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SHAPE OPTIMIZATION IN RECTANGULAR ROOMS FOR A CORRECT MODAL DISTRIBUTION AT LOW FREQUENCIES BASED ON PSYCHOACUSTICAL MODEL

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Abstract. Resonances in small rooms may lead to inadequate frequency responses. In rooms where the exigencies on the listening conditions are important, these resonances cause non wanted coloration effects, which implies a non desirable sound quality. Choosing the correct dimensions it is possible to reduce the audible effects of these resonances. The presented methodology aims to determine the form and size of small rectangular rooms based on the finite elements method for modeling the room's physical acoustic behavior; a neural network for loudness estimation and genetic algorithm uses to achieve the optimal dimensions. A comparison with previous techniques used to choose the dimension of rectangular room is also presented.

1 INTRODUCTION

The sound that is listened to in recording studios is the result of the interaction of the system of audio and the acoustics of the room. The pitch answer and the balance in the timbre depend on the geometry and the position of the sonorous sources and the listener. This article is centered in the design of the room in order to diminish the effects of the resonances based on the room dimensions and the human the auditory system response.

The main problems in low frequencies are due to a relative low modal density. Most of the developed solution methods previously developed to solve this problem are directed to rectangular rooms, and they are based on the election of the proportions, positioning of the sources and the use of resonators. The primary target of this article is to present a different perspective from this problem when approaching the solution from the field of psychacoustics instead of the architectural and physical acoustics.

In this aspect the method to present is characterized to search for a room dimensions whose to produce equal loudness in the low frequency band. This means that the sound pressure level must, as far as possible, to agree with some of the equal loudness curves (see Figure 1) curves represent the response of human the auditory system based on the sound pressure and the frequency, giving the sensation of equal sound amplitude (*Zwicker and Fastl, 1990, Suzuki and Takeshima, 2004*).

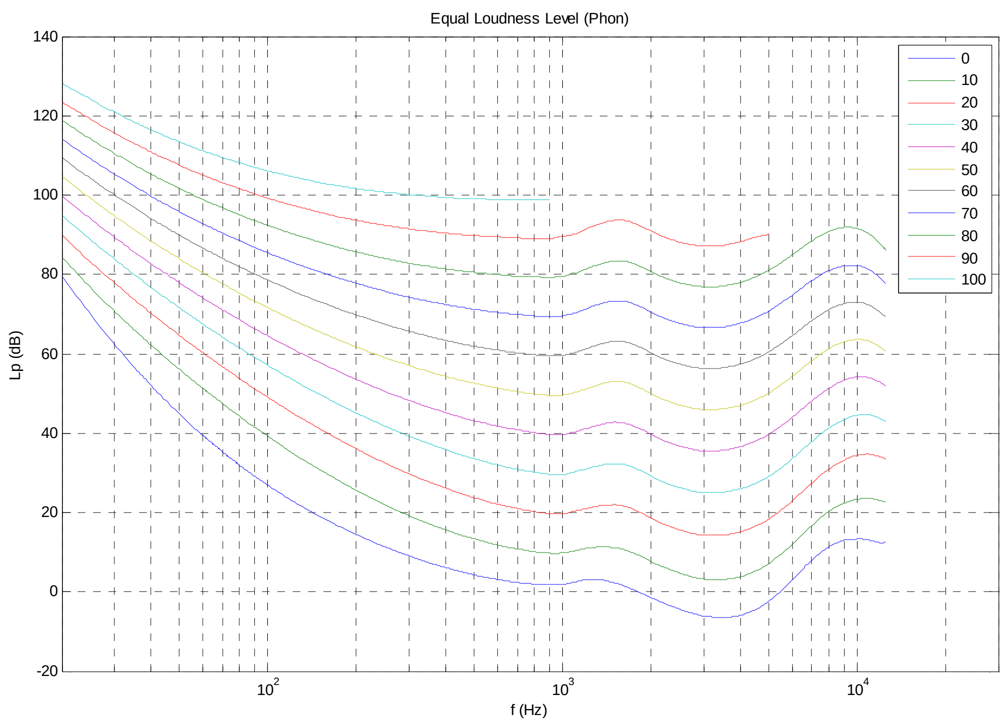

Figure 1 Equal Loudness Level Curves

2 BIBLIOGRAPHICAL ANTECEDENTS

2.1 Previous Works

Most of the methodologies to diminish and to avoid the colorations in the design rooms based on rectangular enclosures and mainly they consist of the election of the proportions adapted between wide and high length. Essentially those methods try to avoid degenerated modes where multiple frequencies of resonance fall one narrow frequency band.

From the equation 1 that determines the frequencies of resonance in a rectangular enclosure have been developed diverse methods to determine the proportions an enclosure

$$f_{n_y n_y n_z} = \frac{c}{2}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right]^{\frac{1}{2}} \tag{1}$$

f_{n_x,n_y,n_z}	:	Natural frequencies
L_x, L_y, L_z	:	Dimensions of the rectangular room
n_x, n_y, n_z	:	Modal numbers
c	:	Speed of sound

Bolt (1946) produced design graphs based on the average distance between resonant frequencies, the known proportions more are 2: 3: 5 and 1: 21/3: 41/3. *Louden (1971)*.developed a set of more exact and preferred proportions based on the standard deviation of space between modes and not the distance average, producing the good well-known radius 1: 1.4: 1:9. *Bonello (1981)* developed a criterion based on the fact that the modal density must never decrease as the frequency increases. These and other methods (*Guilford, 1979, Walker, 1996*) have their limitations, first is that they are applicable to rectangular halls with perfectly reflecting surfaces. The absorption not only influences the amplitude of the sound pressure in the modes, also is responsible of the displacement for the resonance frequencies. Another effect of the absorption is that it acts of different form if the ways are axial, tangential or oblique. *Cox, D'Antonio and Avis (2004)* developed a new methodology using optimization techniques for rectangular enclosures, using the method of images and approximated factors of reflection, being able to flatten the frequency response of the room by the minization of the quadratic difference between the frequency response and the straight line of obtained linear regression. They found the following optimal dimensions, 1: 2.19: 3 and 1: 1.55: 1.85.

