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RESONANT FREQUENCIES IN A SPHERICAL ELEVATED CONTAINER PARTIALLY FILLED WITH WATER: FEM AND MEASUREMENT

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Keywords: Fluid-structure interaction, spherical storage tanks, dynamic response.

Abstract. In this paper, a numerical-experimental study about the dynamic response of coupled fluid-structure systems of spherical elevated tanks is presented. The main objective is to gain insight in the physical response of this particular structural typology that is widely used in the petrochemical industry as LPG containers.

Experimental tests on a model were performed identifying the main fundamentals parameters involved in the physical response and determining the natural frequencies that contribute to the response in the range of 1-5 Hz, for different liquid levels. Next, a numerical model that takes into account the coupling between fluid and structure were developed and validated against the experimental results. A very good agreement was obtained between experimental and numerical results.

The obtained results indicate that the sloshing has a significant effect on the dynamics characteristics of the systems and, in the analyzed case, a two mass model it is insufficient to describe the overall dynamic response. In order to obtain a simple lumped mass model, a minimum of three masses and then three degree of freedom is suggested.

1 INTRODUCTION

The dynamic behaviour of elevated liquid storage containers is mainly studied because of the interest in their response to seismic loads (e.g., in petrochemical industry) or in the connection with the structural integrity and reliability analysis of diverse shell components (e.g. in nuclear reactors).

Many researchers have considered the topic of dynamic behaviour of liquid filled tanks in the last five decades, mainly on cylindrical and rectangular storage tanks. A significant amount of experimental and theoretical effort has been invested on studies about the understanding and predicting the seismic behaviour of ground-supported cylindrical tanks. One of the principal works was published in the early 1960s by Housner (1963), who considered cylindrical rigid tanks subjected to horizontal translation, and considered that the dynamical response can be idealized as the contribution of an impulsive (bulging) mass rigidly attached to the container wall and a sloshing mass (convective) that is connected to the wall by means of springs. The impulsive component was attributed to the part of the liquid that vibrate jointly with the tank, while the sloshing component, which was characterized by long-period oscillations, and correspond to the liquid around of the free surface. To represent these effects, Housner (1963) considered a model with two masses and developed equations to compute the impulsive and sloshing liquid masses, along with their location above the tank base and the stiffness of the convective mass spring. Although only, for practical design, one convective mass is normally considered, additional lumped-masses may also be included by Bauer's model (Bauer 1964, Livaoglu and Dogangun 2006). Haroun and Housner (1981) considered the tank-wall flexibility into account by a three-mass model for this type of tanks. Other works concerning natural modes and frequencies of a clamped-free cylindrical storage tanks by means of an experimental and numerical approach are reported by Mazúch (1996), by analytical procedures Han and Liu (1994), and using a general purpose FE program by Goncalves and Ramos, (1996) and Virella et al. (2006). In disagreement with the findings reported by others authors that proposed a cantilever beam type mode for the response to horizontal base motion, Natchigall et al. (2003), stated that shell modes with circumferential wavy pattern to model tank-liquid systems need to be used.

With regard to dynamic analysis of rectangular tanks several works have been reported, e.g. Dogangun et al.(1996), Koh et al.(1998), Kianoush and Chen (2006). On the other hand, Livaoglu (2008) evaluates the dynamic behaviour of fluid-rectangular-tank-soil-foundation system with a simple seismic analysis procedure, based on Housner's two mass approximations, showing that the displacements and base shear forces generally decreased, with decreasing soil stiffness. However, embedment, wall flexibility, and soil–structure interaction (SSI) did not considerably affect the sloshing displacement.

The sloshing of this type of containers has attracted some interest too, e.g. Chount and Yun (1996), Faltinsen et al. (2002, 2003), Celebi and Akyildiz (2002).

Only some few studies have been carried out related to seismic behaviour of elevated tanks (Haroun and Ellaithy 1985 and Rai 2002). A remarkable paper on simplified seismic analysis procedures for elevated tanks considering fluid-structure-soil interaction was presented by Livaoglu and Dogangun (2006) considering different models with an added mass (lumped or distributed) approach for the fluid–structure interaction, and the mass-less foundation and substructure approaches for the soil–structure interactions. Moreover, Livaoglu and Dogangun (2007) investigated the embedment effects on the seismic response of fluid-elevated tank-foundation-soil systems and the fluid-structure interaction was taken into account using Lagrangian fluid FE approximation implemented in a general purpose computer program. The seismic performance of elevated tanks damaged during the 1999

Kocaeli, Turkey earthquake and the parameters influencing the dynamic behaviour were reported by Sezen et al. (2008), in which a simplified three mass model and a finite element model of the tanks are used including the effect of liquefied gas–structure interaction.

