Fluid-structure interaction with a staged algorithm

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Multiphysics

- **Complex coupled problems involving many fields. Some coupling not *a priori* known.**
- **Interaction of acoustics with flexible structures.**
- **Magneto-Hydrodynamics devices**
- **Micro-Electro-Mechanical devices**
- **Thermo-Mechanical problems, like continuous casting.**
- **Fluid-Structure interaction (wing flutter, flow induced vibrations...).**

This article is concerned with the numerical integration of this type of problems when they are coupled in a *loose* or *strong* manner.

Monolithic vs. Partitioned

- **For simple structural problems with few vibrational degrees of freedom it is possible to combine the fluid and the structure in a single formulation. Then the full system can be integrated with a explicit or implicit scheme. These *"monolithic"* methods can be very robust but are in general not modular and parallel efficiency is difficult to reach.**
- **An efficient alternative is to solve each subproblem in a partitioned procedure where time and space discretization methods could be different. Such a scheme simplifies explicit/implicit integration and it is in favor of the use of different codes specialized on each sub-area. In this work a staggered fluid-structure coupling algorithm is considered.**

Basic (weakly coupled) partitioned FSI algorithm

- **(i) transfer the motion of the wet boundary of the solid to the fluid problem,**
- **(ii) update the position of the fluid boundary and the bulk fluid mesh accordingly,**
- **(iii) advance the fluid system and compute new pressures (and the stress field if it is necessary),**
- **(iv) convert the new fluid pressure (and stress field) into a structural load, and**
- **(v) advance the structural system under the flow loads.**

Staged (strongly coupled) partitioned FSI algorithm

States: u **structure,** w **fluid,** X **mesh.**

```
Initialize variables:
for n = 0 to n_step do {Main time step loop}
    t^n = nΔt,
    X^n = CMD(u^n) {run CMD code}
    u^{(n+1)P} = u^{(n+1,0)} = predictor(u^n, u^{n-1}) { compute predictor }
    for i = 0 to n_stage do {stage loop}
        X^{n+1,i+1} = CMD(u^{n+1,i})
        w^{n+1,i+1} = CFD(w^n, X^{n+1,i+1}, X^n) {Fluid solver CFD}
        compute structural loads (w^n, w^{n+1,i+1})
        u^{n+1,i+1} = CSD(u^n, w^n, w^{n+1,i+1}) {Structure solver CSD}
    end for
end for
```

Stability, weakly vs. strong (staged)

- **Stage loop is a fixed point iteration to the monolithic (strong coupled) integration, so that if the stage loop is iterated and converged the algorithm has the stability properties of the monolithic one. ($\Delta t_{\mathrm{crit,staged}} \gg \Delta t_{\mathrm{crit,weak}}$)**
- **However, time step may be limited by convergence of the stage loop, i.e. it may happen that for a given Δt the fixed point stage loop does not converge.**
- **Computational cost is increased by the number of *stages*.**

This is an ongoing research. So far, we have no analytic results about stability (estimations for critical time step).