2.2 Description of the New Method

The methodology presented in this article consists of a modification of the work of Cox et. al. using shape optimization in order to determine the dimensions of the room in such a way that it does not have the flattest response possible, if not that their frequency response is isophonically flat, that means that the frequency response of the enclosure must simulate the way in which the auditory system perceives the sound, this will be made within the frequency band between 20 Hz until 200 Hz. In this first stage of the work will not consider the sonorous absorption of the surfaces of the enclosure and it will be only restricted to rectangular enclosures.

The shape optimization consists in causing geometric or structural changes in order to obtain a wished response of the structure that is being designed. That set of modifications must be restricted in order to satisfy other requirements and with which the changes are physically attainable. In this case the characteristics of the room are modeled like a multidimensional function called objective function, function of cost or fitness function, that depends on the design variables; also a region is due to establish search that characterizes the restrictions. In such way it looks to minimize this function in order that it produces the optimal answer according to the design parameters.

3 MATHEMATICAL MODEL OF A RECTANGULAR ROOM

The enclosure that is desired to optimize is rectangular, for this reason is not necessary to create the problem in three dimensions, similarly has chosen the finite elements method to be able to extrapolate this methodology to rooms with arbitrary shape.

3.1 Formulation of the Problem and Application of the Method of Separation of Variables

The enclosure rectangular and is excited by a point source of flat spectrum. This can be represented using the following equation partial differential and its respective boundary conditions. In order to simplify the problem the stationary solution in the frequency domain will be only studied. The wave equation and the boundary conditions are:

$$\begin{aligned} \nabla^2 p &= \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \\ \nabla p \cdot \hat{n} &= 0 \end{aligned} \tag{2}$$

p	:	Sound pressure
c	:	Sound speed
$\bar{u}$	:	Particle velocity
$\hat{n}$	:	Normal vector outward the room

When considering an harmonic solution we obtain the Helmholtz's equation.

$$\nabla^2 P + k^2 P = 0 \tag{3}$$

Using the method of by separation of variables we have the following equations and boundary condition:

$$P(x,y,z) = P_{xy}(x,y)P_z(z) \tag{4}$$

For the dependency in z

$$\begin{aligned} &\frac{\partial^2 P_z(z)}{\partial z^2} + k_z^2 P_z(z) = 0 \\ &\left(\frac{\partial P_z(z)}{\partial z}\right)_{z=0} = \left(\frac{\partial P_z(z)}{\partial z}\right)_{z=L_z} = 0 \end{aligned} \tag{5}$$

For the dependency in x, y

$$\frac{\partial^2 P_{xy}(x,y)}{\partial x^2}+\frac{\partial^2 P_{xy}(x,y)}{\partial y^2}+k_{xy}^2=0$$
$$\nabla P_{xy}(x,y)\cdot\hat{n}=0 \tag{6}$$

Is due to fulfill that:

$$k^2=k_{xy}^2+k_z^2 \tag{7}$$

k	:	Wave number.
k_{xy}	:	Wave number in the plane x, y .
k_z	:	Wave number in the axis z .

3.2 Application of the Method of Finite Elements in the Solution of the Equations

The equation and the boundary condition (5) have a widely known solution (*Fahy and Walker, 1998*). While the equation (11) and its respective boundary condition (12) can be solved using the Finite Elements Method (*Zienkiewicz and Taylor, 1991*). Specifically it is possible to interpret these equations like a membrane with of Neumann boundary conditions. This leaves us to a system of equations of ordinary differentials to us and to their corresponding eigenvalue problem.

$$\mathbf{M}\ddot{\mathbf{p}}_{xy}+\mathbf{K}\mathbf{p}_{xy}=\mathbf{0} \tag{8}$$

$$\mathbf{K\Phi}=\mathbf{M\Phi}\Lambda \tag{9}$$

$$\mathbf{\Phi}=[\boldsymbol{\varphi}_1,\boldsymbol{\varphi}_2,\boldsymbol{\varphi}_3,\ldots,\boldsymbol{\varphi}_N] \tag{10}$$

$$\Lambda=diag\left(k_{xy1}^2,k_{xy2}^2 k_{xy3}^2,\ldots,k_{xyN}^2\right) \tag{11}$$

k_{xy}^2	:	Eigenvalue (Wave number).
$\mathbf{p}_{xy}$	:	Sound pressure.
$\boldsymbol{\varphi}$	:	Eigenvector.
$\mathbf{\Phi}$	:	Modal matrix.
$\mathbf{\Lambda}$	:	Eigenvalue matrix.
$\mathbf{M}$	:	Mass matrix.
$\mathbf{K}$	:	Stiffness matrix.

Then the natural frequencies can be calculated as:

$$\omega_{n_{xy}n_z}=c\sqrt{k_{xy}^2+k_z^2} \tag{12}$$

Finally the sound pressure for any point $\vec{\mathbf{r}}$ within the given enclosure a source located in $\vec{\mathbf{r}}_0$ at the frequency ω, is the result of the combination of the solutions of the equations 5 y 6.

$$p(\vec{\mathbf{r}},\vec{\mathbf{r}}_0,\omega)=\sum_{n_{xy}}^{\infty}\sum_{n_z}^{\infty}\frac{A_{n_{xy}n_z}(\vec{r},\vec{r}_0,\omega)}{\omega^2-\omega_{n_{xy}n_z}^2} \tag{13}$$

$$A_{n_{xy}n_z}(\vec{\mathbf{r}},\vec{\mathbf{r}}_0,\omega)=jU_0\rho_0c^2\omega\left(\phi_{r,n_{xy}}\cos(k_z z)\right)\left(\phi_{r_0,n_{xy}}\cos(k_z z_0)\right) \tag{14}$$

Finally, ρ_0 and U_0 are the density of the air and the vibratory speed of the surface of the source.