On the other hand, literature describing the dynamic response or dynamic characteristics behaviour of spherical tanks is scarce. The seismic response of a typical spherical liquid storage steel tank equipped with a non-linear viscous bracing system is numerically investigated by Dorosos et al. (2005). A retrofit design scheme utilizing energy-dissipating braces to substitute the existing braces has been proposed by Castellano et al. (2006). In order to quantify the response reduction due to seismic isolation, a number of parametric non-linear time-history analyses on a simplified Housner model of a typical sphere equipped with different types of isolation systems (lead rubber bearings (LRB) and high damping rubber bearings (HDRB), have been carried out by Bergamo et al. (2006).

The effects of liquids sloshing in spherical containers has attracted some interest e.g. McIver (1989); Evans and Linton (1993); Papaspyrou et al. (2003). A recent study that developed a mathematical model for calculating linear sloshing effects in the dynamic response of horizontal-cylindrical and spherical liquid containers under earthquake excitation was presented by Patkas et al. (2007).

However, because most of the few investigations on the dynamic behaviour, taking account the sloshing of spherical liquid filled containers are numerical, special attention must be paid to experimental studies which are essential to compare the results in order to obtain reliable conclusions about the accuracy and applicability of different theories. Then, the main objective of this paper is to gain insight in the physical response of spherical elevated tanks, with emphasis in the frequencies and mode shapes changes due to the stepwise increase in the liquid level. The major interest is to show changes of which frequencies corresponding to modes with the larger modal participation on the response to a horizontal base motion, without taking account specific modes of the shell (circumferential wavy patterns) and liquid. By mean of experimental tests the frequencies of structure in the range of 1-5 Hz for different water levels were determined. In conjunction, the frequencies and mode shapes of vibration were computed by means of a Frequency Response Function analysis on a FE model. The agreement between numerical and experimental results is excellent.

2 DYNAMIC TESTS

2.1 Case study

Sometimes to compare numerical results obtained with different theories leads to an undefined problem and another point of view is necessary in order to solve differences and show the accuracy of these theories. As it was pointed out in the point 1, experimental evidences about vibration of elevated spherical tanks are rather scarce in the literature. Then, series of free vibration test were carried out about this structural typology.

The plastic spherical shell tested, have a radius R = 81.3 mm, wall thickness e = 3 mm and mass density ρs = 980 kg/m^3, (see figure 1). It was mentioned before that it was not necessary to include the flexibility of shell (shell wavy modes), because this study emphasizes the behaviour of structure as a whole. The sphere is supported by two legs with a length L = 260mm, a cross section of 3 x 35 mm^2, and the following material characteristics: Young's modulus E = 2.35 e9 Pa, Poisson ratio v = 0.3 and mass density ρl = 980 kg/m^3. The contained liquid is water with a density of 1000 kg/m^3 and bulk modulus of 2.25 GPa. The legs were clamped at the base.

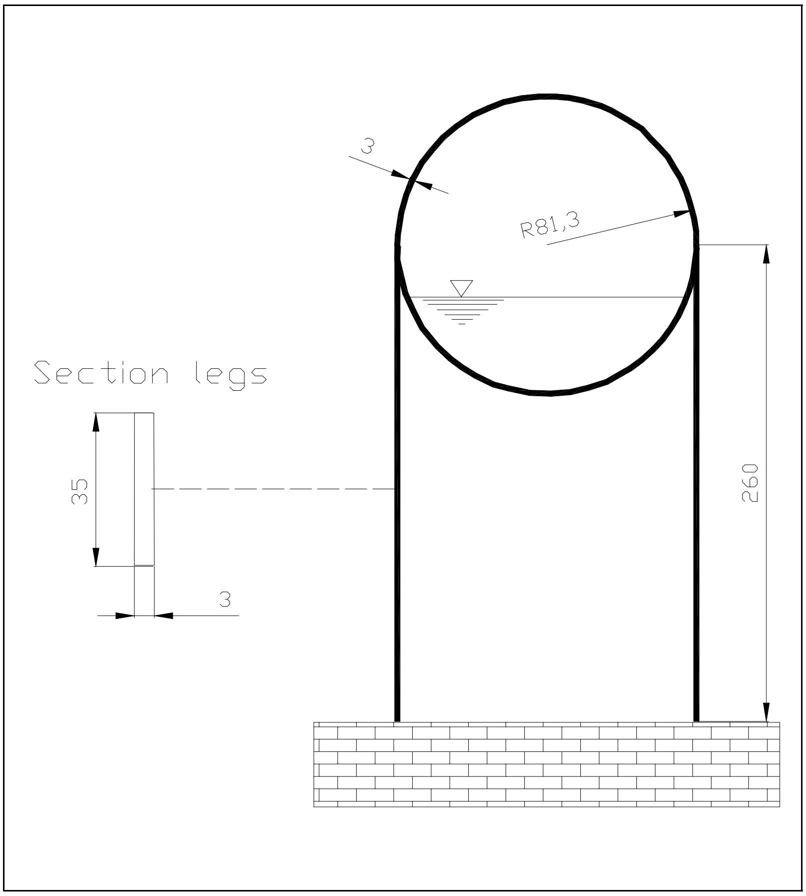


Figure 1. Spherical container.