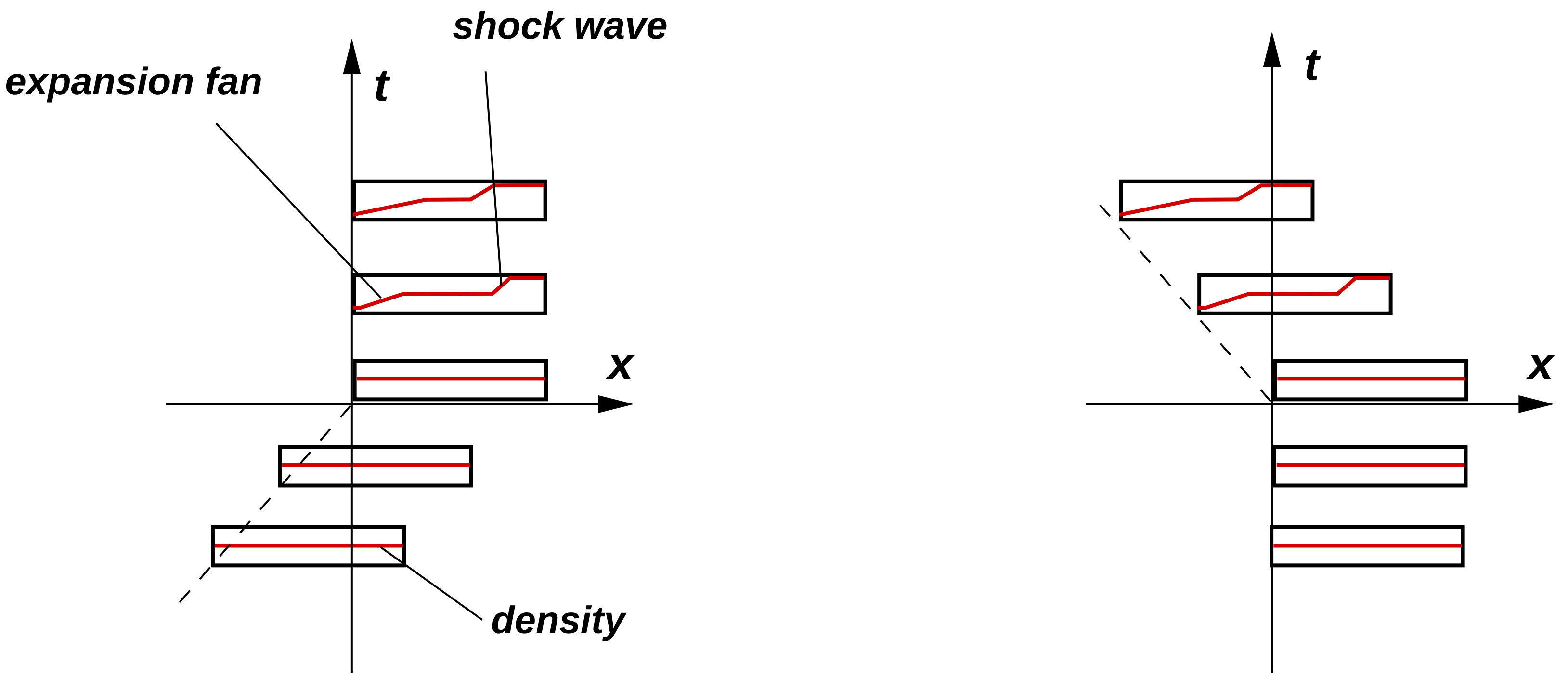
ALE invariance

A key point in fluid-structure interaction problems is the use of the *"Arbitrary Lagrangian Eulerian formulation"* (ALE) , which allows the use of moving meshes. As the ALE convective terms affect the advective terms, some modifications are needed to the standard stabilization terms in order to get the correct amount of stabilization. Also boundary conditions at walls (slip or non-slip) and absorbing boundary conditions must be modified when ALE is used.

ALE invariance (cont.)

- **Discrete equations are not invariant under an arbitrary Galilean transformation, mainly because the importance of the advective terms are relative to the frame of reference.**
- **For instance, a fluid which is at rest in frame S does not need stabilization, whereas in a frame S' with relative velocity $\mathbf{v}$ it may have a high Pèclet number and then it will need stabilization.**
- **However, when using ALE formulations with moving domains, stabilization is based on the velocity of the *fluid relative to the mesh*. With this additional degree of freedom introduced with moving meshes a physical problem can be posed in different Galilean frames and in such a way that the velocity of the fluid *relative to the mesh is the same*. Then the question can be posed of whether discrete stabilized equations give the same solution (after appropriate transformation laws) in these equivalent situations. If the scheme is not invariant then great chances exist that the scheme adds more diffusion in one frame than in other, and then to be unstable or too diffusive. If the discrete formulation pass the test we say that it is *"ALE invariant"*.**