3.3 Determination of the Loudness Levels Using Neuronal Networks

The loudness may be defined as the sensation that corresponds most closely to the sound intensity of a stimulus (*Zwicker and Fastl, 1990*). An equal-loudness contour is a curve that ties up sound pressure levels having equal loudness as a function of frequency. In other words, it expresses a frequency characteristic of loudness sensation.

In this work a loudness model, implemented by using artificial neural network, has been developed from the equal-loudness-level contours data presented in (*Suzuki and Takeshima, 2004*) and following the procedure employed by *Espinoza, Venegas and. Floody (2006)*. The presented model aims to perform an accurate loudness calculation in low frequencies; this objective is different to the objective of the model presented previously which is a loudness model for wide frequency range.

The artificial neural network (*Gupta, Jin and Homma, 2003*) was trained by using the Quasi Newton Backpropagation algorithm with 3000 epochs and an objective goal of 10^{-5}. The final configuration corresponds to a three layer feedforward neural network with 5 neurons in the hidden layer and 1 neuron in the output. The transfer function of the hidden layer is sigmoidal hyperbolic tangent and for the output layer is a linear function. The inputs to the neural network are the frequency and sound pressure level and the output is the respective loudness level.

4 OPTIMIZATION USING GENETIC ALGORITHM

The optimization techniques are used to determine the best possible design in engineering problems. In our case it will be used to determine the optimal shape of a small and/or medium rectangular enclosure, in order to obtain the best psychoacustical response. The propose fitness function corresponds to the variance of the loudness level response versus the frequency.

$$f(\vec{\mathbf{x}}) = \frac{1}{N}\sum_{n=1}^{N}\left(L_L(n) - \overline{L_L}\right)^2 \qquad (15)$$

$$\overline{L_L} = \frac{1}{N}\sum_{n=1}^{N} L_L(n) \qquad (16)$$

$L_L(n)$	:	Loudness level in the nth frequency.
$\overline{L_L}$	:	Loudness level average.
$\vec{\mathbf{x}}$	:	Vector of the variables of design that contains the length, wide and the height.

The optimization problem that sets out this characterized by a strong nonlinear interrelation between the variables and the fitness function, also the function has many peaks and dips which makes the solution over sensitive to the dimensions of the enclosure. For this reason, the curve of frequency response has been smoothed and the method of Genetic Algorithm has been chosen (*Goldberg 1989*), which has demonstrated efficient in to be varied problems of acoustics and vibrations (*Keane, 1995, Ratle and Berry, 1998*). These methods work maintaining a population of the competent designs that are combined to find improved solutions. In its basic form each member of the population is represented by a binary sequence that codifies the variables that characterize the design. The search progresses manipulating the sequences in the population to provide in the new generations of better characteristic designs in average, than its processes of predecessors los processes that are used to look for these improved designs imitate those of the natural selection; that is the reason of Genetic Algorithm's name.

5 NUMERICAL SIMULATIONS

5.1 General Considerations

The limits of the optimization in the frequency range is from 20Hz to 200Hz , although this range can be extended, it is due to remember that as increases in the frequency, the exactitude of the model is smaller and than is in this range where the resonances are more annoying. It will be hoped that the main results of the optimization process will consist in the avoiding of degenerated modes. The points of emission and reception are the opposite corners and it will not be tried in this work to optimize position of the sound sources and the listener position. The source has been characterized pint like and of constant speed. The dimensional limits are: $4m \le Lx \le 8m$, $7m \le Ly \le 15m$ and $3m \le Lz \le 5m$. The population size is 15 and the number of generations is 200.

The comparisons with rectangular rooms are made for enclosures that have equal height and the wide and length proposed in this article are obtained through the proposed relations of optimal proportionality (*Bolt, 1946, Louden, 1971, and Cox et. al., 2004*) Also the result will be comparated using the objective function proposed by *Cox et. al., (2004)* (see Figure 2) and the present objective function. (see Figure 3) The advantages of the proposed function in this work by on the developed ones previously are:

- To represent the response of the human auditory system and its interaction with the sound field.
- When diminishing the variance of the loudness level is able to diminish the fluctuations of the resonances when approaching to the sector of the medium frequencies.
- To increase the loudness, therefore the sound pressure level in the sector of the lowest frequencies.

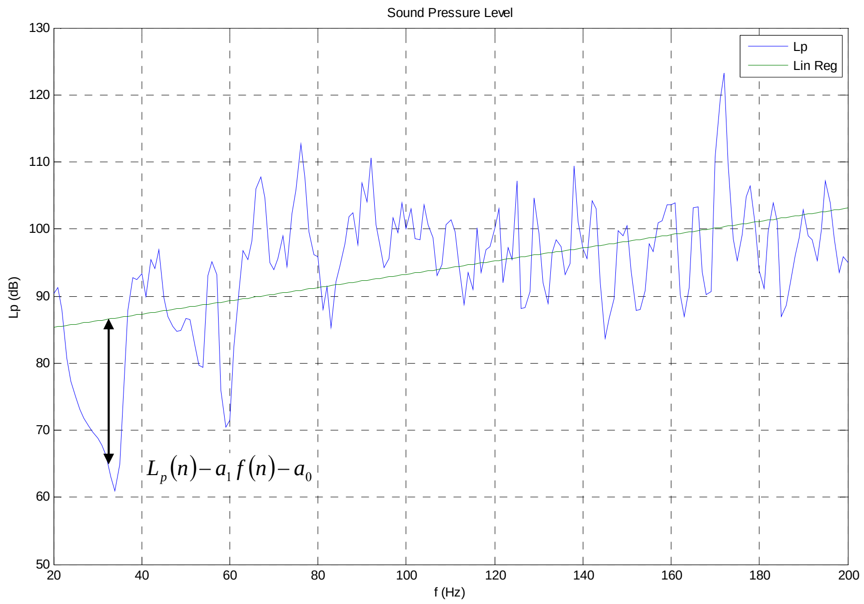

Figure 2 Fitness Function *Cox et. al., (2004)*.

