{0} & ; \\ -\boldsymbol{\eta}_{S,t} + \mathbf{n}_{SS}^{-1}\boldsymbol{\sigma}_S = \mathbf{0} & ; \\ \boldsymbol{\phi}_{S,t} + g\boldsymbol{\eta}_S = \mathbf{0} & ; \end{cases} \tag{2}$$

is obtained, where $\mathbf{n}_{SS} = \text{diag}_{SS}(n_z)$ is a diagonal matrix with the normals n_z of the panels, $\mathbf{u}_1, \mathbf{u}_2$ are the mixed vectors

$$\mathbf{u}_1 = \begin{bmatrix} \boldsymbol{\phi}_B \\ \boldsymbol{\sigma}_S \end{bmatrix} \; ; \; \mathbf{u}_2 = \begin{bmatrix} \boldsymbol{\sigma}_B \\ \boldsymbol{\phi}_S \end{bmatrix} \; ; \tag{3}$$

while the influence matrices $\mathbf{H}, \mathbf{G}$ are obtained by means of a standard panel discretization for the Laplace equation, which leads the sub-matrices

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{BB} & \mathbf{H}_{BS} \\ \mathbf{H}_{SB} & \mathbf{H}_{SS} \end{bmatrix} \; ; \; \mathbf{G} = \begin{bmatrix} \mathbf{G}_{BB} & \mathbf{G}_{BS} \\ \mathbf{G}_{SB} & \mathbf{G}_{SS} \end{bmatrix} \; ; \tag{4}$$

where the sub-indices B, S denote free and body surface, respectively, and also refer the involved matrix dimensions, that is, B panels on the body and S panels on the free surface. This system, in this form, has a drawback that it is not really an ODE' system, but a Differential and Algebraic Equation (DAE) one, in the sense that we have $2S + B$ equations with the unknowns $\boldsymbol{\eta}_{S,t}$, $\boldsymbol{\phi}_{S,t}$ and $\boldsymbol{\phi}_B$, that is, of the type

$$\mathbf{A} \begin{bmatrix} \mathbf{x}_{,t} \\ \mathbf{y} \end{bmatrix} = \mathbf{f}(t) \; ; \tag{5}$$

where $\mathbf{A}$ has a $N \times N$ size, and the vectors $\mathbf{x}, \mathbf{y}$ have the N_x, N_y lengths, respectively, with $N = N_x + N_y$. Then, the vector $\mathbf{y}$ of the last N_y equations can be eliminate and replaced on the first N_x equations, and a system only on $\mathbf{x}_{,t}$ could be obtained. For obtaining a DAE' system we should relax the $\boldsymbol{\phi}_B$ vector. But, we have developed another strategy based on the Stekhlov operator technique in conjunction with a modified Crank-Nicolson scheme [12].

NUMERICAL EXAMPLES

Two simple examples are included as a first validation of our proposed method, skew-oscillations in a rigid U-tube and annular surface gravity waves in a circular tank.

Skew-oscillations in a U-tube

As a first example, a rigid U-tube filled with a liquid is considered. On its free surface, a small perturbation in its hydrostatic equilibrium height is introduced, in such way that skew-symmetric vertical oscillations will develop. If the dissipation effects are neglected, the oscillation state remains in time without attenuation. For the U-tube we choose a vertical semi-toroidal geometry, with external, internal and mean radius R_E,R_I and $R_M = (R_E + R_I)/2$, respectively, and mean perimeter $L_M = \pi R_M$. At the initial time $t = 0$, we impose a small vertical displacement on each free surface, in a constant but skew-symmetric way, that is, $\boldsymbol{\eta}_L = +\varepsilon \mathbf{z}$ at the left free surface, and $\boldsymbol{\eta}_R = -\varepsilon \mathbf{z}$ at the right one, where $0 < \varepsilon < 1$, and z is the vertical, see figure 1, left. The mean wave number of this system is $K_M = 2/L_M$, and the squared of the mean angular frequency is $\omega_M^2 = 2g/L_M$. We have considered 3 meshes of 240, 832 and 1408 panels, see figure 1 (right) for a xz vertical view of one of them. In figure 2 (left), we shown the first period of the height η in the middle sector of the free surfaces, obtained with the 3 meshes. In figure 2, right, 15 periods are plotted, where numerical dissipative-like effects are not perceived. The numerical natural period in the considered meshes has the the succession $T_{BEM} \approx \{17.09, 18.48, 18.87\}$ sec., whereas the mean analytic period is $T_{MED} \approx 19.87$ sec., computed with the mean semi-toroidal perimeter. The relative errors are $e_r \approx \{-0.14, -0.07, -0.05\}$, respectively.

