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A DAUBECHIES WAVELET MINDLIN-REISSNER PLATE ELEMENT

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Abstract. Wavelet multiresolution analysis provides a powerful framework for analyzing functions at various scales. Due to the fact that wavelets have several good properties, such as compact support and vanishing moments, it has gained great interest in solving partial differential equations using the finite element method. In this paper a two-dimensional wavelet finite element is developed in which the scaling functions are adopted as trial functions. Based on the one-dimensional Daubechies wavelet finite element, that we have constructed recently [Mecánica Computacional Vol XXVI, pp.654-666], tensor product is used to calculate the connection coefficients for stiffness matrices and load vectors. Some test problems are studied and the numerical results are in good agreement with the closed-form or traditional finite elements solutions.

1 INTRODUCTION

An important property of wavelet multiresolution analysis is the capability to represent functions at different scales. By means of "two-scale relation", the scale adopted can be changed freely according to requirements to improve analysis accuracy.

In structural analysis, classical and standard numerical methods as the finite element method (FEM), boundary element method (BEM), and Meshless methods have been applied during the last decades. Recently, due to its desirable advantages, researchers are also paying attention to wavelet analysis in FEM. For a wide class of elliptic differential operators, wavelet method was proved to converge [Wei (2000); Chen et al. (2004); Han et al. (2005, 2006)]. In particular, in Ma et al. (2003) and Vampa et al. (2007), Daubechies compactly supported orthogonal wavelets were used to construct one-dimensional beam elements.

In Xiang et al. (2006) , C^0 plate elements are constructed to solve plane elastomechanics and moderately thick plate problems. These finite elements are based on two-dimensional tensor product B-spline wavelet on the interval (BSWI).

In this work, a new class of Daubechies Scaling Wavelet functions finite elements DSCW for Mindlin-Reisner plate model is presented. The wavelet-finite element scheme is constructed in a similar manner to the conventional displacement-based FEM: the Daubechies wavelet functions are used as interpolation functions and the shape functions are expressed by wavelets.

The rest of the paper is organized as follows: Section 2 introduces basic concepts of wavelet analysis including background and a technique for computing connection coefficients; Section 3 presents a Mindlin-Reisner plate finite element formulation and shows a comparison of various numerical test solutions. In Section 4 conclusions are presented.

2 MULTIRESOLUTION ANALYSIS AND DAUBECHIES WAVELETS

Wavelets are functions generated by simple operations of dilation and translation, from one single function called mother wavelet. A mother wavelet ψ gives rise to a decomposition of the Hilbert space $\mathrm{L}^2(\mathbb{R})$, into a direct sum of closed subspaces W_j, $j \in \mathbb{Z}$.

Let $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$ and

$$W_j = \mathrm{clos}_{\mathrm{L}^2}[\psi_{j,k} : k \in \mathbb{Z}] \, . \tag{1}$$

Then,

$$\mathrm{L}^2(\mathbb{R}) = \sum_j W_j = \cdots \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus \cdots \tag{2}$$

and using this decomposition of $\mathrm{L}^2(\mathbb{R})$, a nested sequence of closed subspaces V_j, $j \in \mathbb{Z}$ can be obtained, where

$$V_j = \sum_{l=-\infty}^{j-1} W_l = \cdots \oplus W_{j-2} \oplus W_{j-1} \, . \tag{3}$$

These closed subspaces $\{V_j,\ j \in \mathbb{Z}\}$ of $\mathrm{L}^2(\mathbb{R})$, form a "multiresolution analysis" (Chui, 1992) with the following properties:

1. $\cdots \subset V_{-1} \subset V_0 \subset V_1 \cdots$
2. $\mathrm{clos}_{\mathrm{L}^2}(\bigcup V_j) = \mathrm{L}^2(\mathbb{R})$
3. $\bigcap_j V_j = \{0\}$

4. $V_{j+1} = V_j \oplus W_j$
5. $f(x) \in V_0 \Leftrightarrow f(x-k) \in V_0\ , k \in \mathbb{Z}$
6. $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}\ , j \in \mathbb{Z}$
7. There exists $\phi \in V_0$ that the set $\{\phi(x-k) : k \in \mathbb{Z}\}$ is a Riesz basis of V_0.