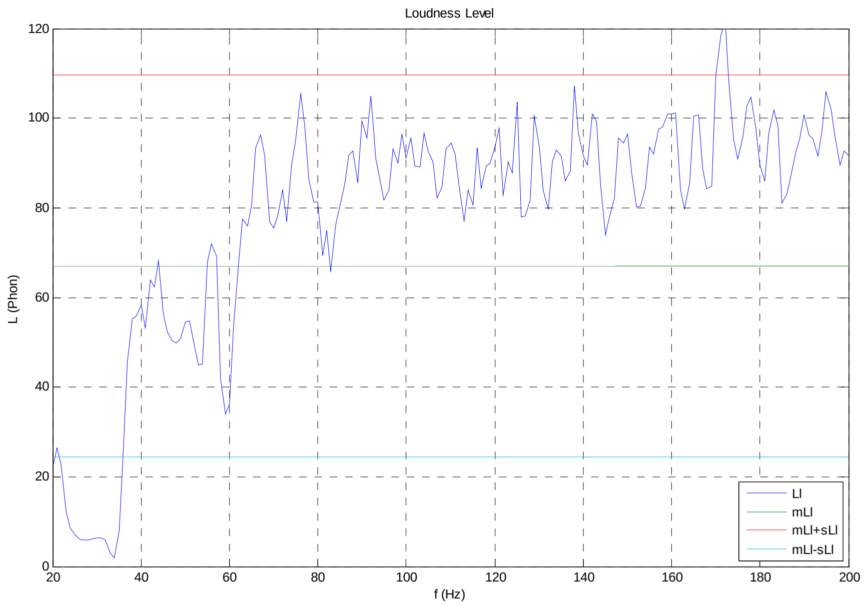

Figure 3 Proposed Fitness Function

5.2 Results

In the Table 1 are the values of the optimal dimensions of the rectangular enclosure obtained by this work. In the Table 2 are the values of the functions of fitness for the different methodologies based on proportions versus the optimal value obtained with this method, being observed a much better performance to the being evaluated with the objective function of this work and the proposed function by *Cox et. al. (2004)*.

Dimension	(m)
Length	13.131
Wide	5.643
Height	3.031

Table 1: Optimal Dimensions of the Rectangular Room.

	$f(\vec{\mathbf{x}})$ Floody - Venegas	$f(\vec{\mathbf{x}})$ Cox et. al.
Bolt (2 : 3 : 5)	1778.652	90.530
Bolt (1: 7/3 : 13/3)	2013.067	65.790
Louden (1 : 1.4: 1.9)	1810.915	63.225
Cox et. al.(1 : 2.19 : 3)	1798.234	70.691
Cox et. al.(1:1.55:1.85)	2159.300	68.902
Optimal Room	1530.137	44.008

Table 2: Values of the Fitness Function.

The Figure 4, Figure 5, Figure 6, Figure 7, Figure 8, Figure 9, Figure 10, Figure 11, Figure 12 and Figure 13 show the differences between the present method and the classic proportions

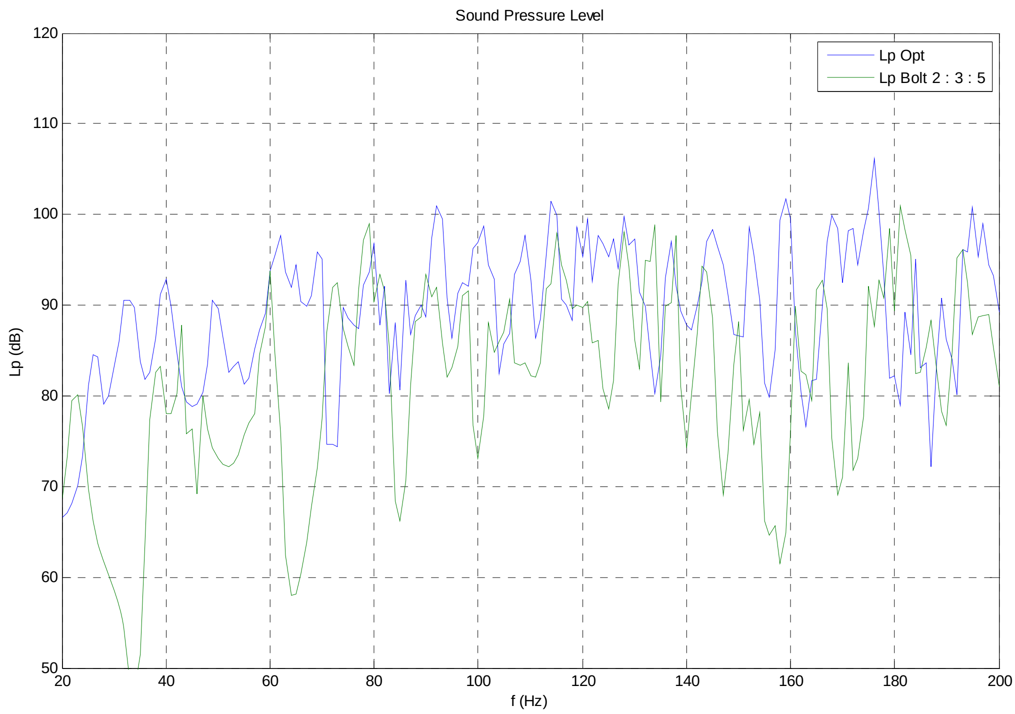

Figure 4: Comparison of the Distribution of the Sound Pressure Level in the Frequency - Optimal Room – *Bolt (1946)* Room 2 : 3 : 5.