ALE invariance test case. Sudden stop of gas container



container initially moving at constant speed is suddenly stopped

container initially at rest is suddenly put in movement with constant negative speed $-u_0$

ALE invariance. SUPG Stabilization term

$$\frac{\partial \mathbf{U}_\mathrm{c}}{\partial t} + \frac{\partial \mathcal{F}_{c,x}}{\partial x} = \frac{\partial \mathcal{F}_{d,x}}{\partial x}; \qquad \text{(gov. eqs. in cons. form)}$$

$$C\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{K}\frac{\partial^2 \mathbf{U}}{\partial x^2}; \qquad \text{(gov. eqs. in quasi-linear form)} \qquad (1)$$

Sufficient conditions for ALE invariance ($\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{v}_\mathrm{mesh}\mathbf{C}$)

$$P = \nabla N \cdot \tilde{\mathbf{A}}\boldsymbol{\tau}\mathbf{C}^{-1}; \qquad \text{(SUPG pert. function)}$$

$$\boldsymbol{\tau} \ \text{ transform as } \mathbf{U}\times\mathbf{U}, \quad \left(\text{i.e. } \boldsymbol{\tau}' = \frac{\partial \mathbf{U}'}{\partial \mathbf{U}}\boldsymbol{\tau}\frac{\partial \mathbf{U}}{\partial \mathbf{U}'}\right). \qquad (2)$$

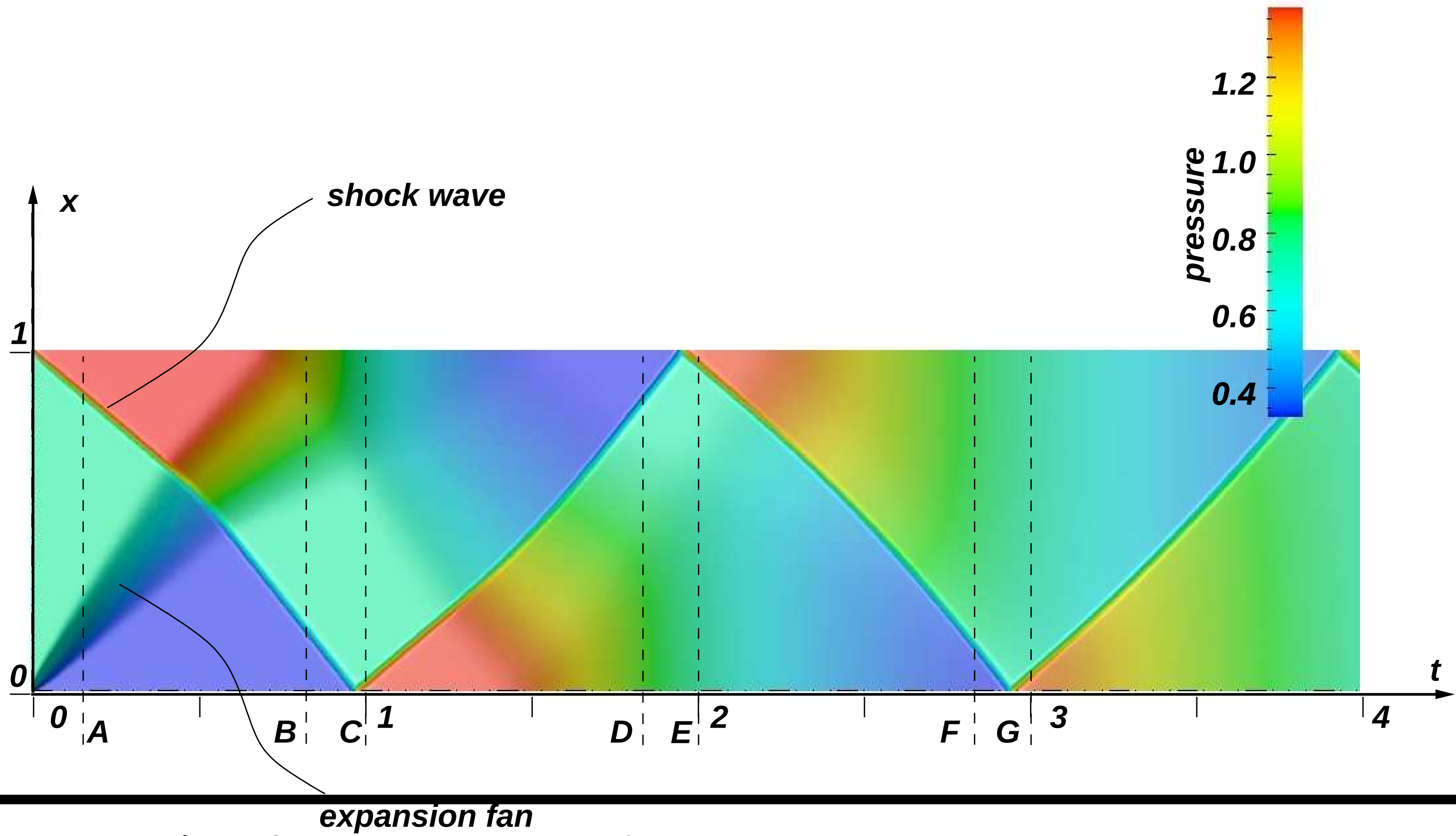
This last is verified if $\boldsymbol{\tau}$ is $f(\mathbf{C}^{-1}\tilde{\mathbf{A}})$, for instance (inviscid case):