The function $\phi \in V_0$ is called "scaling function" and generates the multiresolution analysis $\{V_j\}_{j \in Z}$ of $\mathrm{L}^2(\mathbb{R})$ and by setting

$$\phi_{j,k}(x) := 2^{j/2}\phi(2^j x - k) \tag{4}$$

it follows that, for each $j \in \mathbb{Z}$, the family

$$\{\phi_{j,k} : k \in \mathbb{Z}\} \tag{5}$$

is also a Riesz basis of V_j.

Consequently, a unique sequence $\{p_k\} \in l^2(Z)$ exists, ($l^2(Z)$ denotes the integer space of all square-summable bi-infinite sequences), such that the scaling function $\phi(x)$ satisfies a refinement equation

$$\phi(x) = \sum_{k=-\infty}^{\infty} p_k\, \phi(2x-k), \qquad k \in Z \tag{6}$$

which is also called "two-scale relation".

On the other hand, the wavelet $\psi \in V_1$ is defined from the scaling function by means of a second conjugate sequence $\{g_k\} \in l^2(Z)$

$$\psi(x) = \sum_{k=-\infty}^{\infty} g_k \phi(2x-k), \qquad k \in Z\ . \tag{7}$$

Multiresolution property means that V_j is a subset of V_{j+1}. So each element of V_{j+1} can be uniquely written as the orthogonal sum of an element in V_j and an element in W_j that contains the complementing details, i.e.

$$V_{j+1} = V_j \oplus W_j. \tag{8}$$

As an example of multiresolution analysis, a family of orthogonal wavelets with compactly supported property has been constructed by Daubechies (1992).

In her work, Daubechies (Daubechies, 1988) found and exploited the link between vanishing moments of the wavelet ψ and regularity of wavelet and scaling functions, ψ and ϕ. The wavelet function ψ has M vanishing moments if

$$\int x^k \psi(x) dx = 0 \qquad \text{for} \quad 0 \le k \le M \tag{9}$$

and a necessary and sufficient condition for this to hold is that integer translates of the scaling function ϕ exactly interpolate polynomials of degree up to M. That is, for each k, $0 \le k \le M$ there exist constants c_l^k such that

$$x^k = \sum_l c_l^k\, \phi(x-l) \tag{10}$$

Daubechies introduced scaling functions that satisfying this property have the shortest possible support. Let ψ_N be the wavelet Daubechies function with $N/2$ null moments (where N is an even integer), and ϕ_N the corresponding scaling function. ϕ_N has support in $[0, N-1]$, while ψ_N has support in the interval $[1-N/2, N/2]$ (Daubechies, 1988). Thus, according to equation (10) Daubechies scaling functions of order N can exactly represent any polynomial of order up to, but not greater than $N/2-1$.

2.1 Computation of scaling functions and its derivatives

In using scaling functions of Daubechies wavelets as test functions of finite element method, derivatives of Daubechies scaling functions have to be calculated. As there is no explicit expression for the Daubechies scaling functions, the derivatives can only be obtained on some special points. To evaluate the function or its derivatives, $\phi_N^{(m)}(x) = d^m\phi_N(x)/dx^m$, the two-scale relation is differentiated m times:

$$\phi_N^{(m)}(x) = 2^m \sum_{k=0}^{N-1} p_k \phi_N^{(m)}(2x-k)\ . \tag{11}$$

Evaluating Eq. (11) for all integer values of the interval $[0, N-1]$, gives an homogeneous system of N linear equations which is singular. Thus, a normalizing condition is required in order to determine a unique solution and the following proposed by Beylkin can be considered,

$$\sum_k k^m \phi_N^{(m)}(x-k) = m!\ . \tag{12}$$

This condition is obtained differentiating m times (m is a positive integer number), the important additional property of Daubechies scaling function ϕ_N , (Beylkin, 1992):

$$\sum_k k^m \phi_N(x-k) = x^m + \sum_{k=1}^{m} (-1)^k \frac{m!}{(m-k)!k!} x^{m-k} \int_{-\infty}^{\infty} \phi_N(z) z^k dz\ . \tag{13}$$

Then, solving this new system of inhomogeneous equations, derivatives can be evaluated at integer values of x and used to get the values at the dyadic points.