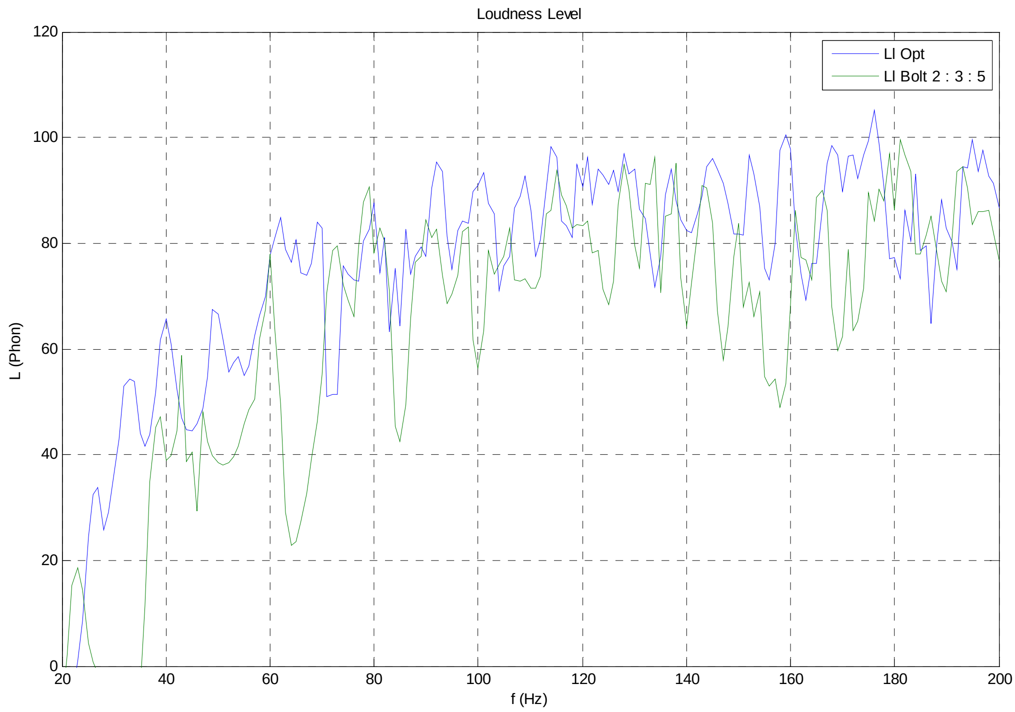

Figure 5: Comparison of the Distribution of the Loudness Level in the Frequency - Optimal Room – *Bolt (1946)* Room 2 : 3 : 5.

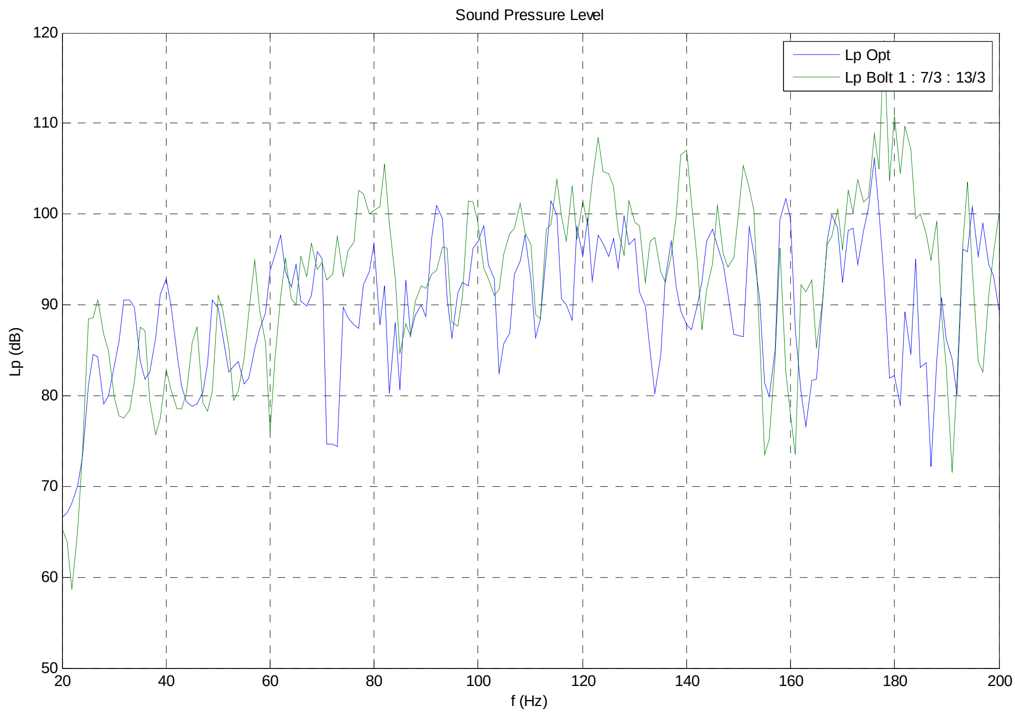

Figure 6: Comparison of the Distribution of the Sound Pressure Level in the Frequency - Optimal Room *Bolt (1946)* Room 1 : 7/3 : 13/3.

