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FAIPA_MDO – A Feasible Arc Interior Point Algorithm for Multidisciplinary Design Optimization

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Abstract. We present a new Numerical Optimization Algorithm for MDO based on FAIPA. FAIPA_MDO employs a Mathematical Programming Model that works with linking variables and equality constraints to introduce the interaction between disciplines. The state variables of each discipline are considered unknowns of the Mathematical Program and the state equations are included as additional equality constraints. Then, the state equations are solved at the same time as the optimization problem. FAIPA_MDO interacts with the Simulation Codes corresponding to each discipline at each of the iterations of the optimization process.

1 INTRODUCTION

Multidisciplinary Design Optimization, MDO, "can be described as a methodology for design of complex engineering systems that are governed by mutually interacting physical phenomena and made up of distinct interacting subsystems"[1].

Modern design techniques require numerical models of each of the parts of the system and each of the interacting physical phenomena. These models were generally developed independently, as well as the simulation codes based on them. From a practical point of view, to be successful, MDO must be based on existing codes, as they are. It is not reasonable to ask engineers and scientists working in the different disciplines to modify their mathematical and numerical models and the corresponding computer codes to adapt them to MDO.

MDO problems are naturally very large. They normally deal with a large number of design variables and include the state variables and constraints coming from all disciplines as well as the interaction between disciplines. Several techniques were developed to overcome this difficulty. Most of them try to decompose the problem into smaller sub problems or to work with reduced models for analysis and/or optimization.

FAIPA_MDO is a numerical optimization algorithm for MDO that works with a model that considers the complete problem without reductions, decompositions or simplifications. This goal is very ambitious due to the size and complexity of the problems, but it can be a way to obtain strong and efficient tools for MDO.

This model works with linking variables and equality constraints to introduce the interaction between disciplines. That is, the objective function and the constraints depend exclusively of the design variables. In the classical approach for Design Optimization, the state equations that represent the systems to be designed are solved at each of the iterations of the optimization process. Our approach also admits the Simultaneous Analysis and Optimal Design technique (SAND), which consists on adding the state variables to the design variables and including the state equation as additional equality constraints. Then, the state equation is solved at the same time as the optimization problem.

FAIPA_MDO is based on FAIPA, the Feasible Arc Interior Point Algorithm[2,3]. It takes advantage of the particular structure of MDO problems and allows classical or SAND optimization. The present algorithm includes several techniques for very large size problems as an extension for constrained optimization of Limited Memory quasi–Newton method and iterative numerical techniques for the solution of the internal systems of FAIPA. In the case of SAND Optimization the size of the problem is reduced by a formulation that requires the solution of the tangent equilibrium equation. The internal solvers of Engineering Analysis codes can be employed to do these computations.

2 A MODEL FOR MULTIDISCIPLINARY OPTIMIZATION

Let us consider the Optimal Design of Engineering Systems represented by a State Equation $e(x,u)=0$, where $e \in R^r$. The equation depends on the parameters $x \in R^n$, that we call design variables, being $u \in R^r$ the state variables**.** In most applications of Structural Optimization, the State Variables are the nodal displacements and the State Equation is given by the Equilibrium.

The classical model for this problem can be represented by the Nonlinear Program

$$\left.\begin{aligned} &\underset{x}{\text{Minimize}} \quad f(x,u(x)) \\ &\text{subject to} \quad g(x,u(x)) \le 0 \\ &\qquad\text{and} \quad h(x,u(x)) = 0, \end{aligned}\right\} \tag{1}$$

where $u(x)$ solves the state equation for x, f is the Objective Function, $g \in R^m$ and $h \in R^p$ are the sets of inequality and equality constraints respectively. We assume that f, g and h are continuous, as well as their first derivatives.

The problem (1) is solved iteratively. At each of the iterations, the state equation must be solved and the sensitivity of the state variables must be computed. If the solution of the state is iterative, the whole process can be very painful. This is the case of Nonlinear Structures Optimization.