Using the two-scale relation once again the values of $\phi_N^{(m)}(x)$ at $x = \frac{i}{2^n}$, with $n \in Z$, for $i = 1, 3, 5, \ldots, \{2^n(N-1)-1\}$ can be determined. Therefore, the functions are first evaluated at the integer points $\{0, 1, \ldots, N-1\}$, then at half integers and so on, increasing the value of n from 0 to the desired resolution.

2.2 Computation of Connection Coefficients

When the wavelet-finite element method is applied to solve one-dimensional differential equations, different types of connection coefficients are required to form stiffness matrices and load vectors (Latto et al., 1995), such as:

$$\Gamma_{i,j}^{d_1 d_2} = \int_0^1 \phi^{(d_1)}(\xi - i)\, \phi^{(d_2)}(\xi - j)\, d\xi\ , \tag{14}$$

$$R_i^{(s)} = \int_0^1 \xi^s\, \phi(\xi - i)\, d\xi\ , \tag{15}$$

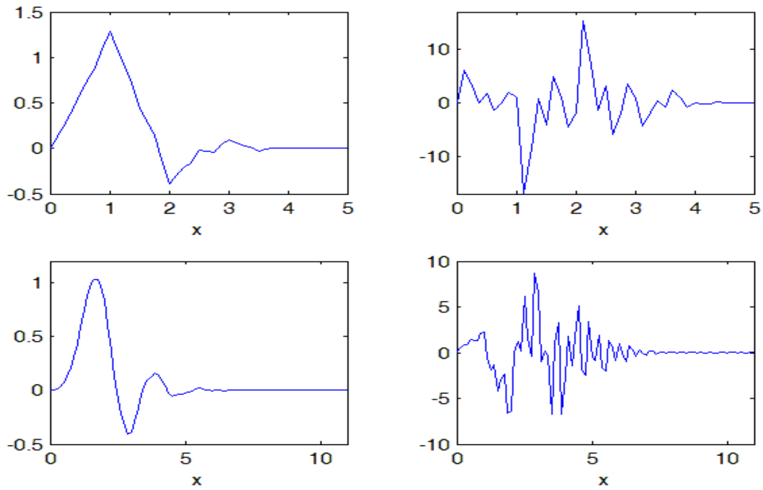

Figure 1: Daubechies scaling functions. Left:$\phi(x)$. Right: $\phi''(x)$. Top: $N = 6$. Bottom: $N = 12$.

where $i, j \in \mathbb{Z}$, ϕ denotes the basis function and the superscripts d_1 and d_2 refer to differentiation orders.

The typical problem that arises using Daubechies wavelets is how to calculate these connection coefficients when ϕ is a Daubechies-wavelet scaling function. In first place, the difficulty is due to the lack of an explicit Daubechies scaling function expression. Moreover, the highly oscillatory nature of the Daubechies basis functions makes standard numerical quadrature impractical for computing connection coefficients. To show this, we present the scaling functions and their second derivative for N=6 and N=12, in Fig. 1. The numerical calculations are in general unstable and it is necessary to provide an alternative method.

