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A SIMULTANEOUS DYNAMIC OPTIMIZATION APPROACH TO ADDRESS RESTORATION POLICIES IN RESERVOIRS

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Abstract. In this work, we develop a dynamic optimization model for a Paso de las Piedras reservoir, which supplies drinking water for more than 400,000 inhabitants and for industrial purposes, to determine restoration policies for algal growth control. A biogeochemical model has been formulated within a simultaneous dynamic optmization framework to solve the optimal control problem whose solution provides limiting nutrient inflow profiles to the lake and in-lake biomanipulation profiles. The water quality model comprises a set of partial differential algebraic equations in time and space. Spatial discretization has been performed in two layers. Simultaneous approaches proceed by discretizing control and state variables by collocation over finite elements and solving the large scale nonlinear program with a successive quadratic programming algorithm. Numerical results provide profiles for both tributary derivation through a wetland and zooplanktivorous fish removal rate.

1 INTRODUCTION

Biogeochemical models provide a representation of major physical, chemical and biological processes that affect the biomass of plankton and nutrients in water bodies. They represent ecological processes through a set of complex nonlinear differential algebraic equations. Significant progress in the development and application of mechanistic lake water quality models has occurred during the last decade (Hamilton and Schladow, 1997; Zhang *et al*., 2004). The main objective in the development of these simulation models is to provide a tool for the proposal of remediation policies to improve water quality.

In this work, we have developed a rigorous eutrophication model within a dynamic optimization framework for the determination of optimal control policies to decrease the eutrophication state of a water body. The objective function is the minimization of the offset between phytoplankton concentration and a desired concentration in the water body along the first year, subject to a large-scale partial differential algebraic equations (PDE) system resulting from temporal and spatial dynamic mass balances in phytoplankton; dissolved oxygen and nutrients. Algebraic equations represent profiles for main forcing functions, as temperature, solar radiation, river inflows, etc. The PDE is transformed into an ordinary differential equation system by spatial discretization (Rodriguez and Diaz, 2007). The DAE optimization problem is then transformed into a large nonlinear programming (NLP) problem by representing state and control variables profiles by polynomial functions over finite elements in time. The solution to the optimal control problem provides the optimal external loading of nutrients through the lake tributaries, with the associated flow rate of the inflows that has to be derived to a nearby wetland for remediation, together with optimal concentrations of zooplankton required to decrease phytoplankton concentration in the lake, associated to fish removal.

The present study has been performed on Paso de las Piedras Reservoir. The discretized NLP problem has been solved with a reduced successive quadratic programming algorithm (Biegler *et al*., 2002).

2 PROCESS DESCRIPTION AND INPUT DATA

Paso de las Piedras reservoir is located in Buenos Aires (Argentina) at 38° 22´ S and 61° 12´ W and it was built to supply drinking water for more than 400,000 inhabitants from Bahía Blanca and Punta Alta and for industrial purposes at a petrochemical complex nearby. The lake has a coastline perimeter of 60 km and a mean depth of 8.2 m. The trophic level of this water body is currently eutrophic and it undergoes algal blooms dominated by potentially toxic cyanobacteria (e.g. *Anabaena circinalis* and *Microcystis aeruginosa*) during warm months (Parodi *et al.*, 2004).

The external forcing functions, such as temperature and solar radiation were approximated with sigmoid functions. River inflows and associated nutrient loading, as well as outflow data have also been approximated with polynomials. The Stream El Divisorio and Sauce Grande River are the two tributaries of the lake (Fig. 1) and runs through the one of the most important agricultural area in Argentina and has a high content of phosphate as result of agricultural and livestock activities. To decrease the external loading of nutrients from El Divisorio Stream, a wetland has been built prior to the lake to retain phosphates and nitrates (Lopez *et al.*, 2007).