The Simultaneous Analysis and Optimal Design (SAND) method consists on adding the state variables to the design variables and including the state equation as additional equality constraints. Then, the state equation is solved at the same time as the optimization problem. This is very advantageous in the case of nonlinear systems but, on the other hand, the size of the Mathematical Program is greatly increased. The Nonlinear Program for SAND Optimization is stated as follows:

$$\left.\begin{aligned} &\underset{x,u}{\text{Minimize}} \quad f(x,u) \\ &\text{subject to} \quad g(x,u) \le 0 \\ &\qquad\qquad h(x,u) = 0 \\ &\text{and} \qquad e(x,u) = 0. \end{aligned}\right\} \tag{2}$$

We consider now the MDO of an Engineering System integrated by “ne” subsystems and/or disciplines. The State Variables of the subsystems are $u=(u_1, u_2,\ldots,u_{ne})$.. We define a vector of linking variables “z” that represent the physical interactions between all the disciplines. The State Equations of the disciplines can then be represented by $e_1(x,z,u_1)$, $e_2(x,z,u_2)$, $e_3(x,z,u_3)$,…, $e_{ne}(x,z,u_{ne})$. We introduce the equality constraints $h(x,z,u)=0$ that impose compatibility of the interactions between disciplines.

As an example, we consider the case of Structural and Aerodynamics Multidisciplinary Optimization of airplanes and their components. The aerodynamics efforts acting on the structure can be included in "z" as linking variables. These efforts are computed by the aerodynamic analysis code as functions of the aerodynamics state variables "u_{ad}" that represent velocities. In the present model, the equality constraints h(x,z,u)=0 impose that at the Optimal Design the efforts computed by the aerodynamic analysis are the same as the efforts, included in "z", employed as input of the structural analysis

Then, the MDO problem can be formulated as follows:

$$\left.\begin{array}{ll} \underset{x,z,u}{\text{Minimize}} & f(x,z,u) \\ \text{subject to} & g(x,z,u) \leq 0 \\ & h(x,z,u) = 0 \\ & e_1(x,z,u) = 0 \\ & e_2(x,z,u) = 0 \\ & \dots\dots\dots\dots\dots\dots \\ \text{and} & e_{ne}(x,z,u) = 0. \end{array}\right\} \tag{3}$$

FAIPA_MDO solves Muldisciplinary Optimization Problems formulated as in (3). In the next section a view on FAIPA is given.

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3 FAIPA – THE FEASIBLE ARC INTERIOR POINT ALGORITHM

FAIPA makes iterations on the primal and dual variables of the optimization problem to solve Karush - Kuhn - Tucker optimality conditions. Given an initial interior point, it defines a sequence of interior points with the objective monotonically reduced. At each point, a feasible descent arc is obtained and an inexact line search is done along this one.

At each of these iterations, to compute a feasible arc, FAIPA solves three linear systems with the same matrix. There is classical a quasi-Newton version of FAIPA and also a Limited Memory quasi-Newton algorithm[4]. FAIPA_SAND is an algorithm for Simultaneous Analysis and Optimization that solves the problem in a very efficient way and takes advantage of numerical tools normally included in Engineering Analysis software.

To describe the computations of FAIPA, we consider the basic nonlinear optimization problem with equality and inequality constraints:

$$\left.\begin{array}{rl} \underset{x}{\text{Minimize}} & f(x) \\ \text{subject to} & g(x) \leq 0 \\ \text{and} & h(x) = 0, \end{array}\right\} \tag{4}$$

with f, g and h defined as above.

FAIPA computes at each of the iterations a descent direction d_0 by solving

$$\begin{bmatrix} B & \nabla g(x) & \nabla h(x) \\ \Lambda\nabla g(x) & G(x) & 0 \\ \nabla h(x) & 0 & 0 \end{bmatrix}\begin{bmatrix} d_0 \\ \lambda_0 \\ \mu_0 \end{bmatrix} = -\begin{bmatrix} \nabla f(x) \\ 0 \\ h(x) \end{bmatrix} \tag{5}$$

where $B \in R^{nxn}$ is a quasi-Newton matrix, λ_0 and μ_0 are estimates of the Lagrange multipliers and G(x)=diag[g(x)] and $\Lambda = \mathrm{diag}(\lambda)$ are diagonal matrices. The linear systems come from a primal dual iteration so solve Karush-Kuhn-Tucker optimality conditions.
The following system with the same matrix gives a "Centering Direction" d_1:

$$\begin{bmatrix} B & \nabla g(x) & \nabla h(x) \\ \Lambda\nabla g(x) & G(x) & 0 \\ \nabla h(x) & 0 & 0 \end{bmatrix}\begin{bmatrix} d_1 \\ \lambda_1 \\ \mu_1 \end{bmatrix} = -\begin{bmatrix} 0 \\ \lambda \\ \mu \end{bmatrix} \tag{6}$$