To calculate the integral in Eq.14, Latto proposed to substitute the two-scale relation given by Eq.6 into Eq.14, which yields

$$\Gamma_{i,j}^{d_1 d_2} = 2^{d_1+d_2} \sum_{k,l} p_k p_l \int_0^1 \phi^{(d_1)}(2\xi - 2i - k)\, \phi^{(d_2)}(2\xi - 2j - l)\, d\xi\ . \tag{16}$$

Doing the adequate transformations the following expression in terms of the original connection coefficients is obtained

$$\Gamma_{i,j}^{d_1 d_2} = 2^{d-1} \sum_{r,s} [p_{r-2i} p_{s-2j} + p_{r-2i+1} p_{s-2j+1}] \Gamma_{r,s}^{d_1 d_2} \tag{17}$$

where $d = d_1 + d_2$ and $-(N-2) \leq i, j \leq 0$.

The last equation can also be written in matrix form, as

$$(2^{d-1} P - I) \vec{\Gamma}^{d_1 d_2} = 0 \tag{18}$$

where $\vec{\Gamma}^{d_1 d_2}$ is a column vector, I is the identity matrix and P is the matrix composed of wavelet coefficients combinations, obtained from Eq.(17).

In order to uniquely determine the connection coefficients $\Gamma_{i,j}^{d_1 d_2}$, sufficient number of inhomogeneous equations can be obtained by using different values of m and n in the following expression (Latto et al., 1995).

$$\frac{mn\ldots(m-(d_1-1))(n-(d_2-1))}{m+n-d+1} = \sum_{k,l} c_k^m c_l^n \Gamma_{k,l}^{d_1 d_2}\,. \tag{19}$$

Adding them to equation (18) connection coefficients can be determined uniquely.

Connection coefficients for load vectors, Eq.15, can be calculated in a similar way (see Chen et al. (2006)). Firstly, for $s=0$, the system to solve is,

$$R_i^{(0)} = \frac{1}{2}\sum_k [p_{k-2i} + p_{k-2i+1}] R_k^{(0)} \tag{20}$$

where $-(N-2) \leq i \leq 0$, and the additional inhomogeneous equation

$$\frac{1}{Q+1} = \sum_k c_k^Q\, R_k^{(0)} \qquad Q \leq N/2 - 1 \tag{21}$$

is required for a unique solution.

On the other hand, connection coefficients for $s>0$ are obtained recursively by solving

$$(2^{s+1}I - B)R_i^{(s)} = \sum_k p_{k-2i+1} \sum_{r=1}^{s} \binom{s}{r} R_k^{(s-r)} \tag{22}$$

where

$$B_{l,k} = p_{l-2k} + p_{l-2k+1} \tag{23}$$

3 THE CONSTRUCTION OF DAUBECHIES MINDLIN-REISSNER PLATE FINITE ELEMENT

The plate element formulation is based on the theory of plates with the effect of transverse shear deformations included (like Timoshenko beam theory). This theory, due to E.Reissner and R.D.Mindlin, needs only C^0 continuity and uses the assumption that particles of the plate, originally on a straight line that is normal to the undeformed middle surface remain on a straight line during deformation, but this line is not necessarily normal to the deformed middle surface. With this assumption, (in small displacement bending theory) the displacement components of a point of coordinates x, y and z are

$$u = -z\theta_x(x,y) \qquad\qquad v = -z\theta_y(x,y) \qquad\qquad w = w(x,y) \tag{24}$$

where u and v are inplain displacements, w is the transverse displacement (or called deflection) and θ_x and θ_y are the rotations of the midplane about y and x axes, respectively (see Fig. 2).

According to Mindlin-Reissner theory, the elemental generalized function of potential energy for Mindlin-Reissner plate bending problem in linear static analysis is,

$$\pi = \frac{1}{2}\int_{\Omega_e} \kappa^T C_b \kappa \; dxdy \quad + \frac{1}{2}\int_{\Omega_e} \gamma^T C_s \gamma \; dxdy \quad - \int_{\Omega_e} q\, w \; dxdy \tag{25}$$

where

$$\kappa = \left\{ \frac{\partial \theta_x}{\partial x}, -\frac{\partial \theta_y}{\partial y}, \frac{\partial \theta_x}{\partial y} - \frac{\partial \theta_y}{\partial x} \right\}^T \qquad \gamma = \left\{ \frac{\partial w}{\partial x} + \theta_x, \frac{\partial w}{\partial y} - \theta_y \right\}^T \tag{26}$$

$$C_b = \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad C_s = \frac{Etk}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{27}$$

Ω_e is the elemental solving domain, q is the distributed load, t is the thickness of the plate (assumed constant), E is Young modulus, ν is Poisson's ratio and k is the shear correction factor equal to $\frac{5}{6}$.