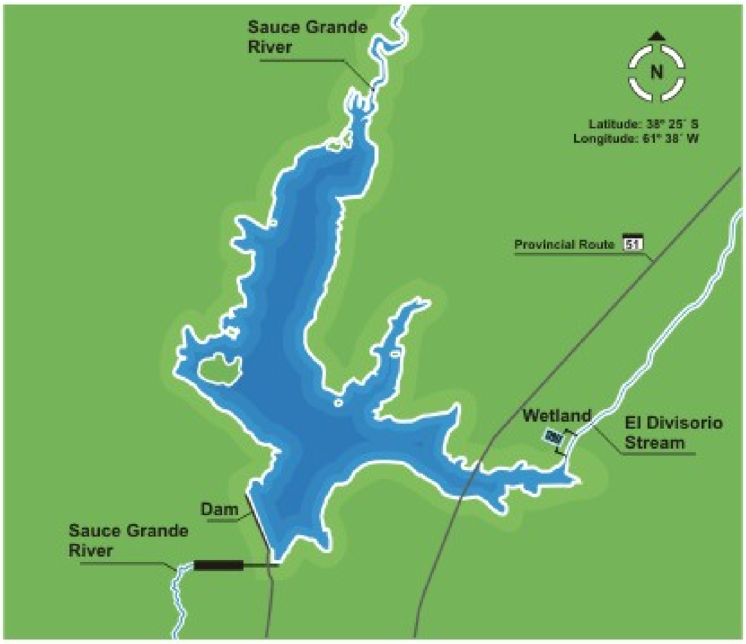

Fig. 1 Paso de las Piedras reservoir indicating the tributaries and the wetland position.

3 DINAMIC OPTIMIZATION PROBLEM

Biogeochemical processes that take place in water bodies can be represented through a set of complex nonlinear partial differential algebraic equations resulting from dynamic mass balances for phytoplankton and nutrients along the water column. The present model is based on horizontally averaged concentrations and its main parameters have been previously estimated (Estrada *et al*., 2007) based on collected data sets of nutrients, phytoplankton, zooplankton, DO, DBO and meteorological ones corresponding to an entire year (Parodi *et al*., 2004). Dynamic mass balances in phytoplankton (in the form of diatoms, green algae and cyanobacteria), dissolved oxygen, biochemical demand of oxygen and nutrients, which include nitrate, ammonium, organic nitrogen, phosphate and organic phosphorus, have been formulated. Biogeochemical processes include growth, respiration, death and grazing processes for phytoplankton, as well as mineralization, nitrification and uptake and release of nutrients. Algebraic equations represent profiles for temperature, solar radiation and river inflows, in addition to the calculation of most factors that affect rate equations, such as effect of solar radiation, nutrients, etc. The resulting partial PDE model is transformed into an ordinary differential equations system by spatially discretization into sets of ordinary differential-algebraic equations (DAE). Available data for the reservoir has allowed discretization in two water layers. We are currently collecting data at eight different levels in the water height to allow for a finer discretization. Equations (1)-(2) represent mass balances for each layer, where subscripts U and L refer to upper and lower layer, respectively. Inflows from tributaries enter the upper layer, as well as the out coming river, while outflows to the potabilization plant and the petrochemical complex flow from the lower layer. The lower layer volume (V_L) has been considered as constant, while total volume variations are reflected in upper layer volume (V_U) and its height (h_U). Equations (3)-(15) show calculation of the generation-consumption term (r_{ij}) for algae and the limiting nutrient (phosphate).

Upper layer

$$\frac{dC_{Uj}}{dt} = \sum_{k=1}^{NIN} \frac{QIN_{U,k}}{V_U} CIN_{Ujk} - \sum_{m=1}^{NOUT} \frac{QOUT_U}{V_U} C_{Uj} + r_{Uj} - \frac{k_d A}{\Delta h_U h_U}(C_{Uj} - C_{Lj}) - \frac{C_{Uj}}{h_U}\frac{dh_U}{dt} \tag{1}$$

Lower layer

$$\frac{dC_{Lj}}{dt} = \sum_{m=1}^{NOUT} \frac{QOUT_L}{V_L} C_{Lj} + r_{Lj} + \frac{k_d A}{\Delta h_L h_L}(C_{Lj} - C_{U,j}) - \frac{C_{Lj}}{h_L}\frac{dh_L}{dt} \tag{2}$$

j= cyanobacteria, diatoms, green algae, nitrate, ammonium, organic nitrogen, phosphate and organic phosphorus, dissolved oxygen, biochemical demand of oxygen, silica.