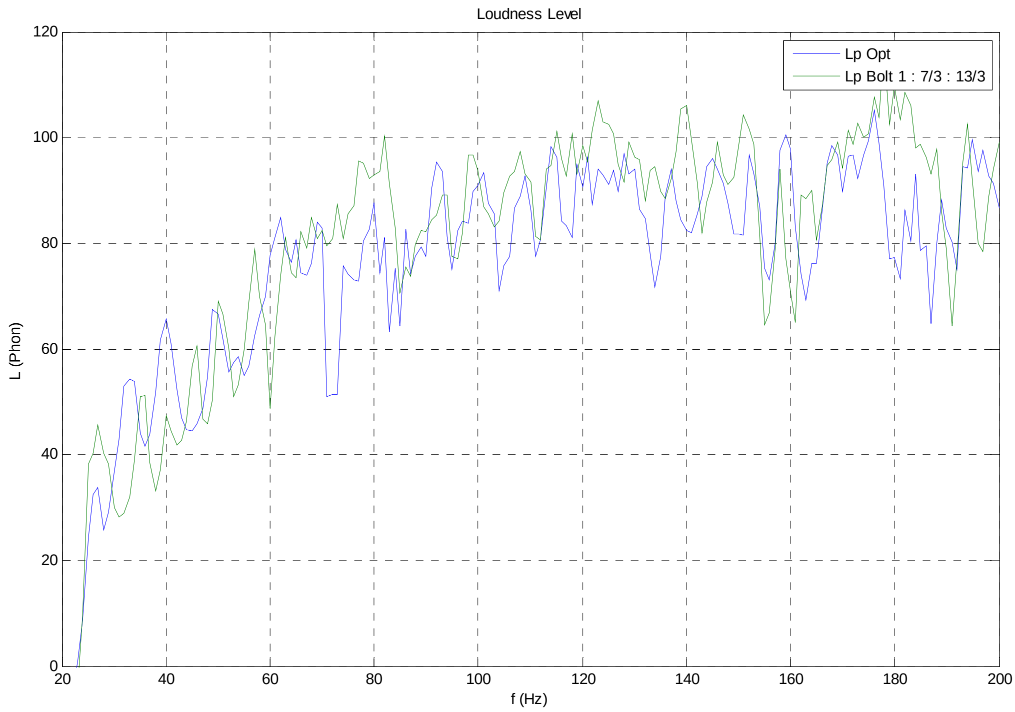

Figure 7: Comparison of the Distribution of the Loudness Level in the Frequency - Optimal Room - *Bolt (1946)* Room 1 : 7/3 : 13/3.

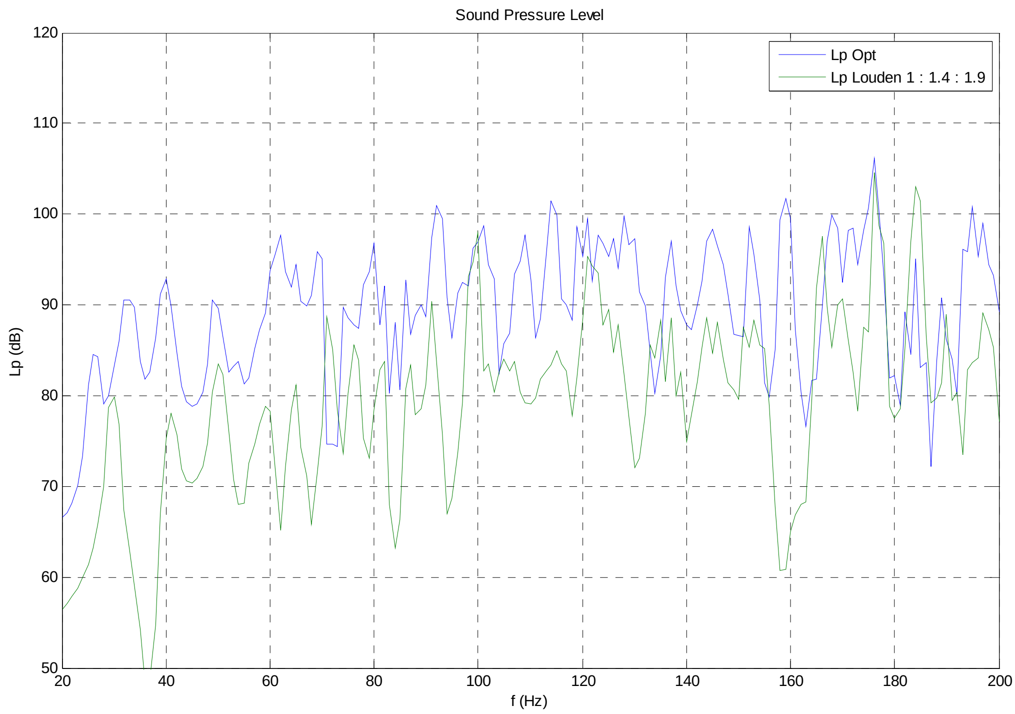

Figure 8: Comparison of the Distribution of the Sound Pressure Level in the Frequency - Optimal Room *Louden (1971)* Room 1 : 1.4 : 1.9.

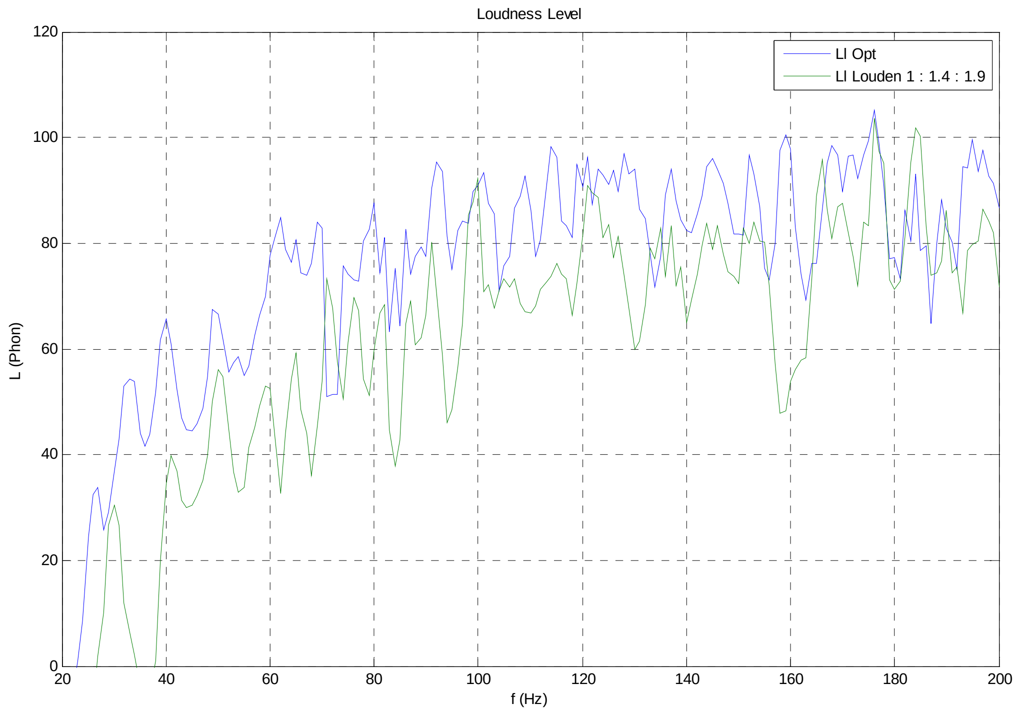

Figure 9: Comparison of the Distribution of the Loudness Level in the Frequency - Optimal Room – *Louden (1971)* Room 1 : 1.4 : 1.9.