One thing to be noted here is that the first term in Eq.25 corresponds to bending energy, while the other is the transverse shear energy and this last term becomes dominant compared to the bending energy as the plate thickness becomes very small compared to its side length.

3.1 Daubechies Mindlin-Reissner plate finite element

Supposing that one-dimensional Daubechies scaling functions $\phi^1(\xi)$ and $\phi^2(\eta)$ generate multiresolution analyses $\{V_j^1\}$ and $\{V_j^2\}$ respectively, the tensor product space of V_j^1 and V_j^2, $j \in Z$, is

$$V_j = V_j^1 \otimes V_j^2 \tag{28}$$

$\{V_j\}$ generates a multi-resolution analysis of $\mathrm{L}^2(\mathbb{R}^2)$. If we call

$$\begin{aligned} \varphi^1 &= \{\phi^1(\xi), \phi^1(\xi+1), \ldots, \phi^1(\xi+(N-2))\} \\ \varphi^2 &= \{\phi^2(\eta), \phi^2(\eta+1), \ldots, \phi^2(\eta+(N-2))\} \end{aligned} \tag{29}$$

the scaling functions of $\{V_j\}$ can be expressed using the tensor product of the wavelets expansions at each coordinate, i.e.:

$$\varphi = \varphi^1 \otimes \varphi^2 \,. \tag{30}$$

The unknown field function $w(\xi, \eta)$ can be expressed as follows

$$w(\xi, \eta) = \varphi\, \alpha \tag{31}$$

where α is the vector of corresponding wavelet coefficients. The elemental transformation matrix T is

$$T = T^1 \otimes T^2 \tag{32}$$

where T^1 and T^2 are the transformation matrices corresponding to one-dimensional problem (Ma et al. (2003); Xiang et al. (2006); Vampa et al. (2007)).

For the plate problem, Eq.(25), independent interpolation is considered and the same shape functions are used for the displacements and slope interpolations. In this way, the elemental displacement functions, Eq.(24), can be replaced by

$$\theta_x = \varphi\, T^{-1}\, \widehat{\theta}_x, \qquad \theta_y = \varphi\, T^{-1}\, \widehat{\theta}_y, \qquad w = \varphi\, T^{-1}\, \widehat{w} \tag{33}$$

where $\widehat{\theta}_x$, $\widehat{\theta}_y$ and $\widehat{w}$, are the physical DOFs of elemental nodes, see Fig.(2).

Then, substituting Eq.(33) into Eq.(25) and according to the stationarity condition of π ($\delta\pi = 0$), we can obtain the elemental stiffness matrix.

Finally, the elemental FEM solving equations can be expressed by:

$$\begin{bmatrix} K^1 & K^2 & K^3 \\ K^4 & K^5 & K^6 \\ K^7 & K^8 & K^9 \end{bmatrix} \begin{bmatrix} \widehat{\theta}_x \\ \widehat{\theta}_y \\ \widehat{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \end{bmatrix}, \tag{34}$$