Rate equations for phytoplankton (r_{ij}; i=upper, lower layer: j= cyanobacteria, diatoms, green algae).

$$r_{ij} = R_{ij,growth} - R_{ij,resp} - R_{ij,death} - R_{ij,sedim} - R_{ij,graz} \tag{3}$$

$$R_{ij,growth} = k_{j,growth} * f(T) * f(I) * f(N) * C_{ij} \tag{4}$$

$$f(T) = \frac{Temp}{Tempj} exp(1 - \frac{Temp}{Tempj}) \tag{5}$$

$$f(I) = \frac{I}{Ij} exp(1 - \frac{Ii}{Ij}) \tag{6}$$

$$f(N) = \frac{C_{iPO4}}{C_{iPO4} + kpj} \tag{7}$$

$$R_{ij,resp} = k_{j,resp} * C_{ij} \tag{8}$$

$$R_{ij,death} = k_{j,death} * C_{ij} \tag{9}$$

$$R_{ij,sedim} = k_{j,sedim} * \frac{1}{h_i} * C_{ij} \tag{10}$$

$$R_{ij,graz} = k_{j,graz} * \frac{C_{ij}}{C_{ij} + K_{graz}} * Zoo_j \tag{11}$$

Rate equations for phosphate (r_{ij}; i=upper, lower layer: j=phosphate)

$$r_{ij} = R_{ij,death} + R_{ij,miner} - R_{ij,uptake} \tag{12}$$

$$R_{ij,death} = \sum_{m=1}^{3} (apc * k_{m,death} * (1 - f_{po}) * C_{im}) \tag{13}$$

$$R_{ij,miner} = k_{miner} * \theta_{miner} * exp(Temp - 20) * \frac{\sum_{m=1}^{3} C_{im} * C_{iOP}}{km_{pc} + \sum_{j=1}^{3} C_{im}} \tag{14}$$

$$R_{ij,uptake} = \sum_{m=1}^{3} (R_{im,growth} * a_{pc} * C_{im}) \tag{15}$$

Additional rate equations have been written for the remaining components. The objective function is the minimization of the offset between total algae concentration in the reservoir and a desired phytoplankton concentration, along a time horizon of one year. This objective can be, in principle, achieved by reducing external loading of nutrients to the lake. An artificial wetland has been built nearby for this purpose and one control variable is the fraction of inlet stream derived to the wetland for nutrient concentration reduction. However, model has shown that external nutrient loading reduction is not enough to achieve an important decrease in phytoplankton concentration throughout one year, due to internal lake mechanisms, chemical and biological, which prevent or delay recovery. An in-lake restoration technique that has been applied in the last decades is the removal of zooplanktivorous fish to reduce the possible top-down control of zooplankton on the phytoplankton (Jeppesen *et al.*, 2005). Therefore, an additional control variable in the present model is zooplankton concentration profile along the time horizon, as a measure of fish removal policies. The dynamic optimization problem has been formulated as:

$$min \int_0^{tf} \left(\sum_{j=phyto} C_j(t) - 0.25 \right)^2 dt \tag{16}$$

st

DAE Eutrophication model

$$0 \le F_{WETLAND} \le 0.5 F_{DIVISORIO} (l/d)$$

$$0.01 \le C_{zoo} \le 5 (mg/l)$$

The dynamic optimization problem, which has been formulated as an index one model, is solved through a simultaneous approach by transforming it into a large-scale nonlinear programming (NLP) problem discretizing state and control variables applying collocation over finite elements (Raghunathan *et al.*, 2004). The discretized NLP model has been solved with a reduced successive quadratic programming algorithm (Biegler *et al.,* 2002).