$$\boldsymbol{\tau} = \frac{h}{\max|\lambda_j|}\mathbf{I}, \quad \lambda_j = \mathrm{eig}(\mathbf{C}^{-1}\tilde{\mathbf{A}}), \qquad \text{(max. eigenv.)}$$

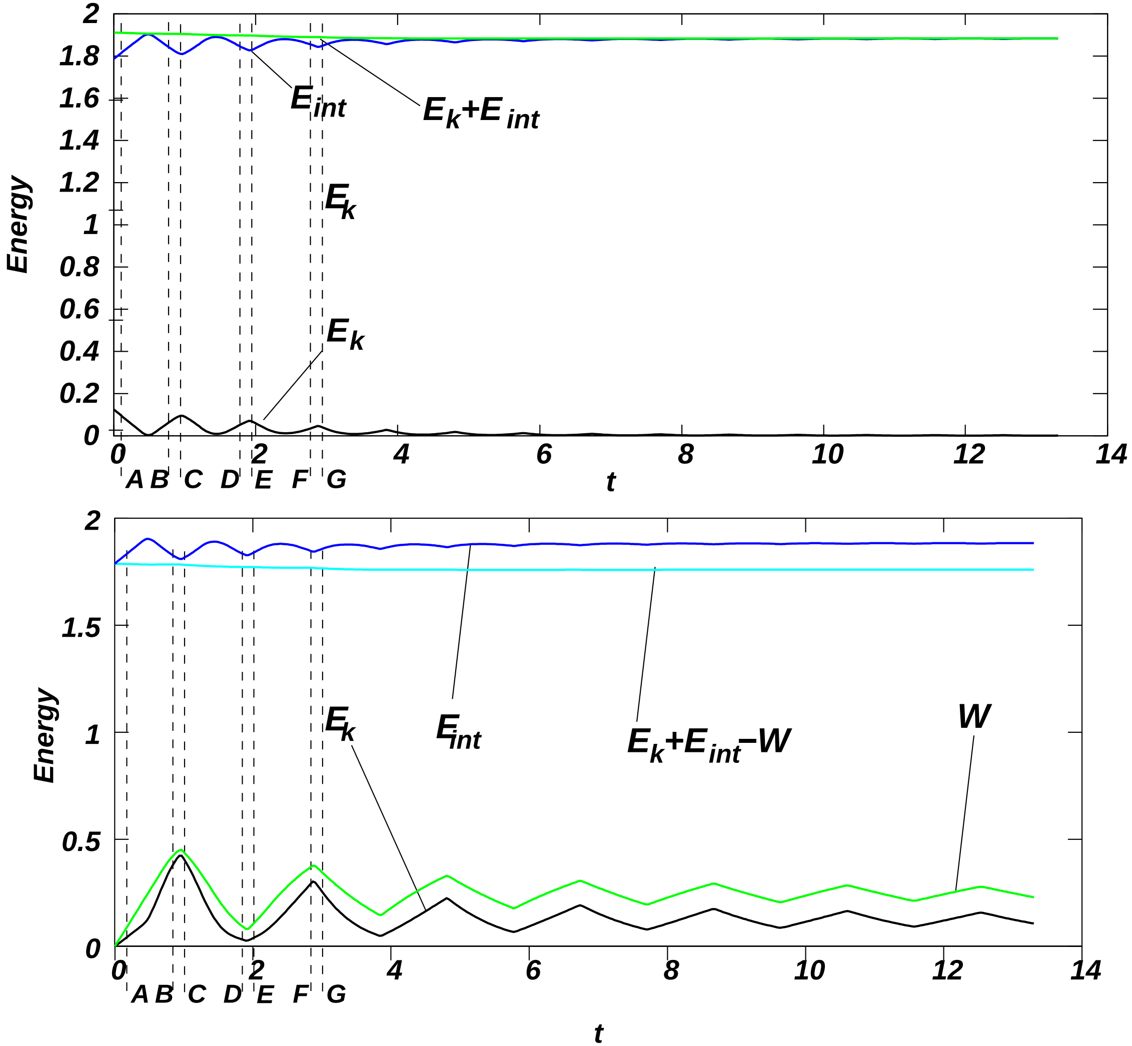
$$\boldsymbol{\tau} = h|\mathbf{C}^{-1}\tilde{\mathbf{A}}|^{-1}. \qquad \text{($|\cdot|$ in matrix sense)} \qquad (3)$$

ALE invariance. Sudden stop of a gas container

$\gamma = 1.4$, $u_0/c_0 = 0.5$. **Results in both reference systems are equivalent to machine precision.**



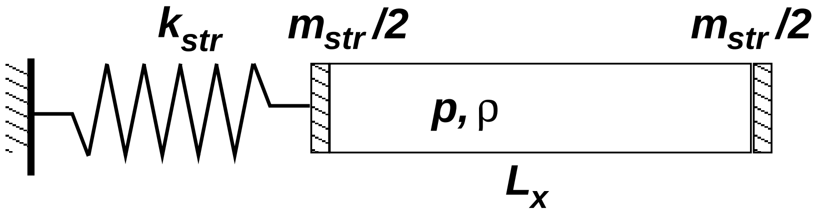
ALE invariance. Sudden stop of a gas container (cont.)



(Up) Energy balance in reference system fixed w.r.t container)
(Down) Energy balance in reference system fixed w.r.t initial gas at rest)

Stability of staged scheme.

Test case: elastically coupled gas container.