2.2 Experimental set-up and instrumentation

In order to determine experimentally the natural frequencies, the free vibration response was measured by means of a PCB Piezotronics capacitive accelerometer (700 mV/g). A data acquisition board Computerboards PCM-DAS16D/16 of 16 bit of resolution and a maximum conversion time of 10 μs (100 KHz) was mounted on a notebook computer in order to record and process the signals by means of the program HP VEE 5.0 1998. The structure was excited with a hammer blow in different points in order to excite different modes.

The signals were sampled with the following parameters: $N = 10000$ (total number of points), $n = 500$ (sampling rate or number of points per second), $T = 20$ s (total time of the sample), $\Delta t = 0.002$ s, (time interval), $\Delta f = 0.05$ Hz (frequency interval), $f_{máx} = 250$ Hz (maximum frequency).

An algorithm to obtain and process the data was programmed in the environment HPVEE 1998. After applying the *Fast Fourier Transform*, the spectrum was calculated using the Welch method (Ewins 2000).

2.3 Dynamic response and experimental results

A total of 16 tests were conducted at various gradually increasing water surface levels defined by the ratio between height of the free surface of water and radius of the container, from $H/R = 0$ (empty) to $H/R = 2$ (full). The correspondence between water surface level and container fullness is listed in table 1. The figure 2 shows the natural frequencies in the range of 1-5 Hz, obtained experimentally by means of the Welch method of free vibration response signals measured at the equator of sphere, for each water level. The decrease in the resonance frequencies (peaks of spectrum) as the water surface level H/R increases are shown in Figure 3.

Sphere volume = 2250 cm3 Radius = 8.13 cm	
H/R	Fullness [%]
0.00	0.00
0.37	8.9
0.54	17.8
0.81	35.6
1.044	53.33
1.29	71.11
1.59	88.9
2.0	100.0

Table 1. Correspondence between water surface level and container fullness

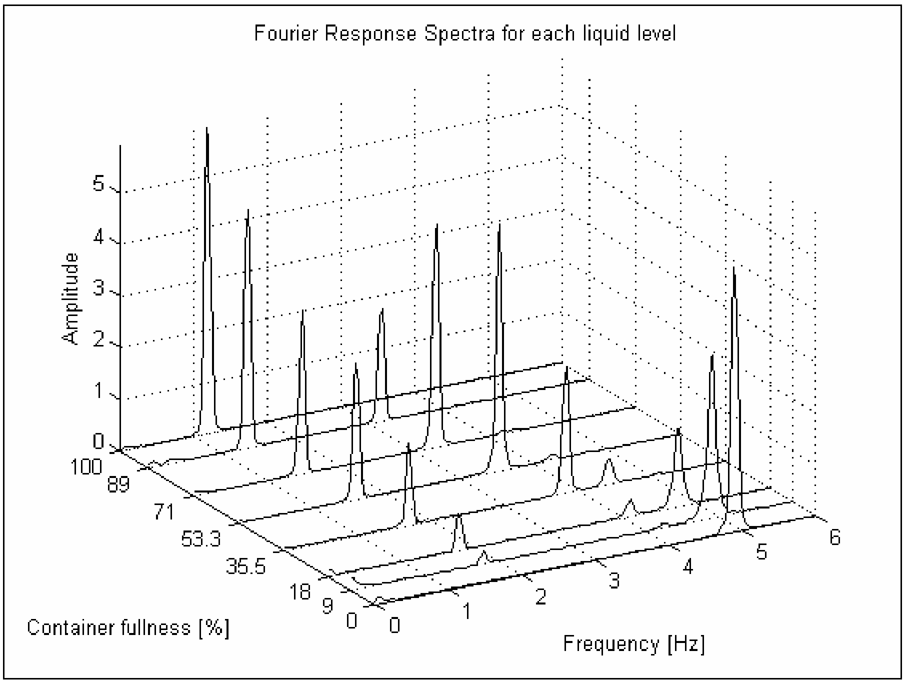


Figure 2. Spectrum of measured free vibration response, for 8 water surface levels of the spherical container.

Figure 2 provides a general view of the frequencies behaviour due to rise of liquid level.