where

$$\begin{aligned}
P &= ((T)^{-1})^T \int_0^1 \int_0^1 q(\xi,\eta)\varphi^T d\xi d\eta \\
K^1 &= D_0\{A_1^{11} \otimes A_2^{00} + (1-\mu)/2\; A_1^{00} \otimes A_2^{11}\} + C_0\; A_1^{00} \otimes A_2^{00} \\
K^2 &= D_0\{A_1^{10} \otimes A_2^{01} + (1-\mu)/2\; A_1^{01} \otimes A_2^{10}\} + C_0\; A_1^{00} \otimes A_2^{00} \\
K^3 &= -C_0\; A_1^{01} \otimes A_2^{00} \\
K^4 &= (K^2)^T \\
K^5 &= D_0\{A_1^{00} \otimes A_2^{11} + (1-\mu)/2\; A_1^{11} \otimes A_2^{00}\} + C_0\; A_1^{00} \otimes A_2^{00} \\
K^6 &= -C_0\; A_1^{00} \otimes A_2^{01} \\
K^7 &= (K^3)^T \\
K^8 &= (K^6)^T \\
K^9 &= C_0\; A_1^{11} \otimes A_2^{00} + A_1^{00} \otimes A_2^{11}
\end{aligned} \tag{35}$$

and $D_0 = \frac{E\ t^3}{12(1-\nu^2)}$ and $C_0 = \frac{E\ t\ k}{2(1+\nu)}$. In this formulation,

$$A_s^{d_1 d_2} = l_{e,s}^{1-(d_1+d_2)}\ (T_s^{-1})^T\ \Gamma_s^{d_1 d_2}\ T_s^{-1}, \qquad s = 1,2 \tag{36}$$

where $l_{e,s}$ is the finite element side length, $\Gamma_s^{d_1 d_2}$ is the connection coefficients matrix defined in Section 2.2, (Eq.14) and subscript s denotes the Daubechies scaling function φ^s (Eq. 29) considered.

Adopting two dimensional Daubechies scaling functions with N coefficients to construct elements, Ω can be divided into uniform meshes. One Daubechies Scaling Wavelet element with N coefficients (DSCWN) has $(N-1)^2$ total nodes. As, in this model, each node has three DOFs, one DSCWN Mindlin-Reissner element has $3 \times (N-1)^2$ DOFs.

In the following section the finite element implementation is validated using Daubechies scaling functions of order 6. Numerical solutions obtained with DSCW6 elements are firstly compared with the approximations presented in Xiang et al. (2006), which use B-spline wavelets on the interval (BSWI) for the Mindlin-Reissner plate model. Also, a comparison is made with other wavelet based finite element method and with standard finite elements.

3.2 Applications

The formulation of two-dimensional tensor product Daubechies Mindlin-Reissner element developed in Section 3.1, is applied to a typical numerical example: a square isoparametric plate simply supported on all four edges. Two cases were considered: uniform and concentrated load. Let Poisson's rate μ be fixed as 0.3, t denote thickness and L denote side length.

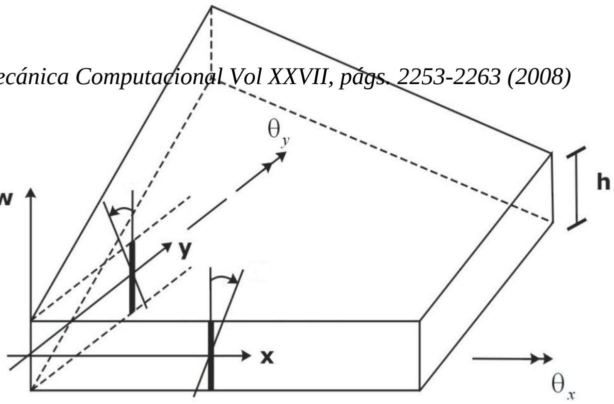

Figure 2: Mindlin-Reissner Plate element

	$w_c/(qL^4/100D_0)$			
t/L	(1x1)DSCW6	(2x2)DSCW6	BSWI23	Exact
0.001	0.3121	0.3441	$1.8\ 10^{-4}$	0.4063
0.01	0.3125	0.3452	0.0173	0.4063
0.05	0.3203	0.3647	0.2174	0.4107
0.1	0.3411	0.4006	0.3510	0.4273
0.15	0.3713	0.4397	0.4152	0.4536
0.2	0.4104	0.4842	0.4678	0.4906
0.3	0.5166	0.5979	0.5861	0.5956
0.35	0.5843	0.6691	0.6579	0.6641

Table 1: Central displacements for simply supported square plate subjected to uniform load $w_c/(qL^4/100D_0)$

Table 1 and 2 show the comparison of central displacements obtained with DSCW6 (with 6 coefficients) Mindlin-Reissner elements with those presented in Xiang et al. (2006), obtained with one BSWI23 element (B-splines of m order, with $m = 2$ and scale $j = 3$) for the thickness-span ratio from 0.001 to 0.35. Also, exact solutions are presented.