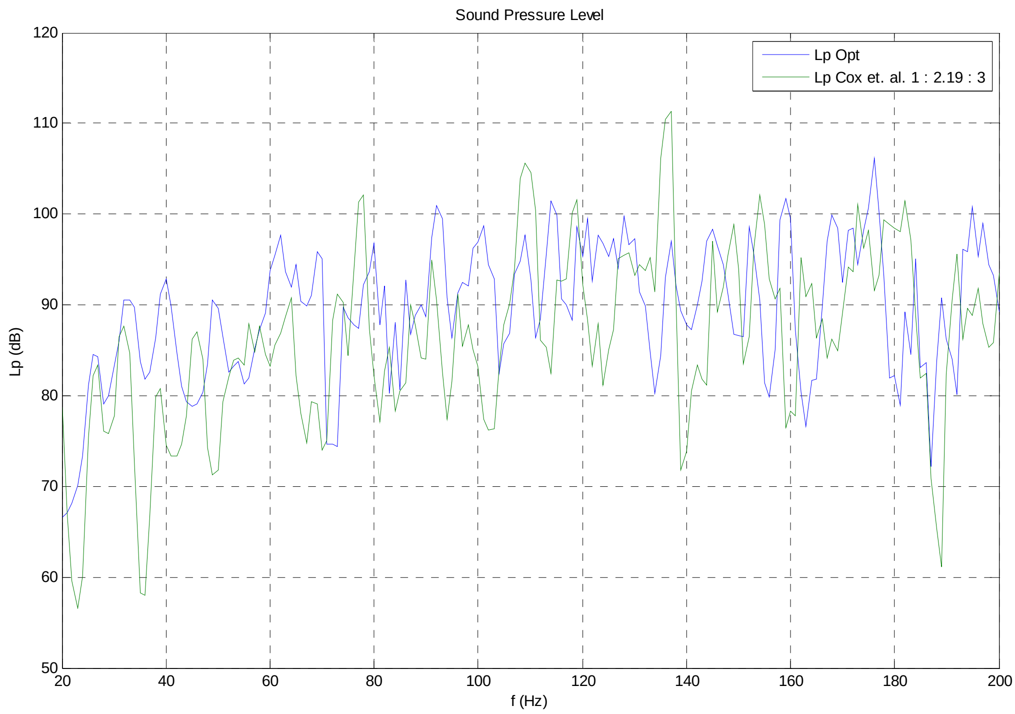

Figure 10: Comparison of the Distribution of the Sound Pressure Level in the Frequency - Optimal Room - *Cox et. al. (2004)* Room 1 : 2.19 : 3.

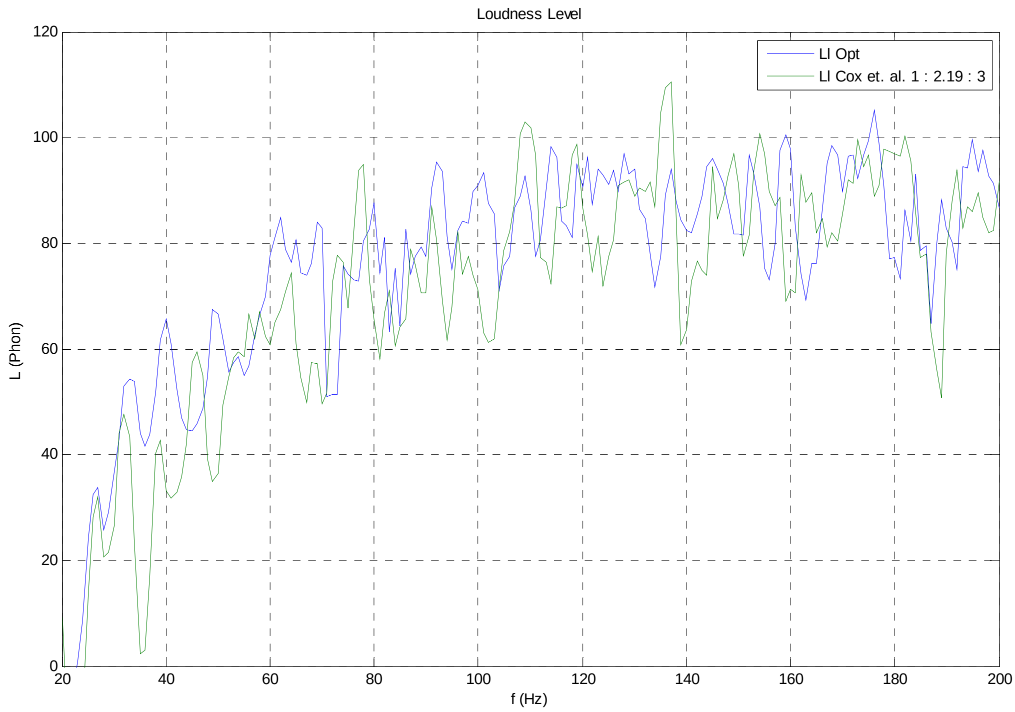

Figure 11: Comparison of the Distribution of the Loudness Level in the Frequency - Optimal Room - *Cox et. al. (2004)* Room 1 : 2.19 : 3.