From figure 3 can be observed that, at the beginning, when the container is empty, only one frequency of 4.8 Hz is identified. When the liquid level is increasing, due to sloshing, two more frequencies appear. As it was expected, the frequencies of partially fluid-filled spherical container decrease with an increasing fluid level up to a fullness of 89% where the highest become imperceptible. Finally, the only frequency that remains is of 1.2 Hz corresponding to full container.

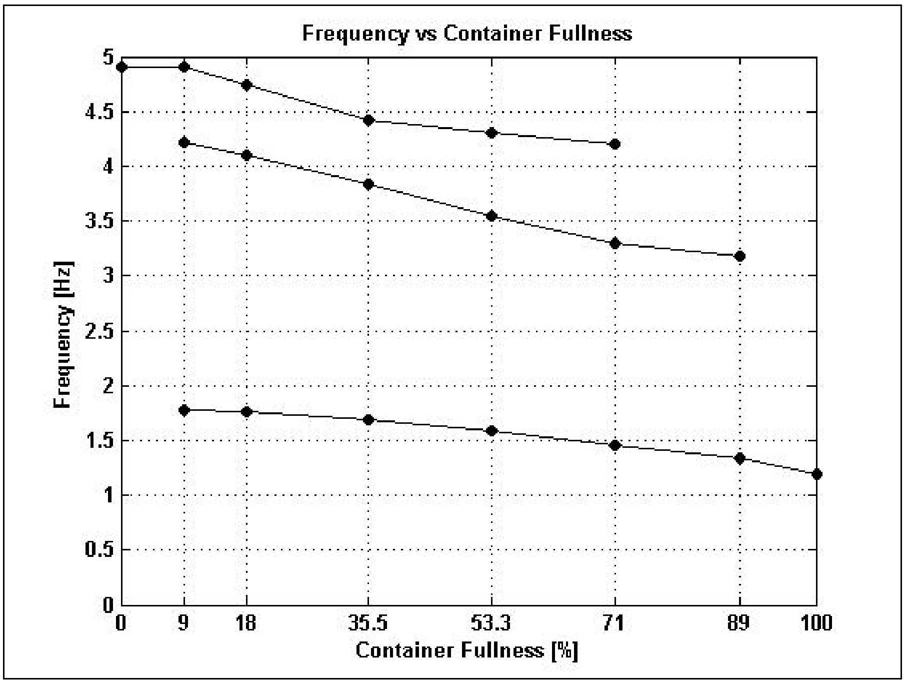


Figure 3. Natural frequencies in the range of 1-5 Hz.

It is important to note that, due to the interaction between fluid and structure, the dependency of the frequency on the fluid level has a singular characteristic. The lower measured mode is less influenced by the liquid mass and decreases in a uniform way with the fluid level, meanwhile, the higher two modes have a greater dependency and present a stepwise profile.

3 NUMERICAL STUDY

3.1 FE model

Fluid–structure interaction can be modelled using different approaches such as simplified methods with Housner’s two mass representation, (Housner 1963), multi mass representation (Bauer 1964), added mass in a “solid” finite element model (FEM), and design code such as the Eurocode-8, API 650 or ACI 350.3. A comparison and evaluation of these methods are presented by Livaoglu and Dogangun (2006). For exhaustive analysis, more complex models incorporating Lagrangian, Eulerian, and Lagrangian–Eulerian formulations should be used. In this study, a harmonic response analysis from a detailed FE model including the effect of liquid–structure interaction based on a Lagrangian approach is adopted. The spherical shell is modelled by four-node shell elements with six degree of freedom per node and the supporting columns by two-node frame elements with six degree of freedom per node. The eight-node solid fluid element with three degree of freedom per node has been chosen to model the incompressible and inviscid liquid contained in the sphere. The element formulation allows acceleration effects such as sloshing. In order to satisfy the continuity conditions between the fluid and solid shell at the spherical boundary, the “coincident” nodes of the fluid and shell elements are constrained to be coupled in the direction normal to the interface, while relative movements are allowed to occur in the tangential directions. The figure 4 shows the FE model used in the analysis.