4 DISCUSSION OF RESULTS

The DAE eutrophication model for Paso de las Piedras reservoir has twenty two differential equations and fifty six algebraic ones, after spatial discretization in two layers. A time horizon of 365 days has been considered. The optimal control problem has two optimization variables corresponding to the fraction of inlet stream that is derived to the wetland and the concentration of zooplankton in the lake to control phytoplankton growth, along the time horizon. The program was first run with only one control variable (profile for stream derived to the wetland) and it rendered an optimal profile equal to the maximum flowrate allowed to remediation in the wetland, which has a global retention of 64% of phosphate (Case 1). The concentration of phosphate in the inflows from El Divisorio has been decreased from 0.09 (average value) to 0.061. However, this decrease in the limiting nutrient loading renders a slight decrease in phosphate concentration in the second half of the year due to internal recycling and an even slighter decrease in phytoplankton concentration. As a second step, the possibility to apply in-lake restoration techniques has been considered by including the concentration of zooplankton as an additional optimization variable (Case 2). This variable can be later associated to an optimal profile for the quantity of zooplanktivorous fish removal. Numerical results show that two important increases in zooplankton concentration are required (Fig. 2), which render an important decrease in phytoplankton peaks corresponding to cyanobacteria and diatoms (from 1.8 to 0.4 and 0.8 to 0.3 mg/l, respectively), as it is shown in Fig. 3. The resulting nonlinear programming (NLP) problem for forty elements and three collocation points has 10342 nonlinear equations and 80 optimization variables. It has been solved with an Interior Point method with reduced Successive Quadratic Programming (rSQP) techniques within program IPOPT (Biegler *et al.*, 2002), in which successive parametric NLP subproblems are solved for decreasing values of the barrier parameter. Initial barrier parameter value has been 0.01.

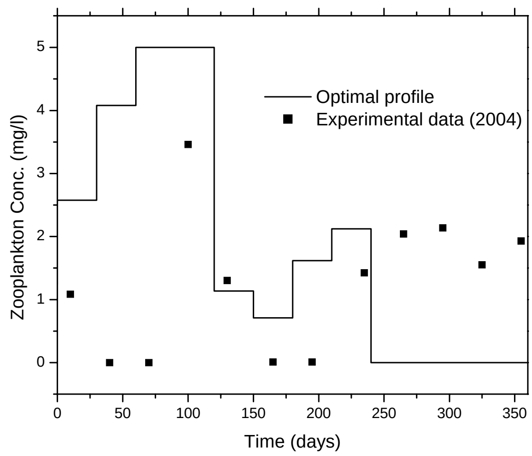

Figure 2. Optimal zooplankton concentration profile for algae growth control.

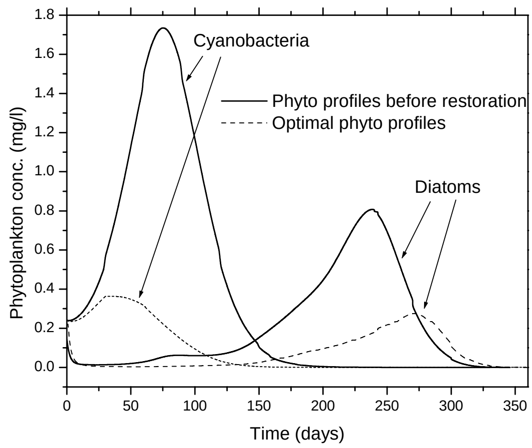

Figure 3. Optimal phytoplankton concentration profiles and profiles before restoration.

5 CONCLUSIONS

A detailed biogeochemical model has been formulated within a simultaneous optimization framework for the determination of optimal restoration policies to improve water quality in a highly eutrophic reservoir. To our knowledge, this is the first time the restoration problem has been formulated as an optimal control case. Numerical results have shown the need for simultaneous external nutrient loading reduction together with in-lake reduction approaches by biomanipulation (fish removal). Optimal control variables profiles have been determined (fraction of inlet stream derived to wetland for nutrient retention and concentration of zooplankton, associated to zooplanktivorous fish removal), as well as spatial and temporal profiles for state and algebraic variables.

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