As it can be observed the method we proposed shows a non-locking behavior: even using scale $j = 0$ and only one element (75 DOFs), our results are better than BSWI23 (243 DOFs) for $t/L \leq 0.05$. With a 2×2 mesh excellent results are obtained for all the thicknesses considered.

We also made a comparison with the multivariable wavelet base finite element method presented in [Han et al. (2005)] to solve bending problems of thick plates. Table 3 shows that the 6×6 mesh DSCW6 elements, yields a more accurate solution.

Regarding thin plates, it is well known that shear locking problems can appear using standard finite elements and a lot of methods have been suggested to alleviate this phenomenon.There are several mixed finite elements methods which present good approximations to the solutions and are free from locking. A successful approach is that of the MITCn elements developed by Bathe and Dvorkin (1985) (MITC stands for mixed interpolation tensorial components and n refers to the number of element nodes). This family of plate bending elements uses mixed interpolation of transverse displacement section rotation and transverse shear strains. In particular, the MITC4 is a reliable and efficient low order plate element.

Table 4 shows the comparison between the results obtained with MITC4 and those obtained with DSCW6 and DSCW10 elements, for $t/L = 0.01$. Also computational time required is

	$w_c/(qL^4/100D_0)$			
t/L	(1x1)DSCW6	(2x2)DSCW6	BSWI23	Exact
0.001	0.7990	0.7991	$0.0504\ 10^{-2}$	-
0.01	0.8009	0.8059	0.0485	1.127
0.05	0.8429	0.9296	0.6325	1.209
0.1	0.9589	1.157	1.0973	1.353
0.15	1.134	1.418	1.416	-
0.2	1.368	1.727	1.752	1.851
0.3	2.016	2.540	2.614	-
0.35	2.432	3.054	3.159	-

Table 2: Central displacements for simply supported square plate subjected to concentrated load $w_c/(qL^4/100D_0)$

	$w_c/(qL^4/100D_0)$		
Mesh	Han et al. (2005)	DSCW6	Exact
6x6	0.3218	0.3224	0.3227

Table 3: Central displacements for clamped square plate subjected to uniform load $t/L = 0.3$

presented. It can be observed that with MITC4, the required CPU time is about four times larger than with DSCW6 element to achieve similar accuracy. On the other hand, with one individual DSCW10 element a very good approximation is obtained and the computational effort is comparable with MITC4. This results confirm that the Daubechies wavelet element proposed performs well.

4 CONCLUSIONS

In this work, we have demonstrated the feasibility and capability of using wavelet bases in the FEM. In particular, for Mindlin-Reissner plate model, Daubechies Scaling Wavelet elements (DSCWN) presented in this paper are efficient to solve plate bending problems. These elements can be easily constructed due to independent interpolation of each displacement function. Due to the orthonormal, compactly supported and nesting properties of the Daubechies wavelets,

			$w_c/(qL^4/100D_0)$			
Mesh	MITC4	CPU(in s)	DSCW6	CPU(in s)	DSCW10	CPU(in s)
1x1	–		0.3125	0.375	0.3570	1.65
2x2	0.3189	1.91	0.3454	0.453		
Thin plate sol.	0.40625					

Table 4: Central displacements and computational time required, for simply supported square plate subjected to uniform load $t/L = 0.01$

results are in good agreement with exact solutions for thick and thin plates, even with coarse meshes.

We are convinced that the wavelet-based methods are a powerful tool to deal with several problems in structural analysis and that more advantages could be obtained increasing j-scale level.

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