In Herskovits(1998)[3] it was proved than taking $\rho > (\alpha - 1)d_o^t\nabla f(x) / d_1^t\nabla f(x)$, with $\alpha \in (0,1)$, then $d = d_d + \rho d_1$ is a "Feasible Descent Direction".
Another centering direction that compensates the curvature of the constraints is given by

$$\begin{bmatrix} B & \nabla g(x) & \nabla h(x) \\ \Lambda\nabla g(x) & G(x) & 0 \\ \nabla h(x) & 0 & 0 \end{bmatrix}\begin{bmatrix} \tilde{d} \\ \tilde{\lambda} \\ \tilde{\mu} \end{bmatrix} = -\begin{bmatrix} 0 \\ \lambda \tilde{\omega}^I \\ \mu \tilde{\omega}^E \end{bmatrix} \tag{7}$$

where

$$\tilde{\omega}_i^I = g_i(x + d) - g_i(x) - \nabla g_i^t(x)d,$$
$$\tilde{\omega}_i^E = h_i(x + td) - h_i(x) - \nabla h_i^t(x)d$$

are estimates of the second derivatives of the constraints on the direction d.

Once computed d, d_0 and $\tilde{d}$ a line search is performed along the Feasible Arc:

$$x(t) = x + td + t^2\tilde{d}$$

to find a new feasible point with a lower cost. . The Feasible Arc is represented in Fig. 1.

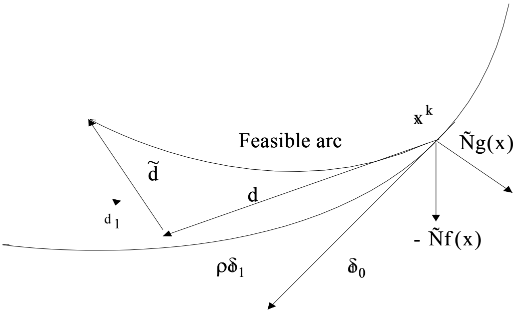

Fig.1- The Feasible Arc

Solving the linear systems (5-7), even if their matrix is the same, require the heaviest internal computations of **FAIPA**. The size of these systems is equal to the number of variables plus the total number of constraints. **FAIPA** includes several techniques to solve them^5. When the classical quasi – Newton method is employed, B is a full matrix. However **FAIPA** can work with limited memory quasi – Newton method avoiding the storage of B. In this the number of equations are equal to the number of constraints only.

In the case when **FAIPA** is applied to solve the SAND Problem (2), the size of the internal linear systems is much increased since the number of degrees of freedom of the state equation is generally much more larger that the number of design variables.
FAIPA_SAND reduces the size of the systems by eliminating the state variables and the state equations. This elimination requires the solution of linear systems obtained by derivation of the state equation. The complete formulation of **FAIPA_SAND** can be found in Herskovits(2001)^6.

4 THE FAIPA_MDO ALGORITHM

FAIPA_MDO extends FAIPA_SAND to the case of the MDO Problem (3) when several state equations corresponding to the subsystems are solved simultaneously with the optimal design problem.

Given initial values for the Design Variables “x”, the State Variables “u_i” corresponding to all the subsystems and the Linking Variables “z”, FAIPA_MDO generates a sequence converging to an optimal point that verifies the State Equations and the Compatibility Equations. The initial point only needs to verify the inequality constraints.

FAIPA_MDO solves at each iteration linear systems with the matrix

$$M \equiv \begin{bmatrix} \bar{B} & [\nabla_{x,z} g(x,z,u) - Du^t \nabla_u g(x,z,u)] & [\nabla_{x,z} h(x,z,u) - Du^t \nabla_u h(x,z,u)] \\ \Lambda[\nabla^t_{x,z} g(x,z,u) - \nabla^t_u g(x,z,u) Du] & G(x,z,u) & 0 \\ [\nabla^t_{x,z} h(x,z,u) - \nabla^t_u h(x,z,u) Du] & 0 & 0 \end{bmatrix} ($$

8)

where $\bar{B} \in R^{n \times n}$ is a reduced quasi – Newton matrix computed as in Herskovits(1996)[7] ,

$$\delta u = \left[\nabla^t_u e(x,z,u)\right]^{-1} e(x,z,u) \qquad (9)$$

and

$$Du = \left[\nabla^t_u e(x,z,u)\right]^{-1} \nabla_{x,z} e^t(x,z,u)\,. \qquad (10)$$