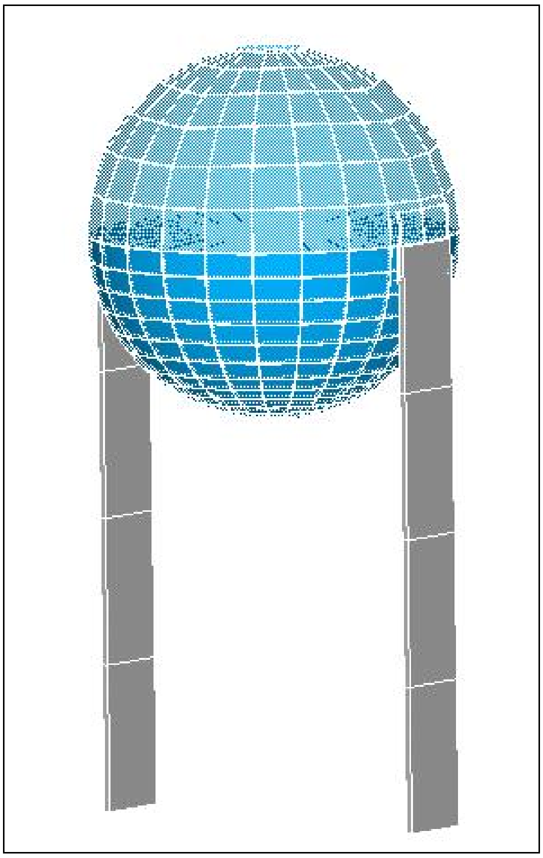


Figure 4. FE model used in the analysis.

3.2 FRF Analysis

In order to obtain the modal shapes and eigenfrequencies, two alternatives are normally used, modal analysis and harmonic response analysis. The former, results in time consuming and computationally expensive analysis due to a lot of strictly liquid modes what make difficult to search the modes where the fluid interacts with the structure; therefore, the latter analysis was adopted. The frequency response function of the fluid-structure system from a detailed FE model was based on the evaluation of the harmonic response analysis in the range of 1-5 Hz with frequency increments of 0.1 Hz. By means of this analysis, only the frequencies and modal shapes excited by a uniaxial horizontal excitation at equator of sphere were determined. The figure 5 shows the peaks that correspond to the resonance eigenfrequencies (local maximum of FRF) of the fluid-structure system for the 8 water surface levels considered.

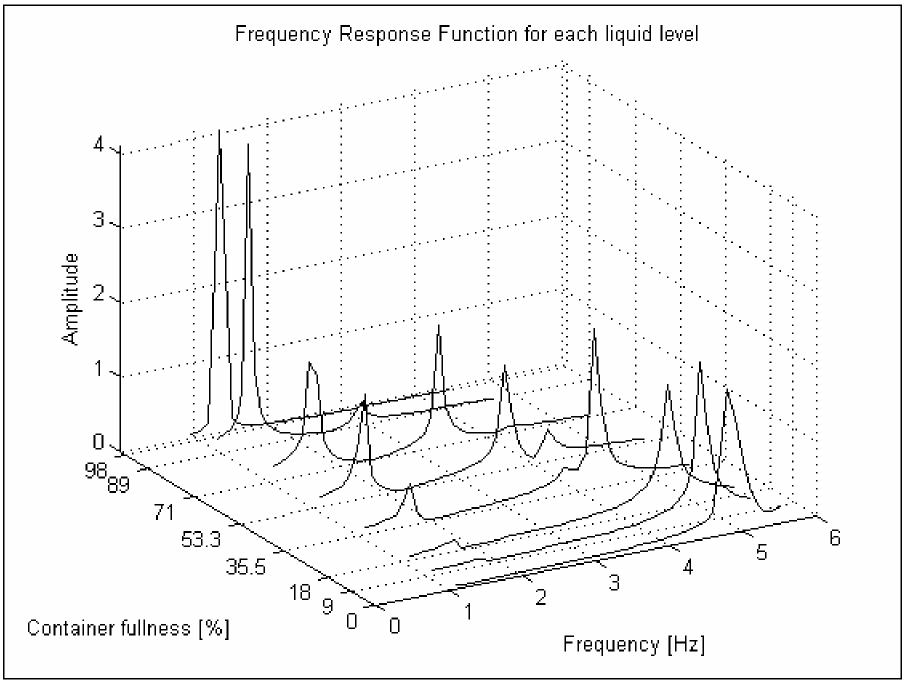
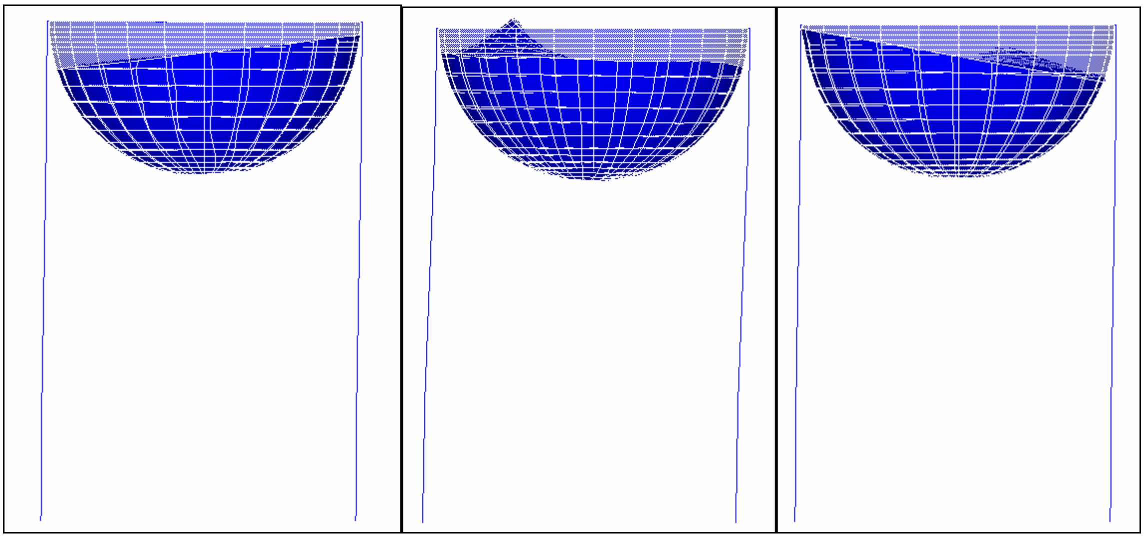