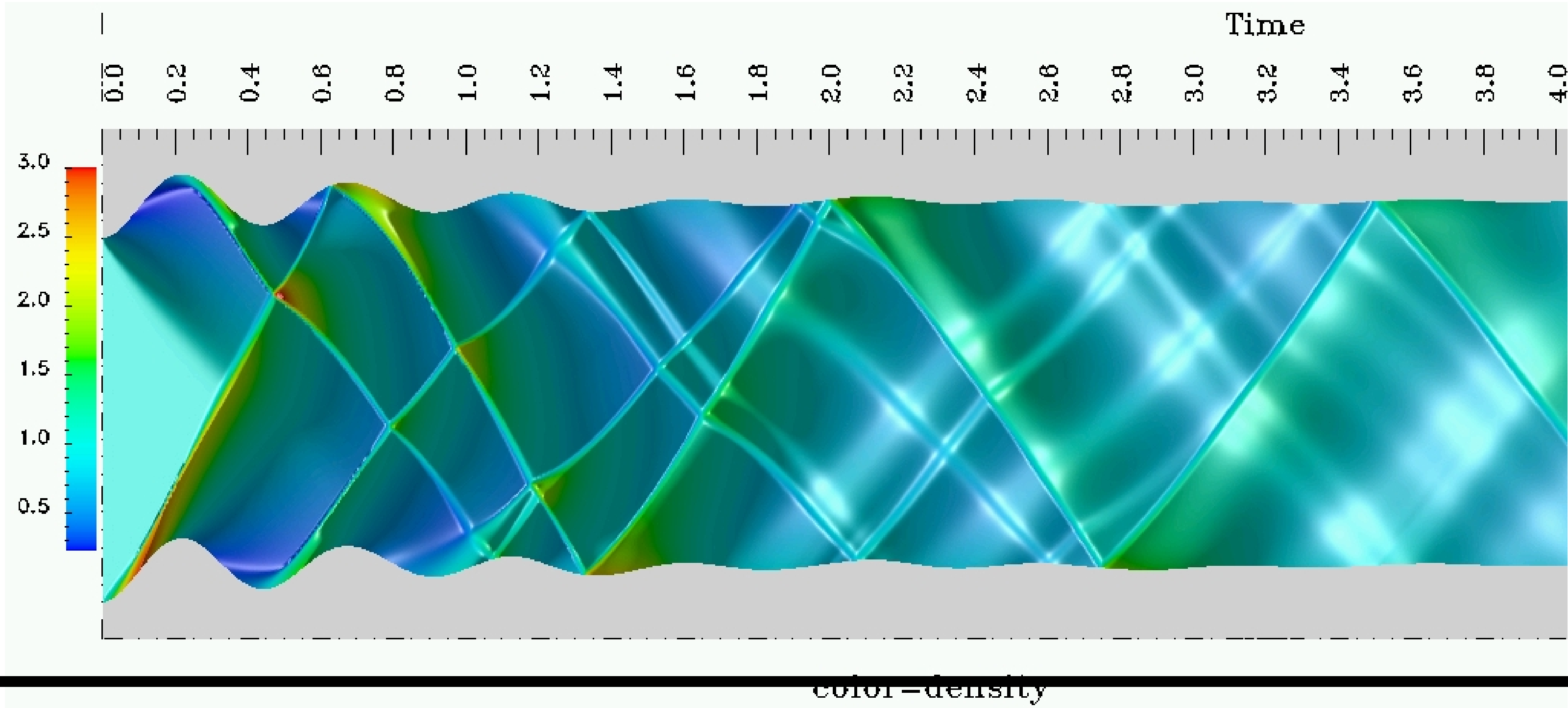
- $m_{\rm str}/(\rho L_x)$, **solid/fluid mass ratio,**
- $T_{\rm str}/T_{\rm acoust}$, **structure/acoustic time ratio,**
- $T_{\rm acoust}/T_{\rm visc}$, **acoustic/viscous time ratio,**

where

- $T_{\rm str} = 2\pi/\omega_{\rm str}$, $\omega_{\rm str} = \sqrt{k_{\rm str}/m_{\rm str}}$, **is the characteristic time of the structure, and**
- $T_{\rm acoust} = L_x/c_0$, $c_0 = \sqrt{\gamma p_0/\rho_0}$, **is the characteristic time for the fluid, i.e. the time needed for the sound speed to travel the length of the container,**
- $T_{\rm visc} = L_x^2/\nu$, **where** ν **is the kinematic viscosity.**

Stability of staged scheme. (cont.)