Gravity annular waves in a circular tank

A circular tank of radius R and depth H enough for neglecting the bottom effects is considered, see figure 3, left. The tank is assumed to be rigid and it is filled with an inviscid (so there is not dissipative effects) and incompressible liquid. When a perturbation is introduced at the center of its free surface, annular out-going surface gravity waves are generated. The radial velocity is null on the vertical wall at $r = R$. Then, exciting in one of the natural modes of the free surface, a standing wave will be obtained. The initial boundary condition at $t = 0$ is a natural mode with radial symmetry. The analytic calculation leads the eigen-functions

$$\Phi_\alpha(r, \theta, z, t) = A J_0(K_\alpha r) e^{K_\alpha z} e^{i\omega t} \quad ; \quad \text{for } z \leq 0 \; ; \tag{6}$$

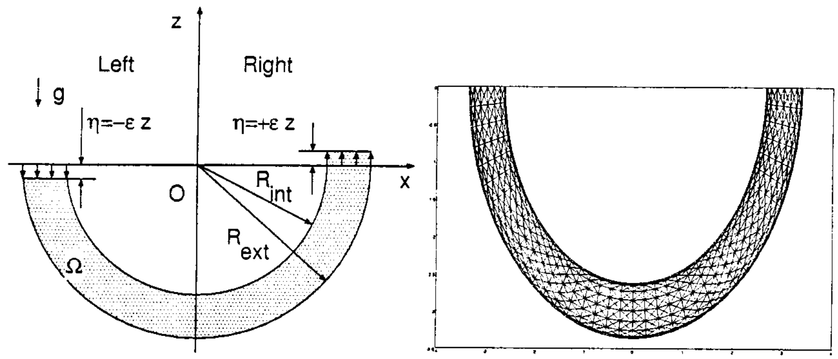

Figure 1: Left: Geometrical description for an U-tube. The vertical oscillation of the free surfaces is skew-symmetric. Right: a xz vertical view of the panel mesh.

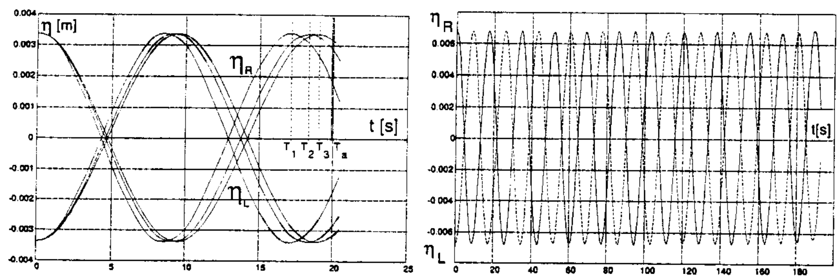

Figure 2: Computed periods of the skew-symmetric heights $\eta_L(t), \eta_R(t)$, at the left and the right on the U-tube: left: the first one; right: the first 15 ones. Numerical dissipative effects are not perceived.