Figure 5. FRF from FE model, for 8 water surface levels of spherical container.

The agreement achieved between the figure 5 (FRF obtained numerically) and figure 2 (Spectrum of the response measured experimentally) is very good.

Figures 6 and 7 show the three modal shapes corresponding to water surface levels of H/R = 0.81 (fullness 35.6%) and H/R= 1.29 (fullness 71.1%), respectively. It seems clear than; in systems as analyzed, in the first two modes of vibration, the sloshing and structure moves in fase and out of fase, respectively, and, in the highest mode, the sloshing out of fase takes a “strange” shape. It is important to note that the corresponding modes for different liquid levels preserve similar profiles.



a) b) c)

Figure 6. Mode shapes for water surface level of H/R = 0.81 (fullness 35.6%); a) 1.7Hz, b) 3.8Hz, c) 4.2 Hz.

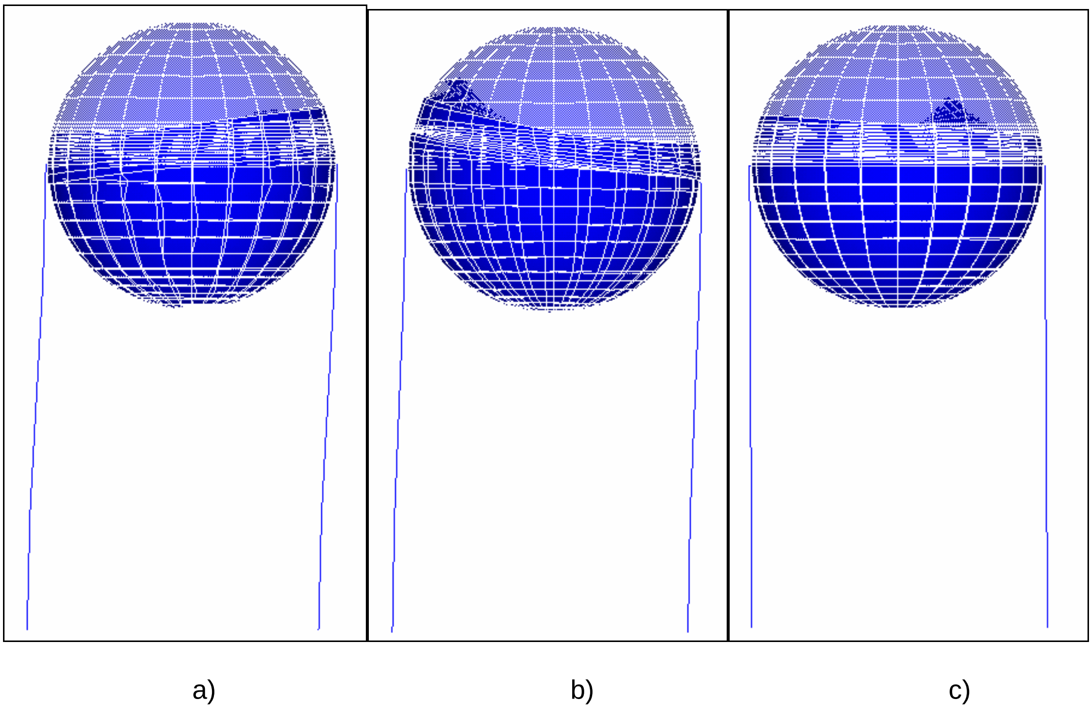


Figure 7. Mode shapes for water surface level of H/R= 1.29 (fullness 71.1%); a) 1.6Hz, b) 3.3Hz, c) 4.15 Hz.