Colormap shows density vs. (x, t).
Params: $L_x = 1$, length of gas domain, $N_x = 200$, number of finite elements, $\rho_0 = 1$, density of gas, $p_0 = 0.71429$, density of gas, $\gamma = 1.4$, adiabatic index, $\nu = 10^{-4}$, kinematic viscosity of gas, $\Delta t c_0/h = 0.5$, Courant number (nondimensional time step), $m_{\rm str} = 1$, mass of container, $k_{\rm str} = 200$, spring constant. Initial displaceement $x(0) = -0.1\, L_x$.



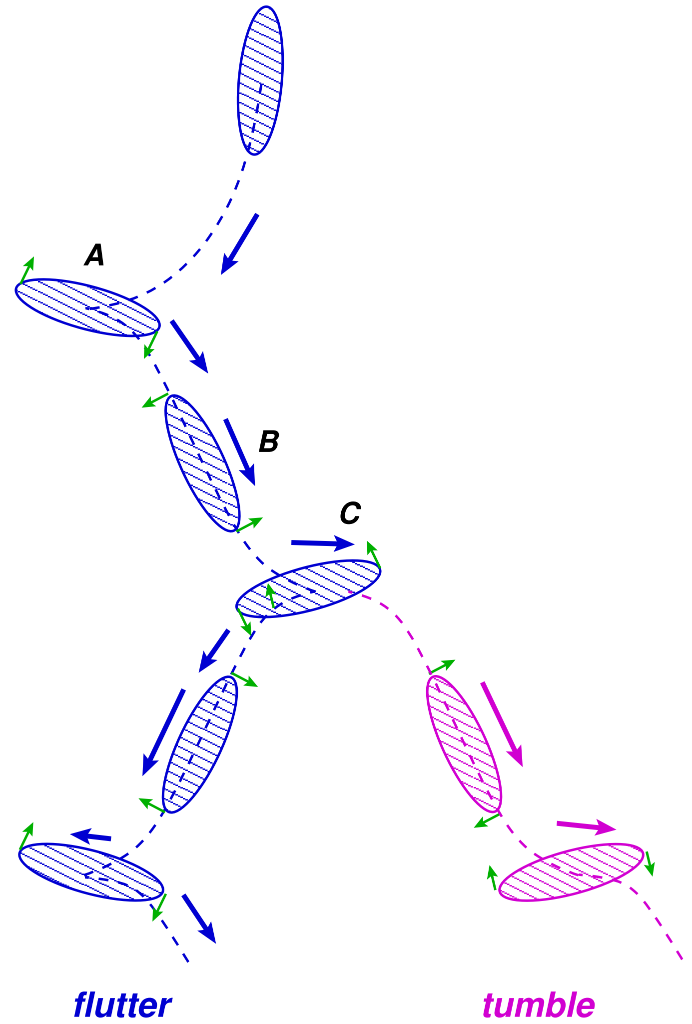
Influence of compressibility, viscosity, and structure

A series of experiments have been conducted in order to determine the stability of the algorithm, and the influence of several physical parameters.

- **If the compressibility of the fluid is high, i.e. $T_{\rm str}/T_{\rm acoust} \sim c_0$ small, then as the container walls compress the fluid a smaller amount of fluid is swept, and the added mass is lower, but the fluid has a certain additional stiffness. Experiments show that compressibility is destabilizing. In all cases stability can be recovered by increasing $n_{\rm stage}$ to 2.**
- **Even low viscosities can have a strong stabilizing effect since when instabilities are produced they have a very short wavelength and viscosity tends to be a prevailing effect for them.**
- **Scaling down $k_{\rm str}, m_{\rm str}$ keeps the characteristic time of the structure unchanged while increasing the force of the fluid onto the structure, and thus the gain of the tholw FSI interaction loop. This has then a strong destabilizing effect.**