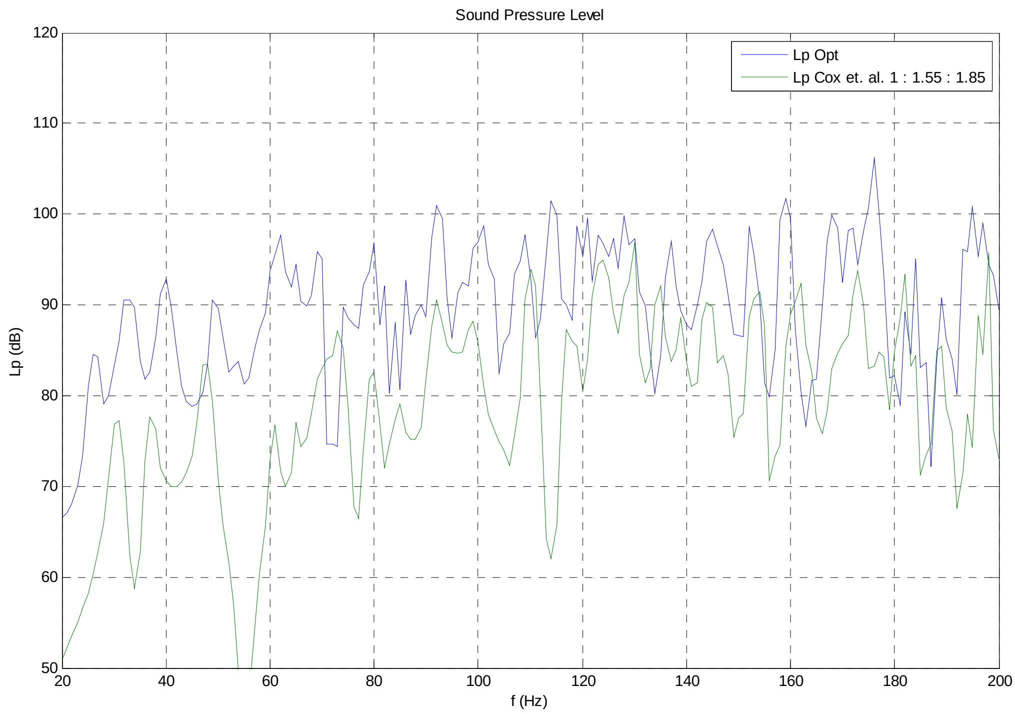

Figure 12: Comparison of the Distribution of the Sound Pressure Level in the Frequency - Optimal Room - *Cox et. al. (2004)* Room 1 : 1.55 : 1.85.

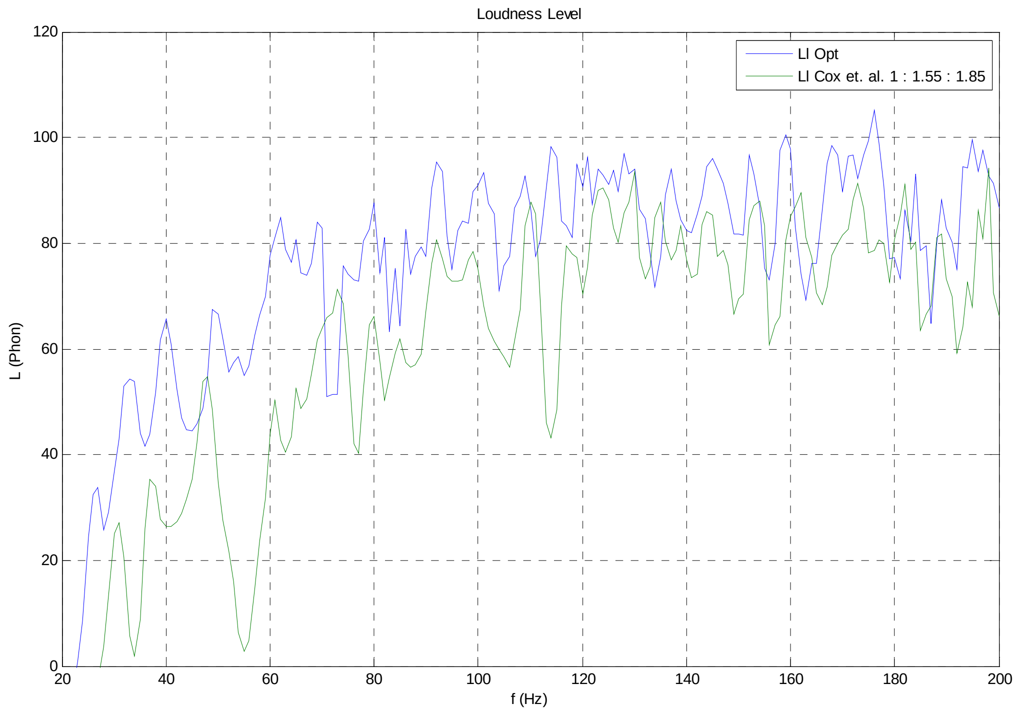

Figure 13: Comparison of the Distribution of the Loudness Level in the Frequency - Optimal Room - *Cox et. al. (2004)* Room 1 : 1.55 : 1.85.

6 CONCLUSIONS

We can see that the proposed method fulfills the objectives drawn up obtaining a better yield than the recommendations of proportions of length, wide and high found in literature.

The main reason is that many of these criteria were constructed on the basis of proportions, is to say that the height of the enclosure was equaled to the unit; although the structure of the phantom would have to stay when it goes to the real dimensions, not always this structure is completes in the frequency band of interest.

In addition the optimization to the three dimensions considers to the height of independent form to the wide one and the length, for this reason will not recommend proportions, because each problem of optimization depends on the space search imposed by the restrictions. On the other hand the criterion of equal loudness proposed in this work is much more demanding that the one of flat frequency response.

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