3.3 Sloshing Effects

In simplified analysis, the sloshing displacement of the fluid is sometime ignored. However, as it is demonstrated in this point, the effects of the contained fluid on the overall dynamic structural behaviour are significant and should be not ignored.

In order to investigate the effect of sloshing on the response of this type of containers, two study-cases were analyzed: a) The contained liquid, in this case water, was considered with its true properties (bulk modulus, density and boundary conditions indicated above) and b) The mass of liquid was assumed as a rigid solid block (model without sloshing, but the same boundary conditions). Figures 8a and 8b show, the FRF for both cases, with a container fullness of 53% (liquid level H/R = 1.044). It was found that, if the sloshing effect is ignored some frequencies do not appear.

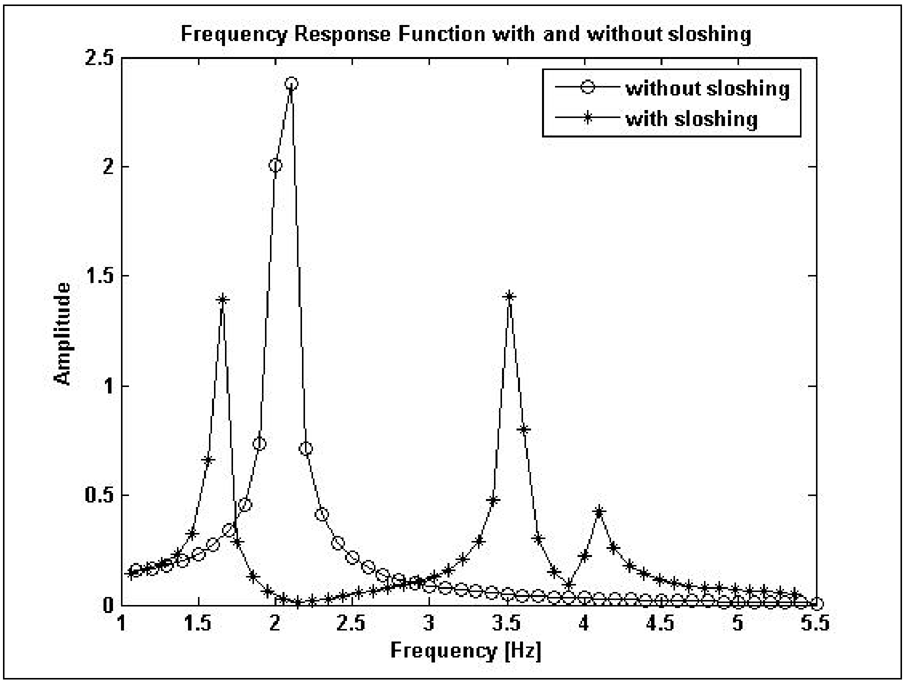


Figure 8. Sloshing effects. Container fullness of 53%.

From figure 8 it is possible to conclude that, although for design purposes a simplified one or two-lumped-mass-model is appropriate (Sezen et al. 2008), for thorough dynamics analysis or for damage assessment, this hypothesis is not valid and models more elaborated are necessary. This statement is confirmed by Livaoglu and Dogangun 2006.

4 COMPARISON OF RESULTS

4.1 Comparison between Experimental and FE model Natural Frequencies

Natural frequencies evaluated from the measured free vibration response for increasing water level H/R are shown and compared in Figure 9 with the corresponding frequencies computed by the FE model. The agreement between the experimental and numerical results is very good.