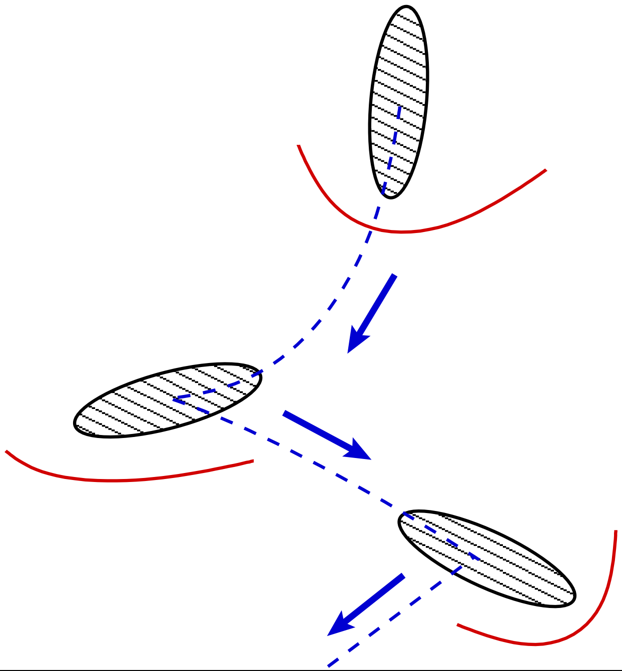
Aerodynamics of a body falling at supersonic speed

Consider, for simplicity, a two dimensional case of an homogeneous ellipse in free fall. As the body accelerates, the pitching moments tend to increase the angle of attack until it stalls (A), and then the body starts to fall towards its other end and accelerating etc... (*"flutter"*). However, if the body has a large angular moment at (B) then it may happen that it rolls on itself, keeping always the same sense of rotation. This kind of falling mechanism is called *"tumbling"* and is characteristic of less slender and more massive objects.



Aerodynamics of a body falling at supersonic speed (cont.)

Under certain conditions in size and density relation to the surrounding atmosphere it reaches supersonic speeds. In particular as form drag grows like L^2 whereas weight grows like L^3, larger bodies tend to reach larger limit speeds and eventually reach supersonic regime. At supersonic speeds the principal source of drag is the shock wave, we use slip boundary condition at the body in order to simplify the problem.



Aerodynamics of a body falling at supersonic speed (cont.)

Params:

- $a = 1$, $b = 0.6$ **(major and minor semi-axes, eccentricity** $e = \sqrt{1 - b^2/a^2} = 0.8$**),**
- $m = 1$, **(mass),**
- $w = 2.5$, **(weight of body),**
- $r = 1$, **(Radius of inertia),**
- $\mathrm{c.m.} = (-0.15, 0.0)$, **(center of mass),**
- $\rho_a = 1$, **(atmosphere density),**
- $p = 1$, **(atmosphere pressure),**
- $\gamma = 1.4$, **(gas adiabatic index** $\gamma = C_p/C_v$**),**
- $R_{\mathrm{ext}} = 10$, **(Radius of the fictitious boundary),**
- $\mathbf{u}_{\mathrm{ini}} = [0, 0, 1.39, 0, 1.3, 0]$, **(ellipse initial position and velocity** $[x, y, \alpha, u, v, \dot{\alpha}]$**),**

Further examples

Presented a this same session

- ***"Vortex-Induced Vibration (VIV) Around a Cylinder at Low Reynolds Numbers: The Lock-In Phenomenon"*, by Germn Filippini, Norberto Nigro, Mario Storti and Rodrigo Paz**
- ***"Flow-Induced Vibration of Elastic Bodies in Supersonic Regime Via Fixed Point Iteration Algorithm"* by Rodrigo R. Paz, Lisandro Dalcín, Mario A. Storti and Norberto M. Nigro**

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We made extensive use of *Free Software* (`http://www.gnu.org`) as GNU/Linux OS, MPI, GNU-Guile, Python, PETSc, GCC/G++ compilers, Octave, Open-DX among many others. In addition, many ideas from these packages have been inspiring to us.