Since the State Equations are uncoupled, to compute δu and Du the following systems have to be solved:

$$\left[\nabla^t_{u_i} e(x,z,u_i)\right] \delta u_i = e(x,z,u_i) \ \text{ for } i = 1,2,\ldots,ne \qquad (11)$$

and

$$\left[\nabla^t_{u_i} e(x,z,u_i)\right] Du_i = \nabla_{x,z} e^t(x,z,u_i) \ \text{ for } \ i = 1,2,\ldots,ne. \qquad (12)$$

The systems (11) and (12) can be solved by employing the Engineering Analysis code corresponding to each discipline. We remark that $\left[\nabla_u \, e^t(x,z,u_i)\right]$ is the so-called Tangent Matrix and that the size of the reduced matrix M is the same as the matrix of system (5), corresponding to Problem (2).

A simplified flow diagram of **FAIPA_MDO** is represented in Figure 2. The step-length is defined by an inexact line search iterative procedure[7]. Each of the iterations requires the calculus of all the functions and derivatives of equality constraints.

The stopping criterion of the main algorithm includes verification, with a given tolerance, of Karush – Kuhn – Tucker optimality conditions and state equations of all the subsystems.

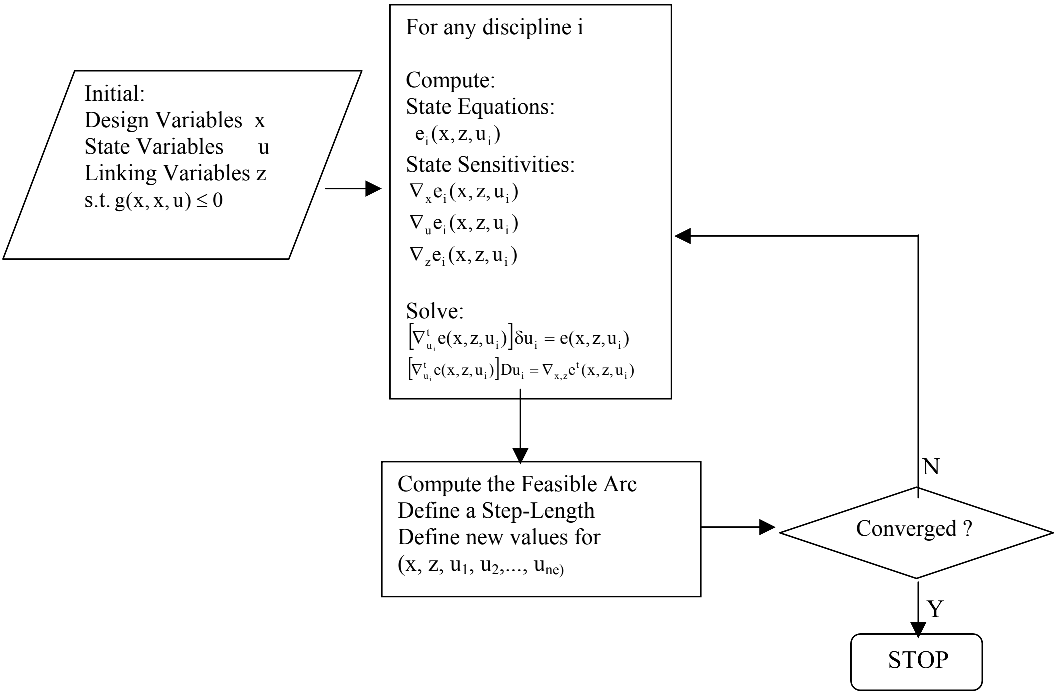

Fig. 2 – Flowchart of **FAIPA_MDO**

5 CONCLUSIONS

FAIPA_MDO is based on the Mathematical Program (3) that represents exactly the aim of Multidisciplinary Design Optimization and it is proved global and superlinear convergence of **FAIPA** for this kind of Nonlinear Optimization Problems. The required computer effort is very competitive. When the state equations are not linear, it is not necessary to solve them at each of the iterations, but merely a linear system with the tangent matrix.

FAIPA_MDO interacts very easily with existing Engineering Analysis techniques and codes. In consequence, with **FAIPA_MDO** is very easy to integrate the Engineering Teams working in the Analysis and Design of the interacting disciplines and subsystems.

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