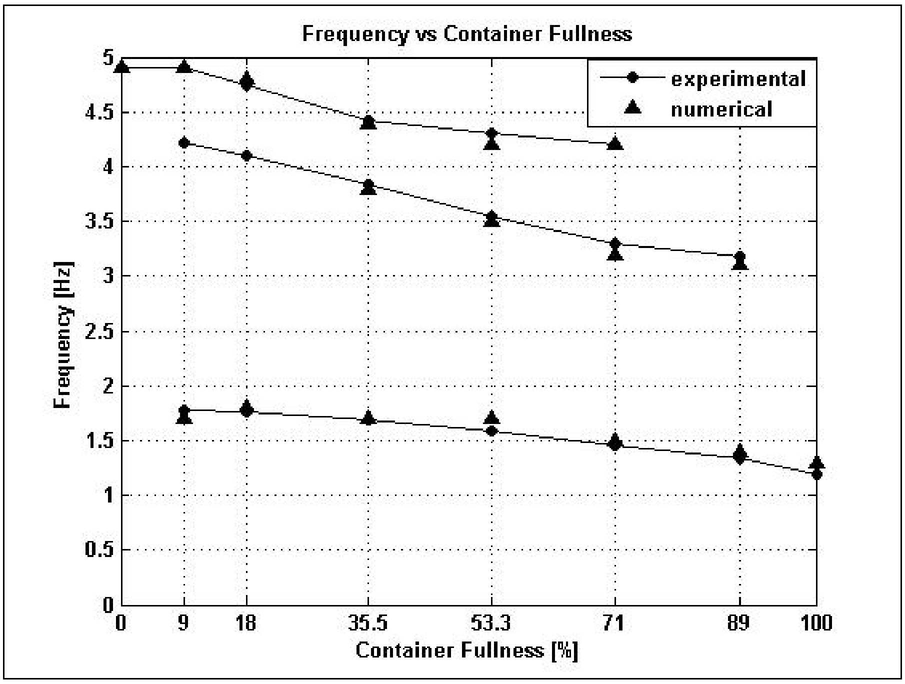


Figure 9. The experimental and computed natural frequencies in the range of 1-5 Hz.

The measured and computed natural frequencies are more exactly compared for local maximum values (peaks) of FRF for each water level in table 2. Insignificant values of FRF (very low peaks) are not indicated. The relative differences included in the table 2 are defined by $d_{ij} = 100\ (f_i - f_j)/f_i$, where "i" correspond to frequencies measured and "j" to frequencies determined by the numerical model. The greatest differences are lesser than 10% and the mean difference is 2.5% (very good agreement).

Fullness [%]	Mode	Frequency [Hz]		
		Experimental	Model	Diff. [%]
0.0	1	-	-	-
	2	-	-	-
	3	4.90	4.90 (4.89)*	0.00 (0.20)
8.9	1	1.78	1.70	-4.60
	2	4.22	-	-
	3	4.90	4.90	0.00
17.8	1	1.76	1.80	2.40
	2	4.10	-	-
	3	4.75	4.80	1.05
35.6	1	1.68	1.70	0.90
	2	3.83	3.80	-0.86
	3	4.42	4.40	-0.45

53.3	1	1.56	1.70	7.25
	2	3.54	3.50	-1.13
	3	4.30	4.20	-2.30
71.1	1	1.45	1.50	3.45
	2	3.30	3.20	-2.90
	3	4.20	4.20	0.00
88.9	1	1.34	1.40	4.24
	2	3.17	3.10	-2.33
	3	-	-	-
100	1	1.19	1.30 (1.16)*	9.24 (2.50)
	2	-	-	-
	3	-	-	-

* Frequencies calculated with equation 1.

Table 2. Comparison between experimental and numerical eigenfrequencies.

Due to the mode shape of the empty and full container cases are of the “shear” type, the fundamental (lowest) natural frequency for those cases, could be easily approximated using the following expression:

$$f_e = \frac{1}{2\pi}\sqrt{\frac{2k_e}{m_e + 0.23m_l}}\sqrt{\left(1 - \frac{P_o}{P_{cr}}\right)} \tag{1}$$

with:

$$k_e = 12\frac{EI}{L^3} \tag{2}$$

In the equation above, fe: fundamental frequency of empty container; k_e: “shear” stiffness of one leg; m_e: empty spherical container mass; m_l: legs mass; EI: flexural stiffness; L: legs length; P_o: leg axial load; P_{cr}: first elastic critical load.

The agreement between analytical and measured fundamental eigenfrequency for empty container is very good as showed in table 2.

5 CONCLUSIONS

The present paper investigates the dynamic response of coupled fluid-structure systems of spherical elevated tanks. First, experimental tests on a model were performed identifying the main fundamentals parameters involved in the physical response and determining the natural frequencies that contribute to the response in the range of 1-5 Hz, for different liquid levels. Next, a numerical model that takes into account the coupling between fluid and structure were developed and validated against the experimental results. A very good agreement was obtained between experimental and numerical results.

As it was expected, the frequencies of partially fluid-filled spherical containers decrease with an increasing fluid level. Due to the interaction between fluid and structure, the dependency of the frequency on the fluid level has a singular characteristic. The lower mode measured is less influenced by the liquid mass and decreases monotonically with the fluid level, meanwhile, the higher two modes have a greater dependency and present a stepwise profile.

The results indicate that sloshing has a significant effect on the dynamics characteristics of the systems; thus, in cases where an accurate dynamics analysis is required, the sloshing should be considered. Moreover, it is clear from the obtained results that, in case of spherical elevated tanks, a two mass model it is insufficient to describe the overall dynamic response. In order to obtain a simple lumped mass model, a minimum of three masses and then three degree of freedom is suggested.

These conclusions help the understanding of the dynamic characteristics variation caused by container shape and fluid level and it is the start point for the development of damage structural assessment methods based on frequency changes.

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