where $J_0(Kr)$ is the Bessel function of first kind and zero order, A is the amplitude of the oscillation, $x = Kr$ is the non-dimensional radial coordinate, x_α are the zeros of the Bessel function of first kind and first order $J_1(x)$, for instance, $x_\alpha = \{3.832, 7.016, 10.173, 13.324, ...\}$. The wave-numbers are given by $K_\alpha = x_\alpha/R$ and $\omega_\alpha = \sqrt{g x_\alpha / R}$. That is, simple annular gravity waves with null radial velocity at $r = R$ and $-H \leq z \leq 0$ are obtained, when the wall is localized at the zeros of $J_1(x)$. The wave-weight $\eta(r,t)$ in the first period, obtained with the proposed method, as a function of the radius r and the time t, is shown in figure 3, right, which are well-compared with the analytical ones. In figure 4, an xy-horizontal view and a xz-vertical one are shown. In figure 5, 1 and 4 periods of the height $\eta(t)$, in the middle zone of the free surface, are included. The natural period estimated by the panel code is $T_{BEM} \approx 4.2943$ sec., while the analytic one is $T_{al} \approx 4.5394$ sec., with a relative error of $e_r \approx -5\%$.

CONCLUSIONS

The next stages in the development should include radiation boundary conditions and a version for a parallel computation on the Beowulf cluster "Gerónimo" of the CIMEC Group.

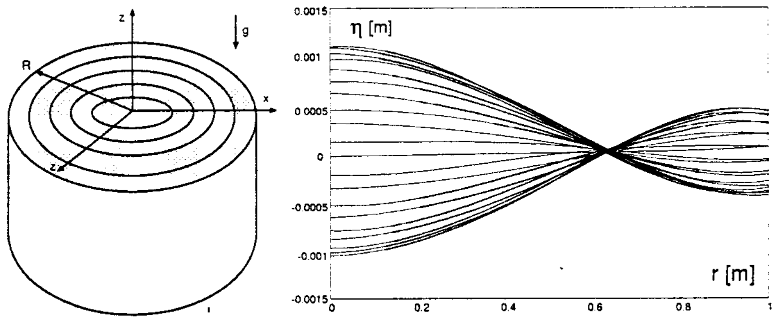

Figure 3: Left: sketch for the annular gravity waves on the free surface of a depth circular tank. Right: wave-heights $\eta(r,t)$, as a function of the radius r and the time t, in the first oscillation mode, computed by the proposed panel method. The analytic ones are proportionals to the Bessel function of first kind and zero-order $J_0(Kr)$.

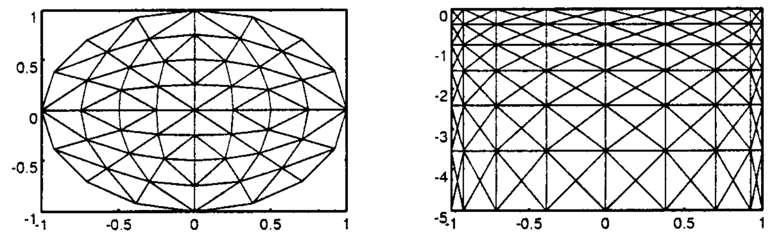

Figure 4: Views of the panel mesh for a circular tank.

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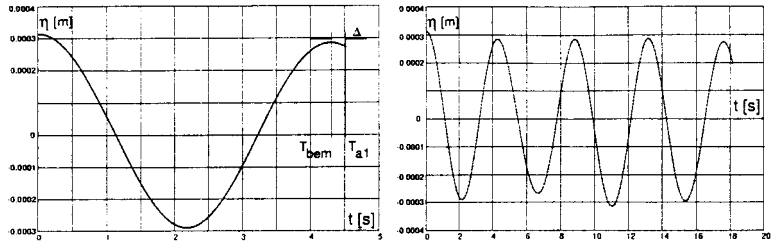

Figure 5: Computed periods of the skew-symmetric heights $\eta_L(t), \eta_R(t)$, at the left and the right on a circular tank: left: the first one; right: the first 4 ones. Numerical dissipative effects are not perceived.

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